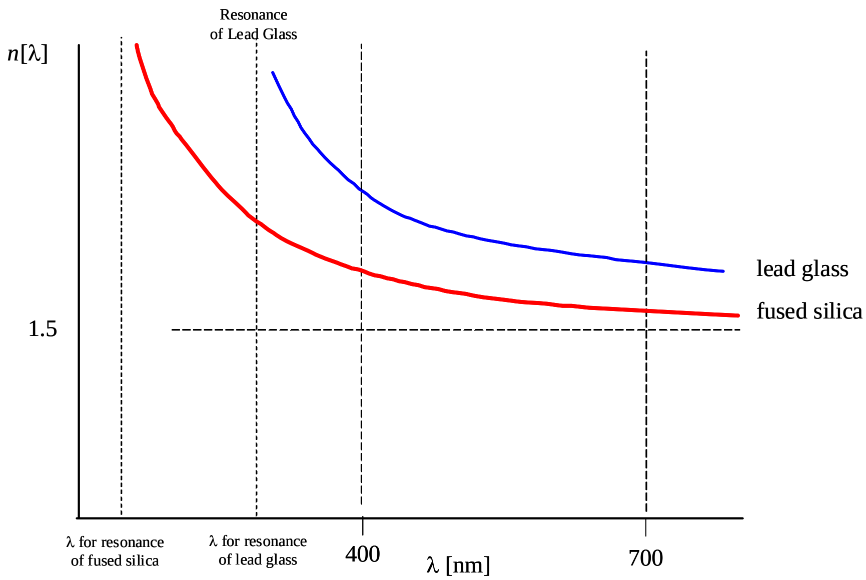
1051-733-20092 Solution Set #4 (Corrected)

1. The resonant frequency for lead glass is in the ultraviolet region fairly near the visible region, whereas the resonant frequency for fused silica is far into the ultraviolet region. Make a rough sketch of $n[\lambda]$ for both cases for the visible region of the spectrum; label significant features of the graphs.

   Since the resonant frequency ω_0 for lead glass is closer to the visible spectrum than that for silica, we expect that BOTH the refractive index n AND the dispersion (variation in n over the range $400\,\text{nm} \leq \lambda \leq 700\,\text{nm}$ to be larger for lead glass than for silica. The graphs of the refractive indices are shown to a first approximation in the graph:



Approximate graph of refractive indices of lead glass and of fused silica, showing that the resonance of the former is closer to the visible region, which means that both the refractive index and dispersion are higher for lead glass than for fused silica.

2. A narrow bundle of white light (a "pencil" of rays) is incident upon a plane-parallel plate of thickness $\ell_0 = 10\,\text{mm}$ and with indices of refraction for red light and for blue light of $n_{\text{red}} = 1.614$ and $n_{\text{blue}} = 1.653$. The angle of incidence is $\frac{\pi}{4}$ radians. Determine the "sideways separation" of these two colors upon exiting the plate.

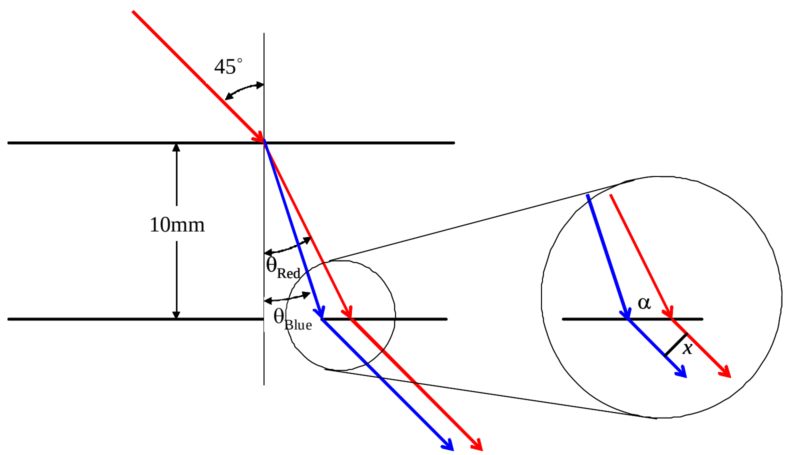
From the figure, we see that the displacements of the rays with the two colors measured from the normal to the surface and upon exiting the glass are:

$$\begin{aligned}
\alpha &= 10\,\text{mm} \cdot (\tan[\theta_{\text{red}}] - \tan[\theta_{\text{blue}}]) \\
&= 10\,\text{mm} \cdot \left(\tan\left[\sin^{-1}\left[\frac{1}{n_{\text{red}}}(\sin[\theta_0])\right]\right] - \tan\left[\sin^{-1}\left[\frac{1}{n_{\text{red}}} \cdot \sin[\theta_0]\right]\right]\right) \\
&= 10\,\text{mm} \cdot \left(\tan\left[\sin^{-1}\left[\frac{1}{1.614} \cdot \sin\left[\frac{\pi}{4}\right]\right]\right] - \tan\left[\sin^{-1}\left[\frac{1}{1.653} \cdot \sin\left[\frac{\pi}{4}\right]\right]\right]\right) \\
&\cong 4.8737\,\text{mm} - 4.7326\,\text{mm} \\
& \boxed{\alpha \cong 0.141\,\text{mm}}
\end{aligned}$$

The displacement x in the figure is obtained from:

$$\frac{x}{\alpha} = \cos\frac{\pi}{4} \implies x = \frac{\alpha}{\sqrt{2}} \cong \frac{0.141\,\text{mm}}{\sqrt{2}} \cong 0.997\,\text{mm}$$

$$\boxed{x \cong 0.1\,\text{mm} = 100\,\mu\text{m}}$$



3. A plano-convex lens is made of borate flint glass for which the Abbé number is $\upsilon = 55.2$. The focal length of the lens for sodium light ($\lambda_D = 589.59\,\text{nm}$) is 762 mm. An image of the sun is formed by this lens; find the distance between the images for red light and for blue light (a differential equation might be helpful).

The equation for the Abbé number is:

$$\upsilon = \frac{n_D - 1}{n_F - n_C} = \frac{n_D - 1}{dn}$$

where the indices are measured at the Fraunhofer C, D, and F lines, respectively. From the notes, we know that $\lambda_C = 656.28\,\text{nm}$ *(red),* $\lambda_D = 589.59\,\text{nm}$ *(green), and* $\lambda_F = 486.13\,\text{nm}$ *(blue). From the study of dispersion, we expect* $n_C < n_D < n_F$*, which means that the focal length in the red should be longer:* $\mathbf{f}_C > \mathbf{f}_D > \mathbf{f}_F$*. We also have the lensmaker's equation:*

$$\frac{1}{\mathbf{f}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where R_1 *and* R_2 *are the radii of curvature of the front and rear surfaces of the thin lens. Since the focal length doesn't change if we exchange the curvatures (though the aberrations DO change), we can set either radius to* ∞*; I'll choose* $R_2 = \infty$*, but this is immaterial, as we shall show:*

$$\begin{aligned}
\frac{1}{\mathbf{f}} &= (n-1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{n-1}{R_1} \\
&\Longrightarrow \mathbf{f}_D = \frac{R_1}{n_D - 1} = 762\,\text{mm} \\
\mathbf{f}_C &= \frac{R_1}{n_C - 1} \\
\mathbf{f}_F &= \frac{R_1}{n_F - 1}
\end{aligned}$$

We want to get an equation with a factor of the variation in the refractive index dn. If we evaluate the logarithm of the lensmaker's equation, we convert the multiplication and division to addition and subtraction:

$$\log[\mathbf{f}] = \log[R_1] - \log(n-1)$$

Differentiate with respect to n, after recalling that:

$$\frac{d}{dx}[\log[x]] = \frac{1}{x} \Longrightarrow \frac{d}{dn}[\log[r]] = \frac{1}{n}\frac{dr}{dn}$$

so that:

$$\begin{aligned}
\frac{d}{dn}[\log[\mathbf{f}]] &= \frac{d}{dn}[\log[R_1] - \log(n-1)] \\
\frac{1}{\mathbf{f}}\frac{d\mathbf{f}}{dn} &= \frac{d}{dn}\log[R_1] - \frac{d}{dn}\log(n-1)
\end{aligned}$$

Since R_1 is constant, its derivative vanishes (which shows that the lens could have been oriented either way):

$$\begin{aligned}
\frac{1}{\mathbf{f}}\frac{d\mathbf{f}}{dn} &= 0 - \frac{d}{dn}\left[\log\left(n-1\right)\right] \\
&= -\frac{1}{n-1}\cdot\frac{d}{dn}\left(n-1\right) \\
&= -\frac{1}{n-1}\cdot 1 \\
&\Longrightarrow \frac{1}{\mathbf{f}}\frac{d\mathbf{f}}{dn} = -\frac{1}{n-1} \\
&\Longrightarrow \frac{d\mathbf{f}}{\mathbf{f}} = -\frac{dn}{n-1}
\end{aligned}$$

Now substitute the the Abbé number:

$$\upsilon = \frac{n-1}{dn} \Longrightarrow \boxed{\frac{d\mathbf{f}}{\mathbf{f}} = -\frac{1}{\upsilon} \Longrightarrow d\mathbf{f} = -\frac{\mathbf{f}}{\upsilon}}$$

This means that the spread of indices of refraction given (between the C and F lines) induces a change in the focal length by the amount:

$$d\mathbf{f} = -\frac{\mathbf{f}}{\upsilon} = -\frac{762\,\text{mm}}{55.2} \cong -13.804\,\text{mm} = -\frac{13.804\,\text{mm}}{25.4\frac{\text{mm}}{\text{in}}} \cong -0.543\,\text{in}$$

$$\boxed{d\mathbf{f} \cong -13.804\,\text{mm}}$$

which means that the spread in focal lengths between the red and green light is about one part in 55. The negative sign means that the focal length decreases as the refractive index increases (which we already knew). Roughly speaking, after taking into account that $n_D - n_F < n_C - n_D$, I estimate that the focal lengths of the lens are $\mathbf{f}_c \cong 768\,\text{mm}$ *and* $\mathbf{f}_c \cong 754\,\text{mm}$ *(not that you needed to evaluate this).*

4. Calculate the reflection and refraction coefficients for both TE and TM polarizations for light incident from air onto glass with index $n = 1.6$ at angle $\theta_0 = \frac{\pi}{6}$.

$$r_{TE} = \frac{n_1 \cos[\theta_0] - n_2 \cos[\theta_t]}{n_1 \cos[\theta_0] + n_2 \cos[\theta_t]} = \frac{n_1 \cos[\theta_0] - \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{n_1 \cos[\theta_0] + \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}$$

$$r_{TM} = \frac{+n_2 \cos[\theta_0] - n_1 \cos[\theta_t]}{+n_2 \cos[\theta_0] + n_1 \cos[\theta_t]} = \frac{+n_2 \cos[\theta_0] - \frac{n_1}{n_2}\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{+n_2 \cos[\theta_0] + \frac{n_1}{n_2}\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}$$

$$t_{TE} = \frac{+2n_1 \cos[\theta_0]}{n_1 \cos[\theta_0] + n_2 \cos[\theta_t]} = \frac{+2n_1 \cos[\theta_0]}{n_1 \cos[\theta_0] + \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}$$

$$t_{TM} = \frac{2n_1 \cos[\theta_0]}{+n_2 \cos[\theta_0] + n_1 \cos[\theta_t]} = \frac{2n_1 n_2 \cos[\theta_0]}{+n_2^2 \cos[\theta_0] + n_1\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}$$

By substitution, we obtain:

$$r_{TE} = \frac{\cos\left[\frac{\pi}{6}\right] - \sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}}{\cos\left[\frac{\pi}{6}\right] + \sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}} \cong -0.2740$$

$$r_{TM} = \frac{1.6 \cdot \cos\left[\frac{\pi}{6}\right] - \frac{1}{1.6}\sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}}{1.6 \cdot \cos\left[\frac{\pi}{6}\right] + \frac{1}{1.6}\sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}} \cong 0.1876$$

$$t_{TE} = \frac{+2\cos\left[\frac{\pi}{6}\right]}{\cos\left[\frac{\pi}{6}\right] + \sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}} \cong 0.7269$$

$$t_{TM} = \frac{2 \cdot 1.6 \cdot \cos\left[\frac{\pi}{6}\right]}{(1.6)^2 \cos\left[\frac{\pi}{6}\right] + \sqrt{(1.6)^2 - \sin^2\left[\frac{\pi}{6}\right]}} \cong 0.7416$$

5. The critical angle for a certain oil is found to be $\theta_c = 33°33'$ of arc. Find the Brewster angle for both external and internal reflections.

Brewster's angle is that for which there is no TM reflection. An "external reflection" is one for which $n_1 < n_2$ *and an internal reflection has* $n_1 > n_2$ *(dense to rare). For an internal reflection at the critical angle, there is no transmission from a dense to a rare medium. From the notes, the expression satisfied by the critical angle is:*

$$\theta_C = \sin^{-1}\left[\frac{n_2}{n_1}\right] = 33°33'$$

For the internal reflection with $n_1 < n_2$*:*

$$\Longrightarrow \quad \frac{n_2}{n_1} = \sin[33°33'] \cong 0.552\,66, \textit{ for internal reflection}$$

$$\Longrightarrow \quad \boxed{\theta_B = \tan^{-1}[\sin[33°33']] \cong 0.504\,88 \textit{ radians} \cong 28.93° \cong 28°56'}$$

For the external reflection, $n_1 > n_2 \implies$

$$\frac{n_1}{n_2} = \frac{1}{\sin[33°33']} \cong 1.8094 \textit{ for external reflection}$$

$$\boxed{\theta_B = \tan^{-1}\left[\frac{n_1}{n_2}\right] = \tan^{-1}\left[\frac{1}{\sin[33°33']}\right] \cong \tan^{-1}[1.8094] \cong 1.0659 \textit{ radians} \cong 61.07° \cong 61°4'}$$

6. Unpolarized light is reflected from a plane surface of fused silica glass of index $n = 1.458$.

$$\begin{aligned} n_1 &= 1 \\ n_2 &= 1.458 \end{aligned}$$

(a) Determine the critical and polarizing (Brewster) angles.

The critical angle is an internal reflection but the Brewster angles are both internal and external

$$\begin{aligned} \theta_C &= \sin^{-1}\left[\frac{n_1}{n_2}\right] = \sin^{-1}\left[\frac{1}{1.458}\right] \cong 0.7558\ radians = \boxed{\theta_C \cong 43.30°} \\ \theta_B\,(internal) &= \tan^{-1}\left[\frac{n_1}{n_2}\right] = \tan^{-1}\left[\frac{1}{1.458}\right] \cong 0.6012\ radians = \boxed{\theta_B\,(internal) \cong 33.44°} \\ \theta_B\,(external) &= \tan^{-1}\left[\frac{n_2}{n_1}\right] = \tan^{-1}\left[\frac{1.458}{1}\right] \cong 0.9696\ radians = \boxed{\theta_B\,(external) \cong 55.55°} \end{aligned}$$

(b) Determine the reflectance and transmittance for the TE mode at normal incidence ($\theta_0 = 0°$) and at $\theta_0 = 45°$

$$R_{TE} = \left(\frac{n_1 \cos[\theta_0] - \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{n_1 \cos[\theta_0] + \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}\right)^2$$

$$\boxed{R_{TE}[\theta_0 = 0°] \cong 0.0347}$$
$$\boxed{R_{TE}[\theta_0 = 45°] \cong 0.0821}$$

$$T_{TE} = \frac{4n_1 \cos[\theta_0]\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{\left(n_1 \cos[\theta_0] + \sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}\right)^2}$$

$$\boxed{T_{TE}[\theta_0 = 0°] \cong 0.9653}$$
$$\boxed{T_{TE}[\theta_0 = 45°] \cong 0.9179}$$

(c) Repeat part (b) for TM light.

$$R_{TM} = \left(\frac{+n_2^2 \cos[\theta_0] - n_1\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{+n_2^2 \cos[\theta_0] + n_1\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}\right)^2$$

$$\boxed{R_{TM}[\theta_0 = 0°] = 0.0347}$$
$$\boxed{R_{TM}[\theta_0 = 45°] = 0.00674}$$

$$T_{TM} = \left(\frac{\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}{n_1 \cos[\theta_0]}\right) \cdot \left(\frac{2n_1 n_2 \cos[\theta_0]}{+n_2^2 \cos[\theta_0] + n_1\sqrt{n_2^2 - n_1^2 \sin^2[\theta_0]}}\right)^2$$

$$\boxed{T_{TM}\left[\theta_0 = 0\right] = 0.965}$$

$$\boxed{T_{TM}\left[\theta_0 = 45^\circ\right] = 0.9933}$$

(d) Calculate and plot the phase difference between the TM and TE modes for internally reflected light at angles of incidence of $\theta_0 = 0^\circ, 20^\circ, 40^\circ, 50^\circ, 70^\circ$, and 90°. (extra credit given for a graph from $\theta_0 = 0^\circ$ to $\theta_0 = 90^\circ$).

The reflection coefficients are generally complex numbers that may be written in the form:

$$r = |r| \exp\left[+i\phi\right]$$

where ϕ is the phase shift of the light evaluated via:

$$\phi = \tan^{-1}\left[\frac{\text{Im}\{r\}}{\text{Re}\{r\}}\right]$$

Note that $-\pi \le \phi < +\pi$, which means that the full inverse tangent must be used (the usual form of the inverse tangent calculates values in the range $-\frac{\pi}{2} \le \phi < +\frac{\pi}{2}$.

In the Hecht convention:

$$\theta_0 = 0 \Longrightarrow \phi_{TE} = 0 \textit{ and } \phi_{TM} = -\pi \Longrightarrow \boxed{\Delta\phi\left[0\right] = \phi_{TE} - \phi_{TM} = +\pi}$$

$$\theta_0 = 20^\circ < \theta_B \Longrightarrow \phi_{TE} = 0 \textit{ and } \phi_{TM} = -\pi \Longrightarrow \boxed{\Delta\phi\left[20^\circ\right] = +\pi}$$

$$\theta_0 = 40^\circ \Longrightarrow \theta_B < \theta_0 < \theta_C \Longrightarrow \phi_{TE} = 0 \textit{ and } \phi_{TM} = 0 \Longrightarrow \boxed{\Delta\phi\left[40^\circ\right] = 0}$$

$$r_{TE}\left[\theta_0 = 50^\circ\right] = \frac{1.458\cos\left[50^\circ\right] - \sqrt{1^2 - (1.458)^2 \sin^2\left[50^\circ\right]}}{1.458\cos\left[50^\circ\right] + \sqrt{1^2 - (1.458)^2 \sin^2\left[50^\circ\right]}} \equiv 0.560 - 0.828i$$

$$\Longrightarrow \phi_{TE}\left[\theta_0 = 50^\circ\right] = \tan^{-1}\left[\frac{-0.828}{+0.560}\right] \cong -0.976 \textit{ radians } \cong -55.928^\circ$$

$$r_{TM}\left[\theta_0 = 50^\circ\right] = \frac{+1\cdot\cos\left[50^\circ\right] - \frac{1.458}{1}\sqrt{1^2 - (1.458)^2 \sin^2\left[50^\circ\right]}}{+1\cdot\cos\left[50^\circ\right] + \frac{1.458}{1}\sqrt{1^2 - (1.458)^2 \sin^2\left[50^\circ\right]}} \cong -0.120\,19 - 0.992\,75i$$

$$\phi_{TM}\left[\theta_0 = 50^\circ\right] = t\left[-0.120\,19, -0.992\,75\right] \cong -1.6913 \textit{ radians} \cong -96.904^\circ$$

$$\boxed{\Delta\phi\left[\theta_0 = 50^\circ\right] = -55.928^\circ - (-96.904^\circ) \cong 0.715\,15 \cong 0.22764 \cong +40.975^\circ}$$

$$r_{TE}\left[\theta_0 = 70^\circ\right] = \frac{1.458\cos\left[70^\circ\right] - \sqrt{1^2 - (1.458)^2 \sin^2\left[70^\circ\right]}}{1.458\cos\left[70^\circ\right] + \sqrt{1^2 - (1.458)^2 \sin^2\left[70^\circ\right]}} \equiv -0.558\,23 - 0.829\,69i$$

$$\Longrightarrow \phi_{TE}\left[\theta_0 = 70^\circ\right] = \tan^{-1}\left[\frac{-0.829\,69}{-0.55823}\right] \cong -2.163 \textit{ radians } \cong -123.94^\circ$$

$$r_{TM}\left[\theta_0 = 70^\circ\right] = \frac{+1\cdot\cos\left[70^\circ\right] - \frac{1.458}{1}\sqrt{1^2 - (1.458)^2 \sin^2\left[70^\circ\right]}}{+1\cdot\cos\left[70^\circ\right] + \frac{1.458}{1}\sqrt{1^2 - (1.458)^2 \sin^2\left[70^\circ\right]}} \cong -0.881\,93 - 0.471\,38i$$

$$\phi_{TM}\left[\theta_0 = 70^\circ\right] \cong -2.6507 \textit{ radians} \cong -151.88^\circ$$

$$\boxed{\Delta\phi\left[\theta_0 = 70^\circ\right] = -123.94^\circ - (-151.88^\circ) \cong 0.15522\pi \ \ radians = 0.48764 \ \ radians \cong +27.93^\circ}$$

$$\begin{aligned}
r_{TE}\left[\theta_0 = 90^\circ\right] &= \frac{1.458\cos\left[90^\circ\right] - \sqrt{1^2 - (1.458)^2\sin^2\left[90^\circ\right]}}{1.458\cos\left[90^\circ\right] + \sqrt{1^2 - (1.458)^2\sin^2\left[90^\circ\right]}} \equiv -1 \\
&\Longrightarrow \phi_{TE}\left[\theta_0 = 90^\circ\right] = -\pi \ \ radians \\
r_{TM}\left[\theta_0 = 90^\circ\right] &= \frac{+1\cdot\cos\left[90^\circ\right] - \frac{1.458}{1}\sqrt{1^2 - (1.458)^2\sin^2\left[90^\circ\right]}}{+1\cdot\cos\left[90^\circ\right] + \frac{1.458}{1}\sqrt{1^2 - (1.458)^2\sin^2\left[90^\circ\right]}} \cong -1 \\
&\Longrightarrow \phi_{TM}\left[\theta_0 = 90^\circ\right] = -\pi \ \ radians \\
&\boxed{\Delta\phi\left[\theta_0 = 90^\circ\right] = 0}
\end{aligned}$$

In the *<u>Pedrotti convention</u>*:

$$\theta_0 = 0 \Longrightarrow \phi_{TE} = 0 \text{ and } \phi_{TM} = 0 \Longrightarrow \boxed{\Delta\phi\,[0] = \phi_{TE} - \phi_{TM} = 0}$$
$$\theta_0 = 20^\circ < \theta_B \Longrightarrow \phi_{TE} = 0 \text{ and } \phi_{TM} = 0 \Longrightarrow \boxed{\Delta\phi\,[20^\circ] = 0}$$
$$\theta_0 = 40^\circ \Longrightarrow \theta_B < \theta_0 < \theta_C \Longrightarrow \phi_{TE} = 0 \text{ and } \phi_{TM} = -\pi \Longrightarrow \boxed{\Delta\phi\,[40^\circ] = \pi}$$

Note that:

$$\Delta\phi_{\text{Hecht}} - \pi = \Delta\phi_{\text{Pedrotti}}$$

$$r_{TE}\,[\theta_0 = 50^\circ] = \frac{1.458\cos[50^\circ] - \sqrt{1^2 - (1.458)^2\sin^2[50^\circ]}}{1.458\cos[50^\circ] + \sqrt{1^2 - (1.458)^2\sin^2[50^\circ]}} \equiv 0.560 - 0.828i$$
$$\Longrightarrow \phi_{_{TE}}\,[\theta_0 = 50^\circ] \cong -0.976 \text{ radians} \cong -55.928^\circ$$
$$r_{TM}\,[\theta_0 = 50^\circ] = -\left(r_{TM}\,[\theta_0 = 50^\circ]\right)_{\text{Hecht}} = +0.120\,19 + 0.992\,75i$$
$$\phi_{TM}\,[\theta_0 = 50^\circ] = +1.4503 \text{ radians} \cong +83.096^\circ$$

$$\boxed{\Delta\phi\,[\theta_0 = 50^\circ] = -55.928^\circ - (+83.096^\circ) \cong -0.772\,36\pi = -2.43 \text{ radians} \cong -139.02^\circ}$$

$$r_{TE}\,[\theta_0 = 70^\circ] = \frac{1.458\cos[70^\circ] - \sqrt{1^2 - (1.458)^2\sin^2[70^\circ]}}{1.458\cos[70^\circ] + \sqrt{1^2 - (1.458)^2\sin^2[70^\circ]}} \equiv -0.558\,23 - 0.829\,69i$$
$$\Longrightarrow \phi_{_{TE}}\,[\theta_0 = 70^\circ] = \tan^{-1}\left[\frac{-0.829\,69}{-0.55823}\right] \cong -2.163 \text{ radians} \cong -123.\,94^\circ$$

$$r_{TM}\,[\theta_0 = 70^\circ] \cong +0.881\,93 + 0.471\,38i$$
$$\phi_{TM}\,[\theta_0 = 70^\circ] = t\,[+0.881\,93, +0.471\,38] \cong +.49085 \text{ radians} \cong +28.124^\circ$$

$$\boxed{\Delta\phi\,[\theta_0 = 70^\circ] = -123.\,94^\circ - (+28.124^\circ) \cong -0.844\,8\pi \text{ radians} \cong -2.654 \text{ radians} \cong -152.06^\circ}$$

$$r_{TE}\,[\theta_0 = 90^\circ] = \frac{1.458\cos[90^\circ] - \sqrt{1^2 - (1.458)^2\sin^2[90^\circ]}}{1.458\cos[90^\circ] + \sqrt{1^2 - (1.458)^2\sin^2[90^\circ]}} \equiv -1$$
$$\Longrightarrow \phi_{_{TE}}\,[\theta_0 = 90^\circ] = -\pi \text{ radians}$$
$$r_{TM}\,[\theta_0 = 90^\circ] = -\frac{+1\cdot\cos[90^\circ] - \frac{1.458}{1}\sqrt{1^2 - (1.458)^2\sin^2[90^\circ]}}{+1\cdot\cos[90^\circ] + \frac{1.458}{1}\sqrt{1^2 - (1.458)^2\sin^2[90^\circ]}} = +1$$
$$\phi_{TM}\,[\theta_0 = 90^\circ] = 0 \text{ radians}$$
$$\boxed{\Delta\phi\,[\theta_0 = 90^\circ] = -\pi \text{ radians} = 180^\circ}$$