IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5” x 8” paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ______________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD
IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes, except everyone must answer all parts of the Graduate Laboratory question. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write the identification letter provided to you this morning, the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use.

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THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD
1. Fourier Methods in Imaging (10 pts)

Consider the 2-D function with constant magnitude and phase that is a linear function of both \(x\) and \(y\), which may be called a “bilinear” phase.

\[ f(x,y) = \exp[+i \cdot 2\pi \cdot xy] \]

(a) Sketch profiles of the function along four axes: (1) \(f(x,0)\), \(f(0,y)\), \(f(x,y = x)\), and \(f(x,y = -x)\)

(b) Find expressions for the spatial frequencies along the same four directions, i.e., find expressions for \(\xi[x,y = 0]\) and \(\eta[x = 0,y]\)

(c) Evaluate the 2-D Fourier transform of \(f(x,y)\)

(d) Use the result of (b) to evaluate the 2-D Fourier transforms of \(\cos[2\pi \cdot xy]\), and \(\sin[2\pi \cdot xy]\).
2. Fourier Methods in Imaging (10 pts)

The input to a 2-D imaging system is

\[ f(x, y) = \left( \frac{1}{2} \cdot 1[x] + \frac{1}{2} \cdot \cos (2\pi \xi_0 x) \right) \cdot 1[y] \]

where \( \xi_0 = 1 \text{cycle/mm} \). The separable transfer function for the 2-D transfer function has profiles \( H[\xi, 0] \) and \( H[0, \eta] \) as shown. The 1-D profiles evaluate to \( H = 0 \) at all frequencies not shown.

Profiles of separable transfer function \( H[\xi, \eta] \)

Evaluate the output \( g(x, y) = f(x, y) \ast h(x, y) \) and sketch its profiles along the \( x \) and \( y \) axes.

(a) Sketch profiles or “top views” of the input function \( f(x, y) \)

(b) Evaluate the output \( g(x, y) = f(x, y) \ast h(x, y) \) and sketch its profiles along the \( x \) and \( y \) axes.
3. Fourier Methods in Imaging (10 pts)

Consider a 1-D three-stage “imaging system” with these steps

(1) \( g_1 [x] = \mathcal{F}_1 \{ f [x]\}_{\xi \rightarrow \frac{\alpha_0}{\alpha_0^2}} \), where \( \alpha_0 \) is a constant parameter with units of length;

(2) \( g_2 [x] = g_1 [x] \cdot p [x] \), where the “system function” \( p [x] \) has compact support;

(3) \( g [x] = \left| \mathcal{F}_1 \{ g_2 [x]\}_{\xi \rightarrow \frac{\alpha_1}{\alpha_1^2}} \right|^2 \), where \( \alpha_1 \) is a constant parameter with units of length;

(a) Evaluate an expression for the output function \( g [x] \) in terms of the input function \( f [x] \), the function \( p [x] \) and the parameters \( \alpha_0 \) and \( \alpha_1 \).

(b) Find an expression for the output \( g [x] \) if \( f [x] \) is composed of a symmetric pair of Dirac delta functions each with unit area located at \( x = \pm x_0 \), \( \alpha_1 = \sqrt{2} \cdot \alpha_0 \), and the system function \( p [x] = RECT \left( \frac{x}{b_0^2} \right) \), where \( b_0 \) is some constant parameter with dimensions of length.

(c) Sketch the approximate output if \( \alpha_0 = 1 \), \( \alpha_1 = \sqrt{2} \), \( x_0 = 1 \) and \( b_0 = 1 \).

(d) State in words the action of the system in part (c)
4. Optics for Imaging (10 pts)

A coherently illuminated infinitely long transparent ribbon has a geometric image given by the expression:

\[ E_s(x_i) = \begin{cases} 
1, & |x_i| < a \\
\exp(i\phi), & |x_i| > a 
\end{cases} \]

where the y-dimension may be ignored, \( \phi = \text{const} \), and \( a >> \lambda \).

(a) Determine the geometric image of the intensity profile in the image plane, \( I(x_i) \).

(b) Determine the angular spectrum of the image as a function of the transverse wavevector \( k_x = kx'/f = k\sin\theta_x \). Hint: let \( 2Lsinc(k_xL) = \delta(k_x) \) in the limit as \( L \to \infty \), where we define \( sinc(\zeta) = \zeta^{-1}\sin(\zeta) \).

(c) Sketch the real and imaginary parts of the angular spectrum as a function of the transverse wavevector \( k_x \) assuming \( \phi = \pi/2 \). Determine the spatial frequencies \( f \) (where \( k_x = 2\pi f \)) at which the angular spectrum is zero valued.

(d) Assuming unity magnification of a lens of focal length \( F \) and f-number, \( f^\# \), determine the coherent cut-off frequency, \( f_{cut} \), expressing it in terms of the f-number and wavelength.
5. **Optics for Imaging (10 pts)**

Design an optical microscope system having a tube length $L = 160$ mm, objective focal length $f_o = 3.0$ mm, and eyepiece focal length $f_e$.

(a) Sketch the rays from the object to the retina of the eye, assuming the pupil of the eye is placed at the exit pupil of the telescope.

(b) At what distance from the objective must the object be placed?

(c) Determine the eyepiece focal length $f_e$ that will provide 1 to 100 imaging of the object size to the size on the retina of the eye, assuming the focal length of the eye is $f_{eye} = 17$ mm. (That is, the image size on the retina is 100 times that of the actual object.)

(d) At what distance is the exit pupil from the eyepiece?
6. Optics for Imaging (10 pts)

Two stars at infinity subtend an angle of $\theta_0 = 0.5^\circ$. You are given three lenses of focal lengths $f_1 = 1.25 \text{ m}$, $f_2 = 30 \text{ mm}$, and $f_3 = -30 \text{ mm}$. Design a Keplerian telescope using two of the lenses such that it produces a larger apparent separation between the stars when viewed with the human eye. Assume the focal length of the eye is $f_{\text{eye}} = 17 \text{ mm}$.

(a) Provide ray-tracing diagrams of the telescope, labeling all the important distances, lenses (including the eye), angles, entrance and exit pupils, distances, etc.

(b) Determine the distance on the retina that separates the two stars without the aid of the telescope. Also determine the distance that separates the two stars on the retina with the aid of the telescope.

(c) Determine the position of the exit pupil of the telescope. Sketch and clearly label the exit pupil on the ray tracing diagram. State whether the exit pupil is real or virtual.

(d) Compare a Keplerian and Galilean telescope. Using ray tracing and a brief description, state which telescope appears to invert the perceived image.
7. Human Visual System (10 pts)

It is said that the human visual system’s optics and cone array are 'ideally matched.'

(a) (4 points) Describe the limits of the optical system and the cone array in the fovea. Include in your description the spatial-frequency limits of each and the reasons for those limits.

(b) (3 points) What would happen (i.e., what would an observer notice) if the performance of the optical system were improved to be significantly 'better' than the cone array could support?

(c) (3 points) What would happen (i.e., what would an observer notice) if the cone array were improved so that it could support a significantly 'better' optical system than the one available?
8. **Human Visual System - An Imaging System (10 pts)**

You are tasked with investigating radiologists’ ability to accurately detect tumors in medical images.\(^1\) Tumors are usually indicated by features in images that vary by size / gray-level, relative to their surroundings. Sometimes they are visually obvious but frequently they are very subtle.

![Brain Image](image)

\(^{1}\) This is 58.3% made up.

(a) (1 point) What sorts of perceptual / psychophysical mechanisms are important here especially as related to human vision?

(b) (6 points) This is, of course, a Signal Detection problem. What responses and errors are possible here? Propose a specific experiment to assess performance on this task. What are the consequences of favoring one sort of error over another and how would you propose to minimize these consequences?

(c) (3 points) The image above shows a suspected tumor, identified by the yellow circle. Given that there might be damage in this part of the brain, what sort of ‘symptoms’ would you expect that would be consistent with problems here? (Note— the ‘front’ of the brain is at the top, the indicated tumor is at the back of the brain, on the left-hand side.)
9. Human Visual System - Limits (10 pts)

Below are two eye charts. On the left, we see a variation of the Snellen eye chart we learned about, called the ‘Tumbling E’ chart. On the right, a chart developed by Denis Pelli and John Robson.

In the Tumbling E test, the subject reports which direction is ‘open’ for each letter, e.g., ‘up’, ‘down’, ‘left’, ‘right’, or some variation of that. On the right hand chart, the subject reads the letters from left to right, top to bottom, until they can’t see any more.

(a) (1 point) What does ‘20/20’ vision mean in this context? Is this a good, objective way of describing visual ability? Why or why not.

(b) (1 point) Why do you think some people prefer the Tumbling E test to the traditional, letter-based Snellen test?

(c) (1 point) What specific aspect of vision is the Tumbling E designed to test?

(d) (1 point) What specific aspect of vision is the Pelli-Robson chart designed to test?

(e) (4 points) You have been tasked with developing a new, high-tech computer-controlled eye test, what would you do to improve on these two charts and why?

(f) (2 points) Further complicating your life, you have been asked to recommend test charts for evaluating a) older adults’ ability to drive at night, and b) young kids’ need for corrective lenses (e.g., glasses) for school. Which chart would you recommend for each? Why, specifically?
You are working in a marine ecosystems laboratory studying the health of the seagrass *Thalassia*, commonly found in subtropical and tropical waters. Freshwater influx from shore often damage these plants, and you are trying to design a detector capable of locating areas that are beginning to show signs of the spectral changes indicating exposure to freshwater has occurred within the seagrass bed. You consult a reference and note that typical spectral differences for varying salinity exposure have been documented in the literature (see Figure 1 on next page). In this figure, the curve for healthy seagrass is labeled 32 ppt, while three other curves showing varying levels of exposure to freshwater. You know that your camera is to be mounted on a submersible vessel looking straight down while an isotropic light source illuminates the surface of the seagrass bed from the back of the vessel as shown in Figure 2 (next page). In this problem, assume that the seagrass bed is dense and that you can treat it as a Lambertian surface. Also assume that extinction in the water column is primarily due to absorption (ignore scattering).

(a) (5 points) Suppose that the light source has radiant intensity $I_{src} \text{ Watts/sr}$, the height of the light and camera are at $h_{survey}$ above the seagrass bed, the submersible vessel has length $l_{vessel}$, your camera observes straight down toward the seagrass bed and has settings: f-number $f_{\#\text{cam}}$, optical transmission $\tau_{\text{cam}}$, with detector elements that are $l_{\text{pixel}}$ on each side and noise equivalent power $\text{NEP}_{\text{cam}}$. Assume that the camera system records light in a narrow band around wavelength $\lambda$, and the absorption of the water column is $\beta_{\text{water}}(\lambda)$. If the healthy leaves (32 ppt case in Figure 1) have reflectance $\rho_{\text{healthy}}(\lambda)$ and impacted seagrass leaves have reflectance $\rho_{\text{unhealthy}}(\lambda)$, find a general expression in terms of these variables allowing you to determine if unhealthy seagrass can be detected.

(b) (3 points) $I_{src} = 1200 \text{ W/sr}$, $h_{survey} = 4 \text{ m}$, $l_{vessel} = 2 \text{ m}$, $l_{\text{pixel}} = 14 \mu\text{m}$, $\tau_{\text{cam}} = 0.93$, $f_{\#\text{cam}} = 2$, $\text{NEP}_{\text{cam}} = 1.4 \times 10^{-11} \text{ W}$. For initial testing, you choose a wavelength of 525 nm and have found that $\beta_{\text{water}}(525 \text{ nm}) = 0.038 \text{ m}^{-1}$. From Fig.1, the four reflectance values are: $\rho_{\text{healthy}}(525 \text{ nm}) \approx 0.075$, $\rho_{16\text{ppt}}(525 \text{ nm}) \approx 0.0675$, $\rho_{\text{brown-leaves}}(525 \text{ nm}) \approx 0.035$, and $\rho_{\text{black-leaves}}(525 \text{ nm}) \approx 0.02$. Show quantitatively which of the three cases 16ppt, brown leaves, and black leaves can be discriminated from the healthy case (32 ppt salinity).

(c) (2 points) Extrapolating the results shown in Figure 1.3 (on next page), if your priority is to detect the difference between the two cases shown in Figure 1 labeled 32 ppt (healthy seagrass, normal salinity) and 24 hour exposure to lower salinity of 16 ppt (labeled 16 ppt), would you be better off using a camera system designed to measure at 900nm rather than 525nm? Provide your answer using a quantitative argument.
FIGURES for RADIOMETRY QUESTION 10.

**Figure from Thorhaug et al., 2006:**

Fig. 1. Spectral reflectance vs. wavelength for measurements of Thalassia plants incubated at 32 parts per thousand (ppt) salinity and 16 ppt for 24 h as well as black (completely dead) and brown (with some color remaining) blades.

**Figure from Wozniak and Dera, 2007:**

**Figure 2.** Typical patch of unhealthy seagrass.
11. **Radiometry (10 pts)**

You are evaluating detectors on a laboratory optical table using a small isotropic light source with flux $\Phi_{src} = 0.62mW$. A small circular detector measures light at $\lambda = 550nm$. The detector has diameter $d = 4cm$ and responsivity $R = 0.37AW^{-1}$ and is positioned at $r_d = 40cm$ away from a Lambertian panel of radius $r_{panel} = 30cm$ and reflectance $\rho = 0.23$ as shown in Figure 2.1 below. The angle of the light source above the panel as measured from the table (as shown in the Figure) is $\theta_t = 30^\circ$ and the distance from the light source to the panel is $r_s = 1.5m$.

(a) (4 points) If you measure the shot noise of the detector to be 141 electrons, what was the integration time?

(b) (2 points) Assuming that the total system noise is due to shot noise and read-out noise, and you record a total noise of 935 electrons, what is the read-out noise? Is the detector shot-noise limited?

(c) (4 points) You decide to compare the detector just considered with a second detector that has the same physical dimensions, but a different responsivity $R_2 = 0.33AW^{-1}$. Where must you place this second detector above the panel in order to obtain the same shot noise level as you observed in the first detector (assume the measurement is repeated in exactly the same way as before, just with the second detector replacing the first detector)?

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**Drawing not to scale**

![Diagram](image)
12. **Radiometry (10 pts)**

You are conducting measurements of leaf transmittance in your laboratory using an integrating sphere as shown in the Figure below. Each leaf is mounted over the small exit port and completely covers the port of radius $r_{leaf}$. For the purposes of this problem you can treat the leaf as a Lambertian diffuser with transmittance $\tau_{leaf}$. A detector is mounted on the other end of your optical table off axis from the midline of the sphere as shown, such that the angle made between the direction of the detector and the sphere midline is $\theta_d$. The detector is above the midline at position $x_{det}$ as shown, and the detector orientation is parallel to the midline. The sphere has radius $r_{sp}$ and the interior surface has reflectance $\rho_{sp}$. The lamp in the sphere has efficiency $\epsilon$ and draws current $i$ from power supply with voltage $V$. The detector has responsivity $R(\lambda)$ and diameter $d_{det}$ and produces signal $S$.

(a) (3 points) Determine an expression for the transmittance of leaf samples in terms of the other given parameters.

(b) (2 points) You remove the detector and leaf apparatus from the table and attach a calibrated irradiance meter to the port directly, and measure an irradiance $E_{port}$. If the sphere operating conditions remain the same as in the leaf experiments, what is the efficiency of the lamp in the integrating sphere?

(c) (2 points) If $i = 2\, \text{Amp}$, $V = 9\, \text{Volts}$, $r_{sp} = 1\, \text{m}$, $\rho_{sp} = 0.95$, and $E_{port} = 3.78\, \text{Wm}^{-2}$, determine the efficiency of the lamp.

(d) (2 points) Having determined the efficiency, now calculate the leaf transmittance if the detector in part (a) registers a signal of $S = 3.49 \times 10^{-5}\, \text{Amps}$, knowing that the detector has $R(\lambda) = 0.6\, \text{AW}^{-1}$, $d_{det} = 8\, \text{cm}$, $x_{det} = 1.5\, \text{m}$, $\theta_d = 20^\circ$, and $r_{leaf} = 10\, \text{cm}$.

(e) (1 point) If the detector described above was only sensitive to light in a narrow band and had quantum efficiency in these measurements of 0.83, what was the wavelength to which the detector was sensitive?
13. **Image Processing and Computer Vision - Principal Component Analysis (10 pts)**

Let $X = [x_1, x_2, \ldots, x_m]$ be a $d \times m$ matrix where each column is a data point in $\mathbb{R}^d$. We want to reduce the dimensionality of these points to $\mathbb{R}^k$, where $k < d$, using Principal Component Analysis (PCA). Let the mean of the data matrix be $\mu$.

(a) (1 point) Assuming you have already computed the PCA transformation matrix $W$, show how to use $W$ to reduce the dimensionality of a data point $x$ in $\mathbb{R}^d$ to $y$ in $\mathbb{R}^k$.

(b) (1 point) If you computed the covariance matrix of the transformed data after applying PCA to it, what property would the new covariance matrix have?

(c) (2 points) Give mathematical steps for computing the $d \times d$ data covariance matrix $C$ from $X$.

(d) (3 points) The algorithm for PCA is often described using the eigendecomposition of $C$; however, due to finite-precision arithmetic on a computer this algorithm for PCA loses a significant amount of precision compared to an algorithm that uses the Singular Value Decomposition (SVD) instead.

Show how to compute $W$ using SVD.

(e) (3 points) Now, show that the method of implementing PCA using SVD is equivalent to the method using the covariance matrix $C$. 
14. Image Processing and Computer Vision - Video Monitoring of a Secret Laboratory (10 pts)

You have been hired to create a surveillance system for a secret research laboratory. The laboratory will have cameras to monitor and log the activity of the people working there. Using cameras in each room, you need to detect and identify the people in the room.

(a) (5 points) You have been told to use the R-CNN algorithm to detect individuals in the building. Explain how the R-CNN algorithm works during detection, and then give the steps for training the R-CNN algorithm.

(b) (3 points) Explain how background subtraction works and how it could be integrated into R-CNN to make it more efficient for processing the video streams from each room.

(c) (2 points) Now that your system works for detecting employees, you have been told to adjust it to identify them as well as to detect people that are not employees. Assume you have a dataset with photos of all of the employees, but you do not have data for the non-employees. Suggest a way to detect the non-employees without training on them.
15. **Image Processing and Computer Vision - Motion Detection with a Moving Camera (10 pts)**

You have been told to process a video stream captured by an unmanned aerial vehicle (UAV) in order to detect moving objects on the ground.

(a) (4 points) Explain how background subtraction works when the camera is not moving in order to determine the foreground objects in the current video frame, given $n-1$ previous frames. Provide pseudo-code in your explanation.

(b) (2 points) What problems arise if you attempt to perform background subtraction from a moving platform?

(c) (4 points) Explain how to automatically compensate for the motion of the camera using solely information from the video frames. Assume that you do not have access to IMU data. Give detailed steps.
16. **Graduate Laboratory** (10 pts)

The project this year was to achieve a markerless avatar rendering system (MARS) that enabled (ideally) real-time capture, rendering and animation of a person. All parts of a question have equal scoring weights.

A. (2 points total) MARS System
   a. Draw a picture of the system, and label all of the elements, including image/point cloud capture devices, scanning mechanics and control, illumination, rendering, and display.
   b. Describe the overall MARS operation.

B. (2 points total) Using the implementation that the class chose to pursue, **estimate** the time that is required for each of the following steps in MARS assuming all of the mechanics etc. work:
   a. capture and scanning,
   b. point cloud registration,
   c. rendering,
   d. display
   e. overall time from initial capture to display of an animated avatar

C. (2 points total) Texture maps
   a. Describe in detail what a texture map is, how it is captured, and how it differs from a conventional photograph.
   b. Was a texture map used in MARS, and if so, at which of the steps listed in part B above?

D. (2 points total) Overall limiting factor
   a. Describe in detail what in the system limited the scanning and capture speed of the ideal system, even if automated scanning was not achieved. Explain whether or not this part of the system was a bottleneck in the overall process time.
   b. Present your ideas on what could be done in the future to eliminate or reduce the time of this step.
   c. Estimate a new overall time if this were implemented.

E. (2 points total) Processing time
   a. Describe in detail the software process or step that consumed the most processing time after capture and before final display. Why was it so slow?
   b. Present your ideas on what could be done in the future to eliminate or reduce this limiting factor.
   c. Estimate a new overall time if your best idea were implemented.