

0.1 Example of Two-Lens System

$$\mathbf{f}_1 = +100 \text{ mm}$$

$$\mathbf{f}_2 = +25 \text{ mm}$$

$$t = +50 \text{ mm}$$

$$\begin{aligned} \frac{1}{\mathbf{f}_{\text{eff}}} &= \frac{1}{\mathbf{f}_1} + \frac{1}{\mathbf{f}_2} - \frac{t}{\mathbf{f}_1 \mathbf{f}_2} \\ &= \left(\frac{1}{100 \text{ mm}} + \frac{1}{+25 \text{ mm}} - \frac{50 \text{ mm}}{(+100 \text{ mm})(+25 \text{ mm})} \right)^{-1} \\ \mathbf{f}_{\text{eff}} &= \frac{100}{3} \text{ mm} = 33\frac{1}{3} \text{ mm} \end{aligned}$$

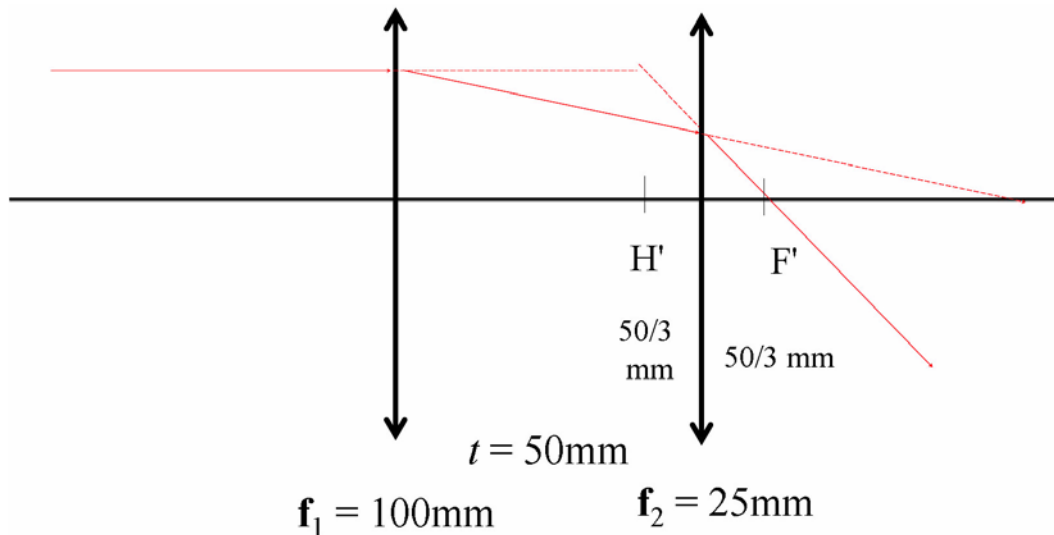
Now find the focal point by cascading through the system. For an object at ∞ , the image distance from the first lens is $\mathbf{f}_1 = +100 \text{ mm}$, so the object distance to the second lens is

$$z_2 = t - \mathbf{f}_1 = 50 \text{ mm} - 100 \text{ mm} = -50 \text{ mm}$$

The image distance from the second lens is:

$$z'_2 = \left(\frac{1}{\mathbf{f}_2} - \frac{1}{z_2} \right)^{-1} = \left(\frac{1}{+25 \text{ mm}} - \frac{1}{-50 \text{ mm}} \right)^{-1} = \frac{50}{3} \text{ mm}$$

which is half the focal length. We can now draw the image space focal and principal points:



Two positive lenses separated by distance $t = 50 \text{ mm}$

To find the image-space focal and principal point, we “turn the system around” and bring in light from the left. The image distance from the “first lens” (actually L_2) is equal to its focal length:

$$z'_1 = f_2 = +25 \text{ mm}$$

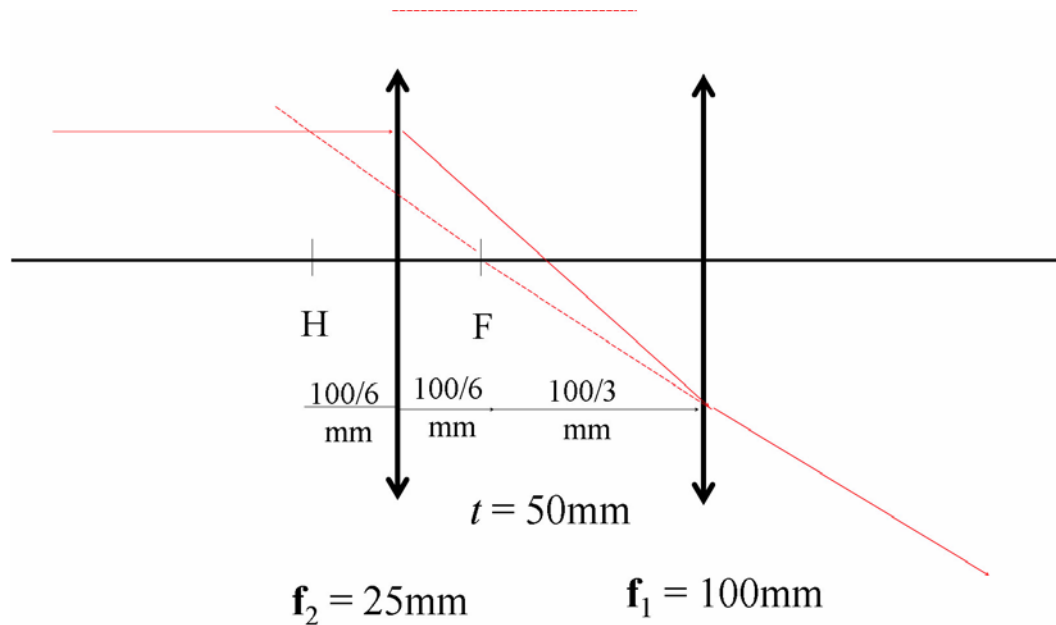
So the object distance to the lens with $f_1 = +100 \text{ mm}$ is:

$$z_2 = t - z'_1 = 50 \text{ mm} - 25 \text{ mm} = +25 \text{ mm}$$

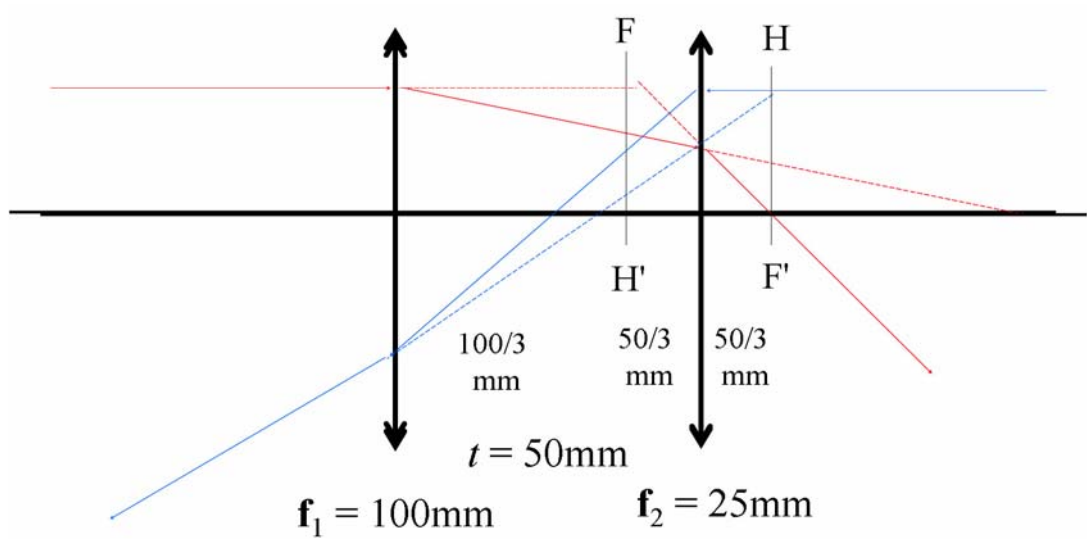
So the distance from this lens to the focal point is:

$$z'_2 = \left(\frac{1}{f_1} - \frac{1}{z_2} \right)^{-1} = \left(\frac{1}{100 \text{ mm}} - \frac{1}{25 \text{ mm}} \right)^{-1} = -\frac{100}{3} \text{ mm}$$

The focal point is *virtual* and the object-space principal is located at the distance f_{eff} behind it in the reversed system.



We can now reverse the second case and plot F, F', H, H' on the same graph:



In this case, the object-space focal point F just happens to coincide with the image-space principal point H' ; the same applies to the object-space principal point H and the image-space focal point F' . This is of no real significance, since the two spaces are independent.