1051-733-20092 Corrected Solution Set #5

1. A narrow band of light of width $\Delta\lambda$ centered about $\lambda_0 = 520\,\text{nm}$ is chopped by a shutter whose on-off cycle is $40\,\text{MHz}$ (pretty fast!). Determine the bandwidth $\Delta\lambda$ of the resulting light.

   The temporal frequency corresponding to the source wavelength is:

$$\nu_0 = \frac{c}{\lambda_0} \cong \frac{3 \times 10^8\,\text{m}\,\text{s}^{-1}}{520\,\text{nm}} \cong 5.77 \times 10^{14}\,\text{Hz}$$

   The bandwidth of $\Delta\lambda$ corresponds to a temporal frequency bandwidth of:

$$\begin{aligned}\Delta\nu &= \left|\Delta\left(\frac{c}{\lambda}\right)\right| = \left|-\frac{c}{\lambda^2}\Delta\lambda\right| = \frac{c}{\lambda^2}\Delta\lambda \\ &= \frac{3 \times 10^8\,\text{m}\,\text{s}^{-1}}{(520\,\text{nm})^2} \cdot \Delta\lambda \cong \left(1.108\,7 \times 10^{21} \cdot \Delta\lambda\,[\text{m}]\right)\,\text{Hz}\end{aligned}$$

   This is the reciprocal of the coherence time τ:

$$\tau = \frac{1}{\Delta\nu}$$

$$\Delta\lambda = \Delta\nu \cdot \frac{\lambda^2}{c} = \frac{1}{\tau} \cdot \frac{\lambda^2}{c} = \frac{1}{\tau} \cdot \frac{(520\,\text{nm})^2}{\left(3 \cdot 10^8\,\frac{\text{m}}{\text{s}}\right)}$$

   The chopper induces a square-wave modulation of the incident light. If we interpret that the on-off cycle $40\,\text{MHz}$ means that the full-cycle time (from "on" to "off" and back to "on" is:

$$T = \frac{1}{\nu} = \frac{1}{40\,\text{MHz}} = 25\,\text{ns}$$

   which implies that the light is "on" for half of that time:

$$\frac{T}{2} = 12.5\,\text{ns}$$

   so the physical length of the "packet" of light is:

$$\ell_0 = c \cdot \frac{T_0}{2} \cong 3 \times 10^8\,\text{m}\,\text{s}^{-1} \cdot 12.5\,\text{ns} \cong 3.75\,\text{m} \cong 12.3\,\text{ft}$$

   which confirms the usefully remembered observation that light travels very close to $1\,\text{ft}$ in $1\,\text{ns}$.

$$\begin{aligned}\Delta\lambda &= \frac{(520\,\text{nm})^2}{\left(3 \cdot 10^8\,\frac{\text{m}}{\text{s}}\right)} \cdot \frac{1}{12.5\,\text{ns}} \cong 7.21 \times 10^{-14}\,\text{m} = 7.21 \times 10^{-5}\,\text{nm} \\ \frac{\Delta\lambda}{\lambda_0} &\cong \frac{7.21 \times 10^{-14}\,\text{m}}{520\,\text{nm}} \cong 1.4 \times 10^{-7} \textit{ (pretty small)}\end{aligned}$$

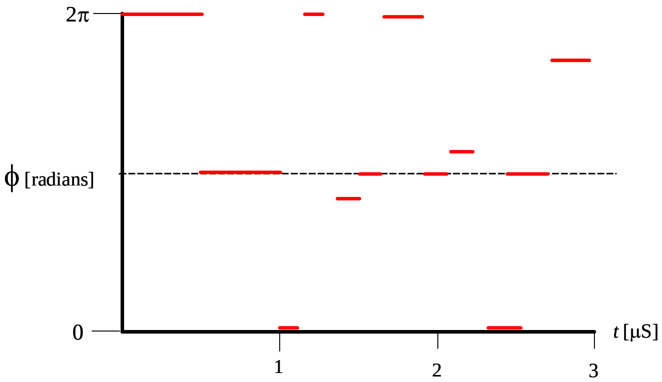
If you interpreted the problem statement to mean that the "half cycle" of the modulation is 40 MHz, *so that the packet of waves is twice as long, then the bandwidth is reduced by half to*

$$\begin{aligned}\Delta\lambda &= \frac{(520\,\mathrm{nm})^2}{\left(3\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}\right)}\cdot\frac{1}{25\,\mathrm{ns}} \cong 3.6\times 10^{-14}\,\mathrm{m} = 3.6\times 10^{-5}\,\mathrm{nm} \\ \frac{\Delta\lambda}{\lambda_0} &\cong \frac{3.6\times 10^{-14}\,\mathrm{m}}{520\,\mathrm{nm}} \cong 7\times 10^{-8} \text{ (pretty small)}\end{aligned}$$

If the source itself has a bandwidth, then each wavelength will be "widened" by approximately the factor of 10^{-7}, *so the bandwidth of the chopped source is still approximately the same as the unchopped source.*

$$\Delta\lambda_{\mathrm{out}} \cong \Delta\lambda_{\mathrm{in}}\times\left(1+\frac{\Delta\lambda}{\lambda_0}\right) \cong \Delta\lambda_{\mathrm{in}}\times\left(1+1.4\times 10^{-7}\right) \cong 1.00000014\cdot\Delta\lambda_{\mathrm{in}}$$

2. A short sample of the optical phase $\phi[t]$ of light emitted by a source is shown in the graph; the phase behaves similarly over time. This light interferes with light from a perfectly coherent source, as in a Young's two-aperture experiment.



(a) The irradiance from the interference is observed on a screen by eye. Will interference be seen? Explain.

The measurement time of the eye is of the order of 10s of milliseconds; since the phase of the light is changing at time scales of the order of microseconds, the interference effects will all "blur out" faster than we can see them.

(b) The irradiance is observed with a sensor whose response time is 1 ns; will interference be observed?

Now the time scale of the measurement is of the order of 10^{-3} *as long as the changes of the phase, so a short exposure with the sensor will see interference effects.*

(c) Use the results from (a) and (b) to make a statement about the coherence of the two sources.

Duh, a rather redundant question, except it is really ONE source, the sensors determine the coherence, as already answered.

3. Consider Young's two-aperture experiment with separation $d_0 = 100\,\mu\text{m}$ and the distance from the aperture plane to the observation screen is $z_2 = 500\,\text{mm}$. Compute the distances between adjacent irradiance maxima for $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 700\,\text{nm}$ and sketch the resulting patterns.

   (zzzzz) This one was hardly worth being called a "problem!" The ratio of the object distance to the separation is

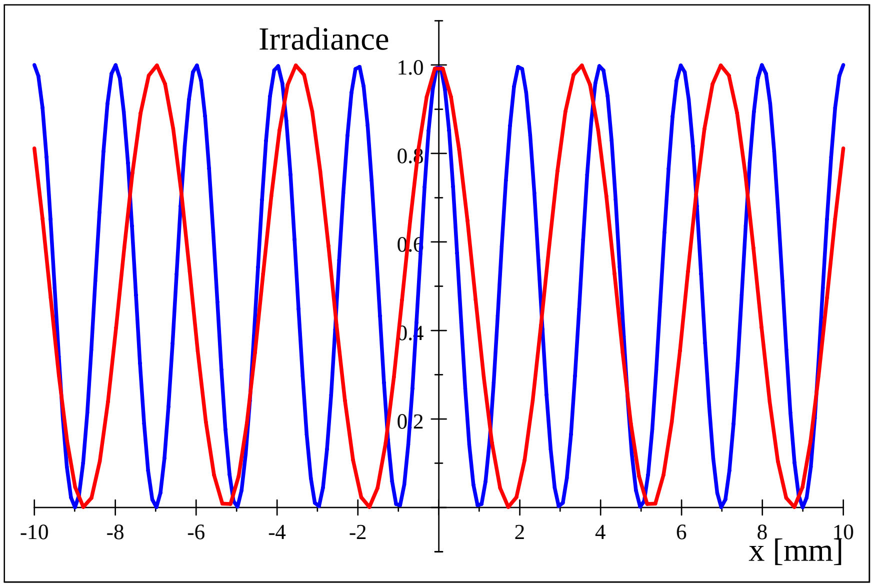
$$\frac{d_0}{z_2} = \frac{1}{5000} = 2 \times 10^{-4}$$

   so the error in the sine, tangent, and linear relationships is:

$$\begin{aligned} \left|\frac{1}{5000} - \sin^{-1}\left(\frac{1}{5000}\right)\right| &\cong 1.33 \times 10^{-12} \\ \left|\frac{1}{5000} - \tan^{-1}\left(\frac{1}{5000}\right)\right| &\cong 2.67 \times 10^{-12} \end{aligned}$$

   so we are comfortably within the Fraunhofer region. The fringes are sinusoidal with periods determined from the relation:

$$\begin{aligned} D_\ell &= \lambda_\ell \frac{z_2}{d_0} = \lambda_\ell \cdot \frac{500\,\text{mm}}{100\,\mu\text{m}} = 5000 \cdot \lambda_\ell \\ \textit{at } \lambda_1 &= 400\,\text{nm},\ D_1 = 5000 \cdot 400\,\text{nm} = 2\,\text{mm} \\ \textit{at } \lambda_1 &= 700\,\text{nm},\ D_1 = 5000 \cdot 700\,\text{nm} = 3.5\,\text{mm} \end{aligned}$$



Irradiance fringes in blue light (in blue, Duh) and in red light.

4. Calculate the irradiance pattern that would be observed at the output plane in problem 3 if a third aperture is added to one side so that the separations of adjacent slits are identically $d_0 = 100\,\mu\text{m}$.

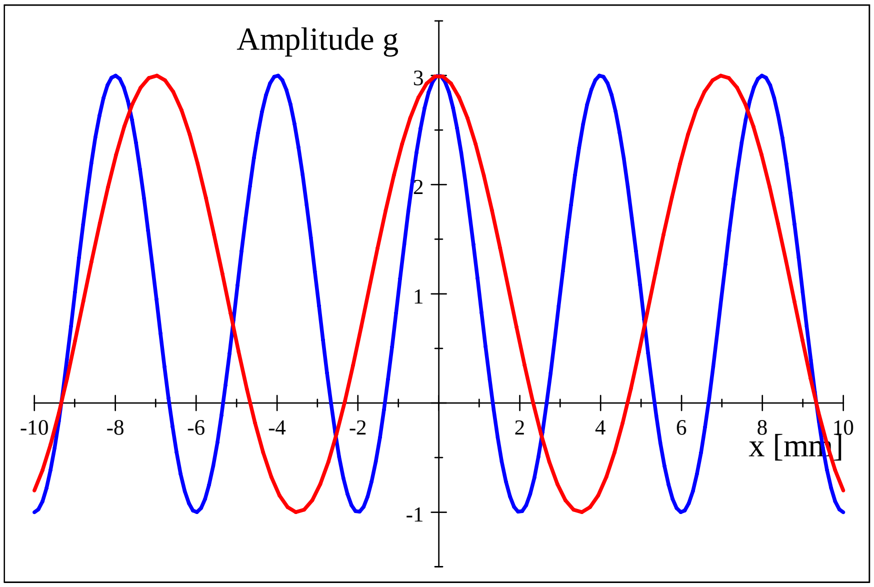
okay, this is more of a problem. The additional aperture changes the input function to:

$$f\left[x,y,z=0\right]=\delta\left[x+\frac{x_0}{50\,\mu\text{m}}\right]+\delta\left[x-\frac{x_0}{50\,\mu\text{m}}\right]+\delta\left[x-\frac{x_0}{150\,\mu\text{m}}\right]$$

We can ignore the translation because we're in the Fraunhofer region – centering the aperture pattern has no effect on the irradiance:

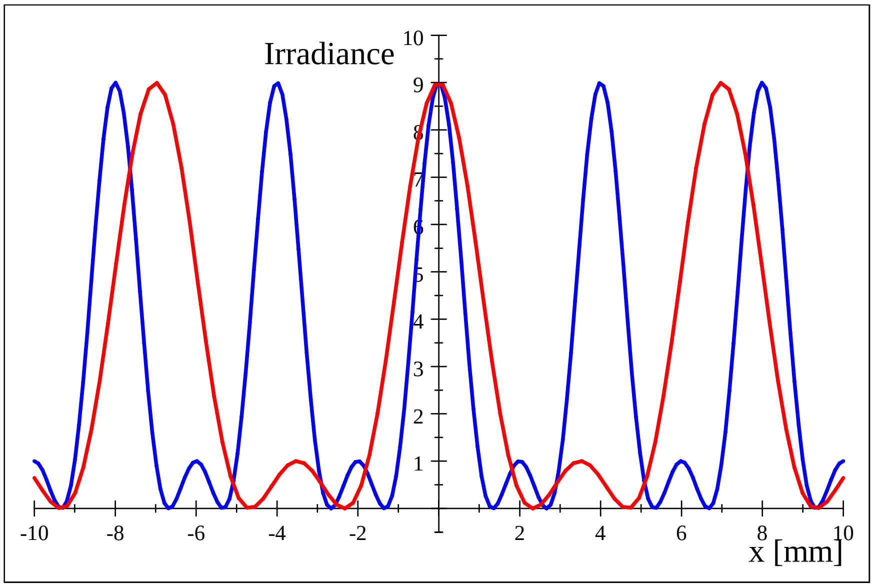
$$\begin{aligned}
f\left[x,y,z=0\right] &= \left(\delta\left[x+\frac{x_0}{100\,\mu\text{m}}\right]+\delta\left[x\right]+\delta\left[x-\frac{x_0}{100\,\mu\text{m}}\right]\right)\cdot 1\left[y\right] \\
F\left[\xi,\eta\right] &= \left(1\left[\xi\right]+2\cdot\cos\left[2\pi\cdot 100\,\mu\text{m}\cdot\xi\right]\right)\cdot\delta\left[\eta\right] \\
F\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right] &= g\left[x,y;z_2\right]=\left(1\left[\frac{x}{\lambda_0 z_2}\right]+2\cdot\cos\left[2\pi\cdot 100\,\mu\text{m}\cdot\frac{x}{\lambda_0 z_2}\right]\right)\cdot\delta\left[\frac{y}{\lambda_0 z_2}\right] \\
&= \left|\lambda_0 z_2\right|\cdot\left(1\left[x\right]+2\cdot\cos\left[2\pi\cdot\frac{x}{\frac{\lambda_0 z_2}{100\,\mu\text{m}}}\right]\right) \\
&= \left|\lambda_0 z_2\right|\cdot\left(1\left[x\right]+2\cdot\cos\left[2\pi\cdot\frac{x}{\lambda_0\cdot 5000}\right]\right)
\end{aligned}$$

So the amplitude pattern is "biased up", at $\lambda=300$



Amplitude observed in the Fraunhofer diffraction region from three equi-spaced slits.

The irradiance pattern is the squared magnitude, which has a little "bump" between the maxima due to the negative amplitude.



Irradiance observed in the Fraunhofer diffraction region from three equi-spaced slits.

5. A two-slit experiment is modified so that the light through one slit passes through a half-wave plate with the fast axis parallel to the slit and the light through the other passes through an identical half-wave plate with the fast axis perpendicular to the slit. The incident light is unpolarized. Determine the positions of the maxima and minima in the irradiance fringe pattern.

   The birefringent material in the waveplate changes the phase of the light emerging from the slits. The phase delay for a halfwave plate of thickness ℓ is:

$$\Delta\phi = \pi = \frac{2\pi\,(n_s - n_f)}{\lambda_0}\cdot\ell$$

   The only thing that matters to us is the phase difference of the emerging light, which is π radians. In other words the aperture function is:

$$\begin{aligned}
f\,[x,y;z=0] &= \left(\delta\left[x+\frac{d_0}{2}\right]+\delta\left[x-\frac{d_0}{2}\right]\cdot\exp\left[+i\pi\right]\right)\cdot 1\,[y] \\
&= \left(\delta\left[x+\frac{d_0}{2}\right]-\delta\left[x-\frac{d_0}{2}\right]\right)\cdot 1\,[y] \\
F\,[\xi,\eta;z=0] &= 2i\cdot\sin\left[2\pi\cdot\frac{d_0}{2}\cdot\xi\right]\cdot\delta\,[\eta] \\
F\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right] &= 2i\cdot\sin\left[2\pi\cdot\frac{d_0}{2}\cdot\frac{x}{\lambda_0 z_2}\right]\cdot\delta\left[\frac{y}{\lambda_0 z_2}\right] \\
I\,[x,y] &\propto \left|F\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right]\right|^2 = 4\lambda_0 z_2\cdot\sin^2\left[2\pi\cdot\frac{x}{\left(2\frac{\lambda_0 z_2}{d_0}\right)}\right]\cdot\delta\,[y] \\
&= 4\lambda_0 z_2\cdot\frac{1}{2}\left(1-\cos\left[2\pi\cdot\frac{x}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right]\right)\cdot\delta\,[y] \\
&= 2\lambda_0 z_2\left(1-\cos\left[2\pi\cdot\frac{x}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right]\right)\cdot\delta\,[y]
\end{aligned}$$

   which is the same sinusoidal fringe pattern except for a translation by half a period.

6. Consider two examples of division-of-wavefront interference created by light from a monochromatic point source with wavelength λ_0 located at a distance z_1 from the aperture plane that is sufficiently large for the Fraunhofer approximation to apply. The aperture is a pair of slits of width d_0 and separation ℓ_0 that are sufficiently long that you may assume that their y-dimensions are infinite. The light then propagates a distance z_2 from the apertures to the observation plane that is also in the Fraunhofer diffraction region.

   In the first example, the monoschromatic point source located on axis and one slit (the "lower one" whose coordinate of its center satisfies $x_0 < 0$) is covered by a thin piece of glass whose thickness t_0 is such that it delays the phase of the light through that slit by $\frac{\pi}{4}$ radians.

   The second system is composed of the same source and the same slits, but the piece of glass has been removed and the point source of light is translated above the optical axis by the distance x_0.

   (a) Derive the amplitude distributions $g\left[x; z_1, z_2\right]$ of the interference patterns in both cases.

   The thin piece of glass has an effect similar to that in problem #5, except that the phase difference is smaller:

$$f\left[x,y;z=0\right] = RECT\left[\frac{x}{d_0}\right] * \left(\delta\left[x+\frac{\ell_0}{2}\right] + \delta\left[x-\frac{\ell_0}{2}\right]\cdot\exp\left[+i\frac{\pi}{4}\right]\right)\cdot 1\left[y\right]$$

$$F\left[\xi,\eta;z=0\right] = d_0 SINC\left[d_0\xi\right]\cdot\left(\exp\left[+2\pi i\xi\cdot\frac{\ell_0}{2}\right] + \exp\left[-2\pi i\xi\cdot\frac{\ell_0}{2}\right]\cdot\exp\left[+i\frac{\pi}{4}\right]\right)\cdot\delta\left[\eta\right]$$

$$= d_0 SINC\left[d_0\xi\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\left(\exp\left[+2\pi i\xi\cdot\frac{\ell_0}{2}\right]\cdot\exp\left[-i\frac{\pi}{8}\right] + \exp\left[-2\pi i\xi\cdot\frac{\ell_0}{2}\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\right)\cdot\delta$$

$$= d_0 SINC\left[d_0\xi\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\cdot 2\cos\left[+2\pi\left(\xi\cdot\frac{\ell_0}{2}-\frac{1}{16}\right)\right]\cdot\delta\left[\eta\right]$$

$$\begin{aligned}
g_1\left[x,y\right] &\propto F\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right] \\
&= d_0 SINC\left[d_0\frac{x}{\lambda_0 z_2}\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\cdot 2\cos\left[+2\pi\left(\frac{x}{\frac{2\lambda_0 z_2}{\ell_0}}-\frac{1}{16}\right)\right]\cdot\delta\left[\frac{y}{\lambda_0 z_2}\right] \\
&= \lambda_0 z_2\cdot SINC\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\cdot 2d_0\cos\left[2\pi\left(\frac{x}{\frac{2\lambda_0 z_2}{\ell_0}}-\frac{1}{16}\right)\right]\cdot\delta\left[y\right]
\end{aligned}$$

$$\boxed{g_1\left[x,y\right] \propto \lambda_0 z_2\cdot SINC\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right]\cdot\exp\left[+i\frac{\pi}{8}\right]\cdot 2\cos\left[\frac{2\pi}{2}\left(\frac{x}{\left(\frac{\lambda_0 z_2}{\ell_0}\right)}-\frac{1}{8}\right)\right]\cdot\delta\left[y\right]}$$

In the second case of the translated source, the illumination has a tilted phase:

$$\begin{aligned}
s\left[x,y\right] &= \delta\left[x-x_0,y\right] \\
S\left[\xi,\eta\right] &= \exp\left[-2\pi i\cdot x_0\cdot\xi\right]\cdot 1\left[\eta\right] \\
S\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right] &= \exp\left[-2\pi i\cdot x_0\cdot\frac{x}{\lambda_0 z_1}\right]\cdot 1\left[\frac{y}{\lambda_0 z_1}\right] \\
&= \exp\left[-2\pi i\cdot\frac{x}{\frac{\lambda_0 z_1}{x_0}}\right]\cdot 1\left[y\right]
\end{aligned}$$

This is sampled by the apertures:

$$f\left[x,y\right] = RECT\left[\frac{x}{d_0}\right] * \left(\delta\left[x+\frac{\ell_0}{2}\right]+\delta\left[x-\frac{\ell_0}{2}\right]\right)\cdot 1\left[y\right]$$

$$\begin{aligned}
&S\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]\cdot f\left[x,y\right] \\
&= \exp\left[-2\pi i\cdot\frac{x}{\frac{\lambda_0 z_1}{x_0}}\right]\cdot\left(RECT\left[\frac{x}{d_0}\right] * \left(\delta\left[x+\frac{\ell_0}{2}\right]+\delta\left[x-\frac{\ell_0}{2}\right]\right)\right)\cdot 1\left[y\right] \\
&\qquad = \exp\left[-2\pi i\cdot\frac{x}{\frac{\lambda_0 z_1}{x_0}}\right]\cdot\left(RECT\left[\frac{x+\frac{\ell_0}{2}}{d_0}\right]+RECT\left[\frac{x-\frac{\ell_0}{2}}{d_0}\right]\right)\cdot 1\left[y\right]
\end{aligned}$$

The Fraunhofer diffraction pattern is proportional to the scaled Fourier transform

$$\begin{aligned}
\mathcal{F}_2\left\{S\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]\cdot f\left[x,y\right]\right\}\Bigg|_{\xi\to\frac{x}{\lambda_0 z_2},\eta\to\frac{y}{\lambda_0 z_2}} &= \left(\delta\left[\xi+\frac{x_0}{\lambda_0 z_1}\right] * \left(d_0 SINC\left[d_0\xi\right]\cdot 2\cos\left[2\pi\xi\cdot\right.\right.\right. \\
&\propto \delta\left[x+\frac{x_0}{\left(\frac{z_2}{z_1}\right)}\right] * \left(SINC\left[\frac{x}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right]\cdot 2\cos\left[2\pi\right.\right. \\
&= SINC\left[\frac{x+\frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right]\cdot 2\cos\left[2\pi\frac{x+\frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\frac{2\lambda_0 z_2}{\ell_0}}\right]
\end{aligned}$$

A translated replica of the previous pattern

$$\begin{aligned}
g_2\left[x,y\right] &\propto SINC\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right]\cdot\left(\exp\left[+2\pi i\frac{\ell_0}{2\lambda_0 z_2}\left(x+x_0\cdot\frac{z_2}{z_1}\right)\right]+\exp\left[-2\pi i\frac{\ell_0}{2\lambda_0 z_2}\left(x+x_0\cdot\frac{z_2}{z_1}\right)\right]\right) \\
&= 2\lambda_0 z_2\cdot SINC\left[\frac{x+x_0\cdot\frac{z_2}{z_1}}{\frac{\lambda_0 z_2}{d_0}}\right]\cdot\cos\left[+2\pi\frac{\ell_0}{2\lambda_0 z_2}\left(x+x_0\cdot\frac{z_2}{z_1}\right)\right]\cdot\delta\left[y\right]
\end{aligned}$$

$$\boxed{g_2\left[x,y\right]\propto SINC\left[\frac{x+\frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right]\cdot 2\cos\left[2\pi\frac{x+\frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\frac{2\lambda_0 z_2}{\ell_0}}\right]\cdot\delta\left[y\right]}$$

(b) Derive AND GRAPH the irradiances $I[x;z_1,z_2] \propto |g[x;z_1,z_2]|^2$ of the patterns.

In the first case, the irradiance is proportional to a biased sinusoidal pattern with period $\frac{\lambda_0 z_2}{\ell_0}$ *that has been translated by the phase of* $\frac{\pi}{4}$*, which may be expanded to show that the translation distance is* $x_1 = \frac{\lambda_0 z_2}{8\ell_0}$*:*

$$I_1[x,y] \propto |g_1[x,y]|^2 \propto \left| SINC\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right] \cdot \exp\left[+i\frac{\pi}{8}\right] \cdot 2\cos\left[\frac{2\pi}{2}\left(\frac{x}{\left(\frac{\lambda_0 z_2}{\ell_0}\right)} - \frac{1}{8}\right)\right]\right|^2 \cdot \delta[y]$$

$$= 2SINC^2\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right] \cdot \left(1 + \cos\left[2\pi\left(\frac{x}{\left(\frac{\lambda_0 z_2}{\ell_0}\right)} - \frac{1}{8}\right)\right]\right) \cdot \delta[y]$$

$$\boxed{I_1[x,y] \propto 2\lambda_0 z_2 \cdot SINC^2\left[\frac{x}{\frac{\lambda_0 z_2}{d_0}}\right]\left(1 + \cos\left[2\pi\left(\frac{x}{\left(\frac{\lambda_0 z_2}{\ell_0}\right)} - \frac{1}{8}\right)\right]\right) \cdot \delta[y]}$$

which is a SINC function modulating a translated cosine pattern.

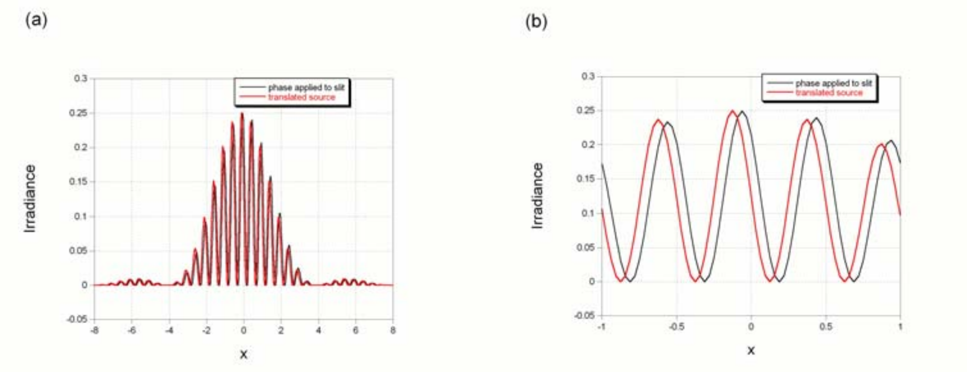
In the second case, the period of the sinusoidal fringe pattern is also $\frac{\lambda_0 z_2}{\ell_0}$*, but the translation is* $x_1 = \frac{z_2}{z_1} \cdot x_0$

$$\boxed{I_2[x,y] \propto |g_2[x,y]|^2 = 2SINC^2\left[\frac{x + \frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right] \cdot \left(1 + \cos\left[2\pi\frac{x + \frac{x_0}{\left(\frac{z_2}{z_1}\right)}}{\left(\frac{\lambda_0 z_2}{\ell_0}\right)}\right]\right) \cdot \delta[y]}$$

We can make the translation distances equal:

$$\frac{\lambda_0 z_2}{8\ell_0} = x_0 \cdot \frac{z_2}{z_1} \implies x_0 = \frac{\lambda_0 z_1}{8\ell_0}$$

In the first example shown, the phase of the "right-hand" aperture was set to $\frac{\pi}{4}$*; in the second example, the source was moved above the axis (towards* $x > 0$*) so that the interference pattern moved "down" (towards* $x < 0$*).*



7. A Michelson interferometer is aligned to show white light fringes. A source of sodium light is substituted that emits equal amplitudes at two wavelengths: $\lambda_1 = 589.0\,\text{nm}$ and $\lambda_2 = 589.6\,\text{nm}$. One mirror is moved farther from the beamsplitter by the distance ℓ until the fringe visibility decreases to a minimum. Determine the value of ℓ.

   The fact that white-light fringes are visible means that the optical path difference between the two beams is zero. As we saw in the optics demo in the lab, we would expect to see of the order of three white-light fringes on either side of the central fringe. If the sodium source is substituted, then the coherence length is substantially longer and the optical path difference can be increased substantially before the fringes "disappear." To get a minimum of the fringe pattern, the optical path difference for the shorter wavelength should be an integer multiple of the number of wavelengths PLUS one half wave, while the optical path difference for the longer wavelength should be an integer multiple of the number of wavelengths; call that number m_0*:*

$$\begin{aligned}
\textit{for } \lambda_1 &= 589.0\,\text{nm} : 2\left(L_1 - L_2\right) = m_0\lambda_1 + \frac{\lambda_1}{2} = \left(m_0 + \frac{1}{2}\right)\lambda_1 = m_0\left(1 + \frac{1}{2m_0}\right)\lambda_0 \\
&\Longrightarrow \frac{2\left(L_1 - L_2\right)}{m_0} = \left(1 + \frac{1}{2m_0}\right)\lambda_0 \\
\textit{for } \lambda_1 &= 589.6\,\text{nm} : 2\left(L_1 - L_2\right) = m_0\lambda_2 \Longrightarrow \lambda_2 = \frac{2\left(L_1 - L_2\right)}{m_0}
\end{aligned}$$

$$\begin{aligned}
\frac{2\left(L_1 - L_2\right)}{m_0} &= \lambda_1\left(1 + \frac{1}{2m_0}\right) = \lambda_2 \\
m_0 &= \frac{\lambda_1}{2\left(\lambda_2 - \lambda_1\right)} = \frac{589.0\,\text{nm}}{2\left(589.6\,\text{nm} - 589.0\,\text{nm}\right)} = 490.83
\end{aligned}$$

$$\begin{aligned}
\lambda_2 &= \frac{2\left(L_1 - L_2\right)}{m_0} \Longrightarrow \left(L_1 - L_2\right) = \frac{m_0\lambda_2}{2} = \frac{490.83 \cdot 589.6\,\text{nm}}{2} \\
&\boxed{\left(L_1 - L_2\right) = 0.1447\,\text{mm}}
\end{aligned}$$