

SIMG-733-20092 Optics for Imaging Solutions to Final Exam

1. An imaging system consists of two identical thin lenses each with focal length $\mathbf{f}_1 = \mathbf{f}_2 = +300$ mm and diameter $d_1 = d_2 = 50$ mm. The lenses are separated by $t = 150$ mm. Between the lenses and at equal distances from each is a single iris with diameter $d_{\text{iris}} = 20$ mm.

(a) Determine the effective focal length of the system.

$$\frac{1}{\mathbf{f}_{\text{eff}}} = \frac{1}{\mathbf{f}_1} + \frac{1}{\mathbf{f}_2} - \frac{t}{\mathbf{f}_1 \cdot \mathbf{f}_2}$$

$$\mathbf{f}_{\text{eff}} = \left(\frac{1}{300 \text{ mm}} + \frac{1}{300 \text{ mm}} - \frac{150 \text{ mm}}{(300 \text{ mm})^2} \right)^{-1} = \boxed{\mathbf{f}_{\text{eff}} = +200 \text{ mm}}$$

- (b) Determine the locations of the focal points \mathbf{F} and \mathbf{F}' and principal points \mathbf{H} , \mathbf{H}' of the system, i.e., determine the distances $\overline{\mathbf{FV}}$, $\overline{\mathbf{V}'\mathbf{F}'}$, $\overline{\mathbf{HV}}$, and $\overline{\mathbf{V}'\mathbf{H}'}$

Trace a ray through the system from an object at infinity:

$$\frac{1}{z_1} + \frac{1}{z'_1} = \frac{1}{\mathbf{f}_1} = \frac{1}{300 \text{ mm}}$$

$$z_1 = \infty \implies z'_1 = 300 \text{ mm} \implies z_2 = -150 \text{ mm}$$

$$\frac{1}{z_2} + \frac{1}{z'_2} = \frac{1}{\mathbf{f}_2} = \frac{1}{300 \text{ mm}} \implies z'_2 = \left(\frac{1}{300 \text{ mm}} - \frac{1}{-150 \text{ mm}} \right)^{-1} = +100 \text{ mm}$$

$$\implies \boxed{\overline{\mathbf{V}'\mathbf{F}'} = +100 \text{ mm}}$$

$$\overline{\mathbf{H}'\mathbf{F}'} = \mathbf{f}_{\text{eff}} = +200 \text{ mm} = \overline{\mathbf{H}'\mathbf{V}'} + \overline{\mathbf{V}'\mathbf{F}'} = \overline{\mathbf{H}'\mathbf{V}'} + 100 \text{ mm}$$

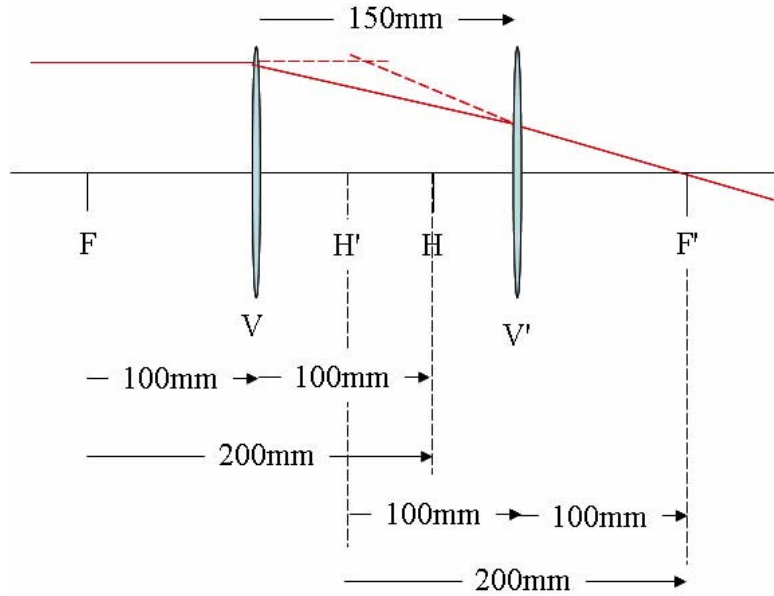
$$\implies \boxed{\overline{\mathbf{H}'\mathbf{V}'} = +100 \text{ mm}}$$

so the image-space principal point is 100 mm in front of the image-space vertex. Since the system is symmetric, the same distances apply in object space

$$\boxed{\overline{\mathbf{FV}} = +100 \text{ mm}}$$

$$\boxed{\overline{\mathbf{VH}} = +100 \text{ mm}}$$

- (c) Sketch the system showing the locations of the vertices, focal points and principal points.



- (d) For an object at infinity, determine which element is the aperture stop of the system.

From the drawing, you can see that the height of the marginal ray at the midpoint (75 mm from the first lens) between the lenses is $3/4$ of the height at the first lens. Since the semidiameter of the first lens is 25 mm, the height of the ray at the iris is 18.75 mm, which is significantly larger than the semidiameter of the iris (10 mm), so the iris is the stop.

- (e) Determine the locations of the entrance and exit pupils of the system for an object at infinity and locate them on the sketch in part (c)

Again, since the system is symmetric, the pupils are symmetrically placed. We need to find the image distance created by the second lens for the object at the iris:

$$\begin{aligned} z_1 &= 75 \text{ mm} \\ \mathbf{f} &= 300 \text{ mm} \\ z_2 &= \left(\frac{1}{300 \text{ mm}} - \frac{1}{75 \text{ mm}} \right)^{-1} = -100 \text{ mm} \end{aligned}$$

So the pupils are virtual, i.e., the exit pupil is “behind” the image-space vertex and so is not accessible. The magnification of the pupils is:

$$M_T = -\frac{z_2}{z_1} = -\frac{-100 \text{ mm}}{75 \text{ mm}} = \frac{4}{3}$$

So the diameter of the entrance pupil is:

$$d_{NP} = \frac{4}{3} \cdot 20 \text{ mm} = \boxed{d_{NP} = \frac{80}{3} \text{ mm} \cong 26.7 \text{ mm}}$$

- (f) Determine the focal ratio ($f/\#$) of the system.

The focal ratio is the ratio of the focal length to the diameter of the entrance pupil:

$$f/\# = \frac{+200 \text{ mm}}{\frac{80}{3} \text{ mm}} = \boxed{f/\# = 7.5}$$

- (g) If the stop is circular with diameter d_{iris} as stated, determine and sketch the the profile of the diffraction spot for wavelength $\lambda_0 = 0.5 \mu\text{m}$

$$D_0 \cong 2.44 \cdot \lambda_0 \cdot \frac{\mathbf{f}_{\text{eff}}}{d_{NP}} = 2.44 \cdot 0.5 \mu\text{m} \cdot 7.5 = \boxed{D_0 \cong 9.15 \mu\text{m}}$$

- (h) Determine the locations of the object and image such that transverse magnification of the system is $M_T = +1$.

trick question, these are at the locations of the principal planes, but these are virtual locations so you can do this by putting a virtual object at \mathbf{H} to get an virtual image at \mathbf{H}' .

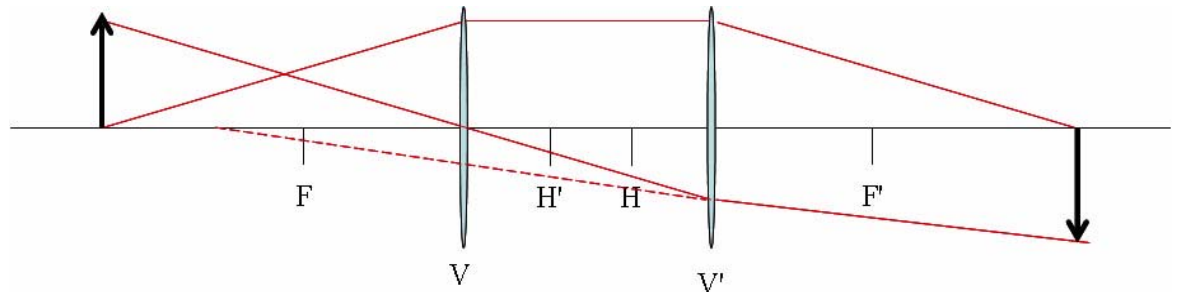
- (i) Determine the locations of the object and image such that transverse magnification of the system is $M_T = -1$.

These are the “equal-conjugate” points:

$$\begin{aligned} \overline{\mathbf{OH}} &= 2\mathbf{f}_{\text{eff}} = 2 \cdot 200 \text{ mm} = 400 \text{ mm} \\ \Rightarrow \overline{\mathbf{OV}} &= \overline{\mathbf{OH}} + \overline{\mathbf{HV}} = \overline{\mathbf{OH}} - \overline{\mathbf{VH}} = 400 \text{ mm} - 100 \text{ mm} = 300 \text{ mm} \\ \Rightarrow \boxed{\overline{\mathbf{V}'\mathbf{O}'} = \overline{\mathbf{OV}} = 300 \text{ mm for } M_T = -1} \end{aligned}$$

So the object is 300 mm “in front” of the object-space vertex. Test this result by finding the image location measured from the image-space vertex:

$$\begin{aligned} \frac{1}{z_1} + \frac{1}{z'_1} &= \frac{1}{\mathbf{f}_1} \Rightarrow z'_1 = \left(\frac{1}{300 \text{ mm}} - \frac{1}{300 \text{ mm}} \right)^{-1} = \infty \\ z_2 &= t - z'_1 = -\infty = +\infty \\ z'_2 &= \mathbf{f}_2 = +300 \text{ mm} \end{aligned}$$



- (j) Determine the locations of the object and image relative to the vertices of the system such that transverse magnification of the system is $M_T = -\frac{1}{4}$.
 If the transverse magnification is $-\frac{1}{4}$, the object distance is four times larger than the image distance:

$$\begin{aligned}
 M_T &= -\frac{1}{4} = -\frac{z_2}{z_1} \implies z_1 = \overline{\mathbf{OH}} = 4z_2 \implies \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{4z_2} + \frac{1}{z_2} = \frac{1}{\mathbf{f}_{\text{eff}}} \\
 \frac{1}{4z_2} + \frac{1}{4z_2} &= \frac{5}{4z_2} = \frac{1}{200 \text{ mm}} \implies z_2 = \frac{5}{4} \cdot 200 \text{ mm} = 250 \text{ mm} = \boxed{\overline{\mathbf{H}'\mathbf{O}'} = 250 \text{ mm}} \\
 &\implies \overline{\mathbf{V}'\mathbf{O}'} = \overline{\mathbf{H}'\mathbf{O}'} - \overline{\mathbf{H}'\mathbf{V}'} = 250 \text{ mm} - 100 \text{ mm} = \boxed{\overline{\mathbf{V}'\mathbf{O}'} = 150 \text{ mm}} \\
 z_1 &= 4 \cdot z_2 = \boxed{\overline{\mathbf{OH}} = 1000 \text{ mm}} \\
 \overline{\mathbf{OH}} &= 1000 \text{ mm} = \overline{\mathbf{OV}} + \overline{\mathbf{VH}} = \overline{\mathbf{OV}} + 100 \text{ mm} \implies \boxed{\overline{\mathbf{OV}} = 900 \text{ mm}}
 \end{aligned}$$

Check it by finding the images from each lens in turn and the associated transverse magnifications:

$$\begin{aligned}
 z'_1 &= \left(\frac{1}{\mathbf{f}_1} - \frac{1}{z_1} \right)^{-1} = \left(\frac{1}{300 \text{ mm}} - \frac{1}{900 \text{ mm}} \right)^{-1} = +450 \text{ mm} \\
 M_1 &= -\frac{450 \text{ mm}}{900 \text{ mm}} = -\frac{1}{2} \\
 z_2 &= t - z'_1 = 150 \text{ mm} - (450 \text{ mm}) = -300 \text{ mm} \\
 z'_2 &= \left(\frac{1}{\mathbf{f}_2} - \frac{1}{z_2} \right)^{-1} = \left(\frac{1}{300 \text{ mm}} - \frac{1}{-300 \text{ mm}} \right)^{-1} = +150 \text{ mm} \\
 M_2 &= -\frac{150 \text{ mm}}{-300 \text{ mm}} = +\frac{1}{2} \\
 M_T &= M_1 \cdot M_2 = \left(-\frac{1}{2} \right) \cdot \left(+\frac{1}{2} \right) = -\frac{1}{4}
 \end{aligned}$$

- (k) If the two lenses are made of the same crown glass with $n_D = 1.5$ at $\lambda_D = 589.59 \text{ nm}$, explain what you expect to be true about the focal length of the system at the F line ($\lambda_F = 486.13 \text{ nm}$) and at the C line ($\lambda_C = 656.28 \text{ nm}$); I'm not looking for numerical values, but rather for the trend.

For blue light, we know that the refractive index will be larger, so the focal length will be shorter. The system focal length will still be:

$$\mathbf{f}_{\text{eff}} = \left(\frac{1}{\mathbf{f}_1} + \frac{1}{\mathbf{f}_2} - \frac{t}{\mathbf{f}_1 \cdot \mathbf{f}_2} \right)^{-1}$$

where t is fixed, but the shorter focal lengths for \mathbf{f}_1 and \mathbf{f}_2 mean that the

system focal length is shorter for blue light and longer for red light

- (l) If the system is set up to image an object at ∞ , what can you say about the depth of field if the diameter of the iris is decreased?

We know that the depth of field goes as the square of the focal ratio, so the depth of field should increase.

2. An object with a square-wave amplitude transmittance with a period of $100 \frac{\text{cycles}}{\text{mm}}$ is imaged by a lens with a circular pupil with $d_0 = 20 \text{ mm}$ and $\mathbf{f} = 100 \text{ mm}$. The distance from object to lens is $z_1 = 200 \text{ mm}$. The wavelength $\lambda_0 = 1 \mu\text{m}$

- (a) Sketch a profile of the measured irradiance at output plane (the detector is sensitive at $\lambda_0 = 1 \mu\text{m}$).

Solution: *The image distance is obviously 200 mm (equal conjugates) so the transverse magnification is $M_T = -1$. The transfer function for coherent illumination will be a cylinder function with scaled frequency:*

$$p(r) = \text{CYL} \left(\frac{r}{d_0} \right) \implies H(\rho) = \text{CYL} \left(-\frac{\lambda_0 z_2 \rho}{d_0} \right) = \text{CYL} \left(\frac{\rho}{\left(\frac{d_0}{\lambda_0 z_2} \right)} \right)$$

The edge of the pupil is located at:

$$r = \frac{d_0}{2}$$

and so the frequency at the edge of the transfer function satisfies the condition:

$$\begin{aligned} \frac{\rho}{\left(\frac{d_0}{\lambda_0 z_2} \right)} &= \frac{1}{2} \implies \rho = \frac{d_0}{2\lambda_0 z_2} \approx 100 \frac{\text{cycles}}{\text{mm}} \\ d_0 &\approx 2 \cdot 100 \frac{\text{cycles}}{\text{mm}} \cdot 1 \mu\text{m} \cdot 200 \text{ mm} \\ &= 2 \cdot 100 \text{ mm}^{-1} \cdot 1 \mu\text{m} \cdot 200 \text{ mm} = 40 \text{ mm} > d_0 \end{aligned}$$

The square wave pattern may be written as:

$$f[x, y] = \left(\text{RECT} \left[\frac{x}{5 \mu\text{m}} \right] * \left(\frac{1}{10 \mu\text{m}} \text{COMB} \left[\frac{x}{10 \mu\text{m}} \right] \right) \right) \cdot 1[y]$$

So its frequency spectrum is:

$$\begin{aligned} F[\xi, \eta] &= (5 \mu\text{m} \cdot \text{SINC} [5 \mu\text{m} \cdot \xi] \cdot \text{COMB} [10 \mu\text{m} \cdot \xi]) \cdot \delta[\eta] \\ &\propto \sum_{k=-\infty}^{+\infty} \delta \left(\xi - \frac{k}{10 \mu\text{m}} \right) \cdot \text{SINC} [5 \mu\text{m} \cdot \xi] \\ &= \sum_{k=-\infty}^{+\infty} \delta \left(\xi - \frac{k}{10 \mu\text{m}} \right) \cdot \text{SINC} \left[5 \mu\text{m} \cdot \frac{k}{10 \mu\text{m}} \right] \\ &= \sum_{k=-\infty}^{+\infty} \delta \left(\xi - \frac{k}{10 \mu\text{m}} \right) \cdot \text{SINC} \left[\frac{k}{2} \right] \end{aligned}$$

This is an array of Dirac delta functions modulated by a SINC function, so the input image may be written as the sum of cosine terms:

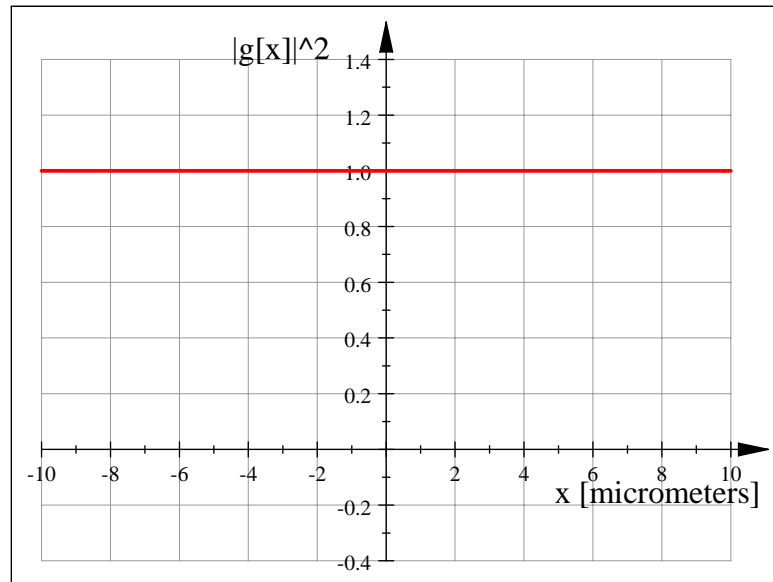
$$\begin{aligned} f[x, y] &= \frac{1}{2} \left(1 + 2 \cdot \text{SINC} \left[\frac{1}{2} \right] \cdot \cos \left[2\pi \frac{x}{10 \mu\text{m}} \right] + 2 \cdot \text{SINC} \left[\frac{3}{2} \right] \cdot \cos \left[2\pi \frac{x}{\frac{10 \mu\text{m}}{3}} \right] + \dots \right) \\ &= \frac{1}{2} + \frac{2}{\pi} \cdot \cos \left[2\pi \frac{x}{10 \mu\text{m}} \right] - \frac{2}{3\pi} \cdot \cos \left[2\pi \frac{x}{\frac{10 \mu\text{m}}{3}} \right] + \dots \end{aligned}$$

The frequency of the first non-zero order of the input spectrum is:

$$\xi_1 = \frac{1}{10 \mu\text{m}} = 100 \frac{\text{cycles}}{\text{mm}}$$

This frequency is (just barely) outside the finite support of the transfer function, so you see NO modulation, just a uniform constant field:

$$g[x] \cong 1[x]$$



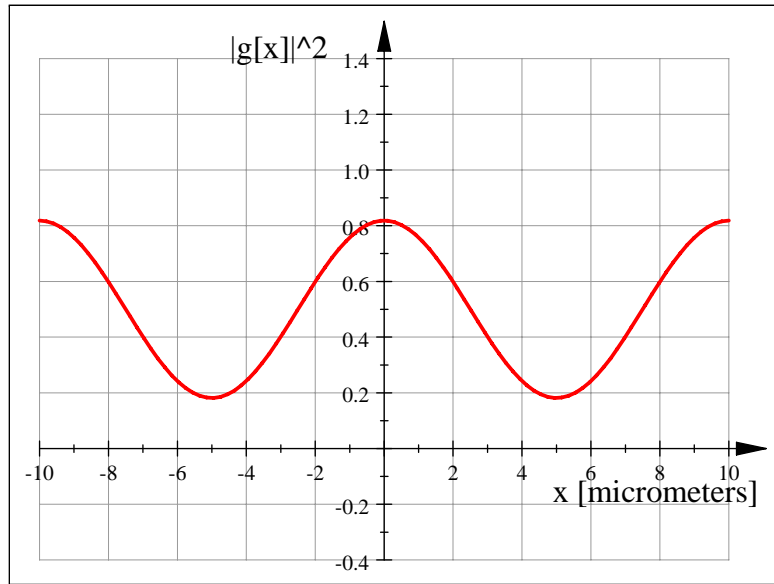
(b) Repeat for wavelength $\lambda_1 = 0.50 \mu\text{m}$.

Solution:

$$2 \cdot 100 \frac{\text{cycles}}{\text{mm}} \cdot 0.5 \mu\text{m} \cdot 200 \text{ mm} = 20 \text{ mm} = d_0$$

At this shorter wavelength, the constant and the fundamental sinusoidal frequency terms are passed to the output, which consists of a biased cosine function:

$$\begin{aligned} g[x, y] &= \frac{1}{2} + \frac{1}{2} \text{SINC} \left[\frac{1}{2} \right] \cdot \cos \left[2\pi \frac{x}{10 \mu\text{m}} \right] \\ &= \frac{1}{2} + \frac{1}{\pi} \cdot \cos \left[2\pi \frac{x}{10 \mu\text{m}} \right] \end{aligned}$$

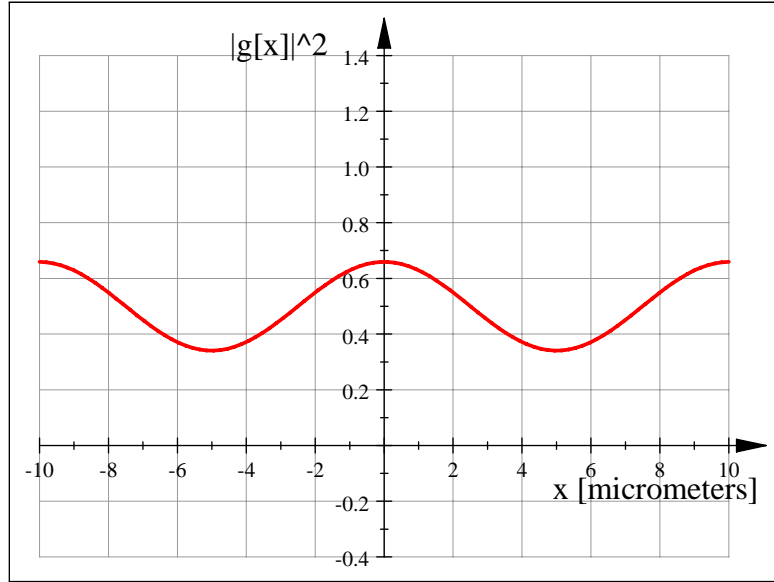


Approximate irradiance profile if using $\lambda_0 = 0.5 \mu\text{m}$, since the fundamental frequency of the square wave is passed after attenuation by $\frac{1}{2}$. The units on the x-axis are μm

- (c) If the same lens diameter in (a) is used for incoherent illumination at $\lambda_0 = 0.5 \mu\text{m}$, sketch the expected irradiance

Solution: *In the incoherent case, the transfer function is the appropriately scaled autocorrelation of the pupil, which in this case is a “circular triangle”. The cutoff frequency is twice as large, but the nonzero components are further attenuated. The output image consists of the unattenuated DC term and a sinusoid with amplitude approximately equal to*

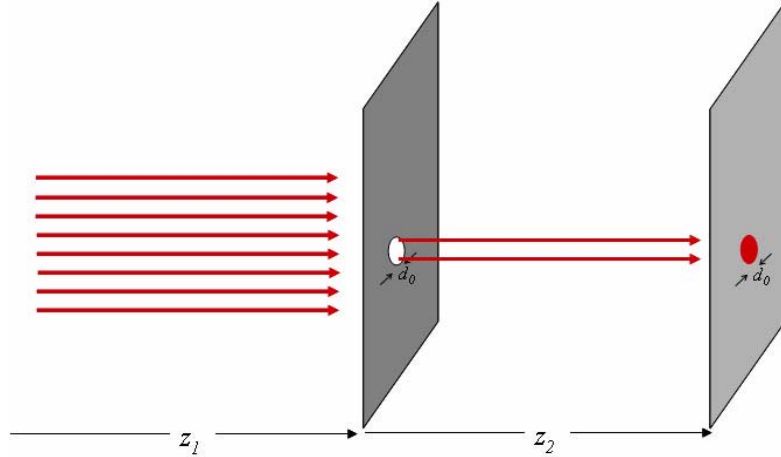
$$g[x] \cong \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{\pi} \cos \left[2\pi \frac{x}{10 \mu\text{m}} \right]$$



Approximate irradiance profile for incoherent light at $\lambda_0 = 0.5 \mu\text{m}$, since the fundamental frequency of the square wave is passed after attenuation by $\frac{1}{2}$. The units on the x-axis are μm

3. Consider a pinhole camera where the pupil function (circular pinhole of diameter d_0) is small compared to the object distance z_1 , which may be approximated as infinite. Assume that the object is illuminated by quasimonochromatic light with mean wavelength λ_μ .

- (a) Assuming that the pinhole is small, but sufficiently large so that the ray optics model is valid, find an expression for the spatial part of the optical transfer function of the camera. Determine the spatial frequency of the first zero of the OTF (which we may call the “cutoff frequency”).



If you see it, this problem is rather easy. Under the ray optics description, then the incoherent impulse response is proportional to the projection of the pinhole onto the observation plane. For an object at infinity, this is:

$$h[x, y; \lambda_\mu, z_2] \propto \text{CYL} \left(\frac{r}{d_0} \right)$$

Note that the ray-optics impulse response depends on neither wavelength λ_μ nor propagation distance z_2 . The OTF is just the appropriately scaled (and, arguably, normalized) 2-D Fourier transform:

$$\mathcal{F}_2 \left\{ \text{CYL} \left(\frac{r}{d_0} \right) \right\} = \pi \frac{d_0^2}{4} \cdot \text{SOMB}(d_0 \rho) = \mathfrak{H}[\xi, \eta; \lambda_\mu, z_2]$$

The normalized OTF is:

$$\frac{\mathfrak{H}[\xi, \eta; \lambda_\mu, z_2]}{\mathfrak{H}[0, 0; \lambda_\mu, z_2]} = \text{SOMB}(d_0 \rho) = \text{SOMB} \left(\frac{\rho}{d_0^{-1}} \right)$$

The cutoff frequency (first zero) is located at the frequencies that satisfy

$$d_0 \rho_{\text{cutoff}} \cong 1.22 \implies \rho_{\text{cutoff}} \cong \frac{1.22}{d_0}$$

which is not a function of wavelength in the ray optics model! Note that the cutoff frequency **decreases** with increasing diameter in the ray-optics model.

- (b) Now assume that the pinhole is so small that Fraunhofer diffraction governs the shape of the psf; calculate the spatial frequency of the first zero of the OTF.

If Fraunhofer diffraction applies, then the incoherent impulse response is proportional to the squared magnitude of the aperture function:

$$\begin{aligned} \mathfrak{h}[x, y; \lambda_\mu, z_2] &\propto \left| \mathcal{F}_2 \left\{ CYL \left(\frac{r}{d_0} \right) \right\} \right|_{\rho \rightarrow \frac{r}{\lambda_\mu z_2}}^2 \\ &= \pi \frac{d_0^2}{4} \cdot SOMB^2(d_0 \rho) \Big|_{\rho \rightarrow \frac{r}{\lambda_\mu z_2}} \\ &= \pi \frac{d_0^2}{4} \cdot SOMB^2 \left(\frac{r}{\left(\frac{\lambda_\mu z_2}{d_0} \right)} \right) \end{aligned}$$

so the impulse response in the Fraunhofer case IS a function of the wavelength λ_μ and of the propagation distance z_2 . The OTF is normalized 2-D Fourier transform:

$$\begin{aligned} \mathfrak{H}[\xi, \eta; \lambda_\mu, z_2] &\propto \pi \frac{d_0^2}{4} \cdot \mathcal{F}_2 \left\{ SOMB^2 \left(\frac{r}{\left(\frac{\lambda_\mu z_2}{d_0} \right)} \right) \right\} \\ &= \pi \frac{d_0^2}{4} \cdot CTRI \left(-\frac{\lambda_\mu z_2}{d_0} \rho \right) = \pi \frac{d_0^2}{4} \cdot CTRI \left(\frac{\rho}{\left(\frac{d_0}{\lambda_\mu z_2} \right)} \right) \end{aligned}$$

The CTRI function has compact support with unit radius, so the cutoff frequency satisfies the condition:

$$\frac{\lambda_\mu z_2}{d_0} \rho_{\text{cutoff}} = 1 \implies \boxed{\rho_{\text{cutoff}} \cong \frac{d_0}{\lambda_\mu z_2}}$$

which is a function of wavelength and which INCREASES with increasing diameter d_0 , as we expect in the wave model.

- (c) OPTIONAL BONUS: Use the results of parts (a) and (b) to estimate an “optimum” size of the pinhole in terms of the various parameters, i.e., the diameter that produces the largest cutoff frequency (of the first zero).

For a pinhole with diameter d_0 that is sufficiently large for geometrical optics to be valid, the cutoff frequency increases with decreasing diameter. Eventually the pinhole diameter will be small enough for the Fraunhofer approximation to be valid. The approximate diameter where the two expressions are both valid is where the cutoff frequencies match:

$$\begin{aligned} \frac{1.22}{(d_0)_{\text{optimum}}} &\cong \frac{(d_0)_{\text{optimum}}}{\lambda_\mu z_2} \\ \implies &(d_0)_{\text{optimum}}^2 \cong 1.22 \lambda_\mu z_2 \\ \implies &\boxed{(d_0)_{\text{optimum}} \cong \sqrt{1.22 \lambda_\mu z_2} \cong 1.104 \cdot \sqrt{\lambda_\mu z_2}} \end{aligned}$$

4. The relationship of the the focal length and the object and image distances is:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{\mathbf{f}}$$

(a) Derive the minimum separation between a *real* object and its *real* image

Solution: *The separation between the real object and the real image is*

$$s \equiv z_1 + z_2 = z_1 + \left(\frac{1}{\mathbf{f}} - \frac{1}{z_1} \right)^{-1}$$

The minimum separation must satisfy the relation:

$$\begin{aligned} \frac{ds}{dz_1} &= 0 \\ \frac{ds}{dz_1} &= 1 - \left(\frac{1}{\mathbf{f}} - \frac{1}{z_1} \right)^{-2} \cdot \frac{d}{dz_1} \left(\frac{1}{\mathbf{f}} - \frac{1}{z_1} \right) \\ &= 1 - \left(\frac{1}{\mathbf{f}} - \frac{1}{z_1} \right)^{-2} \cdot \left(0 + \frac{1}{z_1^2} \right) \\ &\implies \frac{1}{z_1^2} = \left(\frac{1}{\mathbf{f}} - \frac{1}{z_1} \right)^2 \\ \implies \frac{1}{\mathbf{f}^2} + \frac{1}{z_1^2} - \frac{2}{\mathbf{f}z_1} &= \frac{1}{z_1^2} \implies \frac{1}{\mathbf{f}^2} = \frac{2}{\mathbf{f}z_1} \implies z_1 = 2\mathbf{f} \end{aligned}$$

substitute into imaging equation:

$$z_2 = 2\mathbf{f} \implies s_{\min} = 2\mathbf{f} + 2\mathbf{f} = 4\mathbf{f}$$

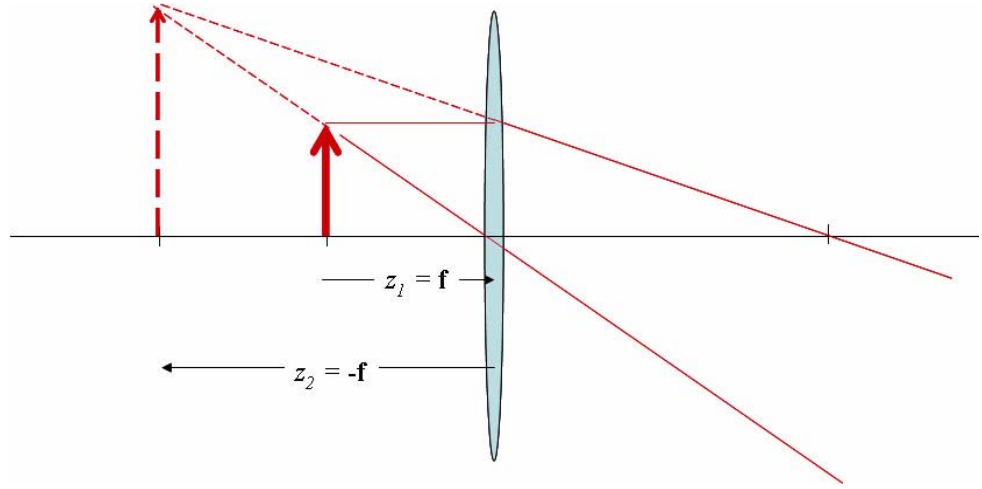
The minimum separation between object and image is $\boxed{s_{\min} = 4\mathbf{f}}$

- (b) An object is located at a distance $z_1 = \frac{f}{2}$ from a lens with focal length $\mathbf{f} = +|\mathbf{f}|$. Locate the image and determine the transverse magnification. Draw the system including the locations of object and image.

$$\frac{1}{\left(\frac{f}{2}\right)} + \frac{1}{z_2} = \frac{1}{f} \implies z_2 = \left(\frac{1}{f} - \frac{2}{f}\right)^{-1} = -f$$

The image is virtual and the transverse magnification is:

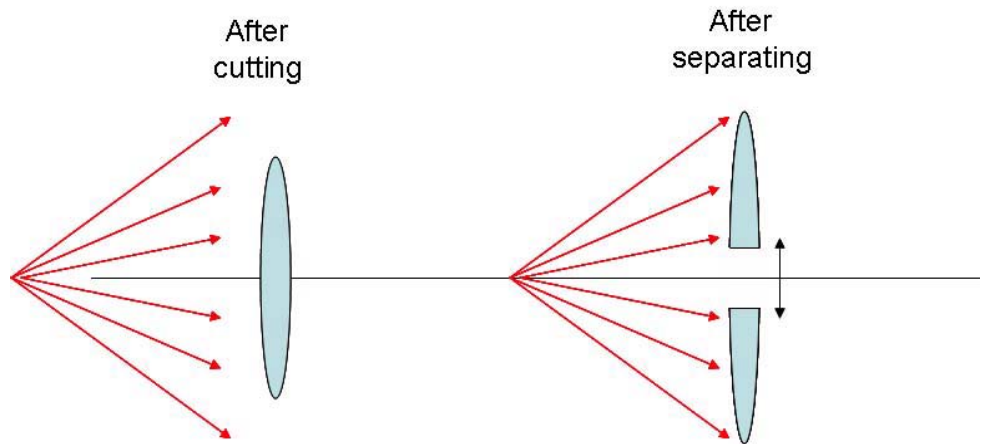
$$M_T = -\frac{z_2}{z_1} = -\left(\frac{-f}{\frac{f}{2}}\right) = +2 \text{ virtual, upright and magnified}$$



5. A lens with $f = +500$ mm is sawn into two pieces through a plane cutting through the optical axis (i.e., the cut is along a diameter). A point source of monochromatic light with $\lambda_0 = 500$ nm is placed on the optical axis at a distance $z_1 = 1000$ mm from the lens. The half lenses are gradually moved apart; each creates an image of the point source that are mutually coherent. The light is observed on a screen placed at a distance $z_2 = 2000$ mm from the lens (in the Fraunhofer diffraction region).

- (a) Determine the period of interference fringes observed on the screen if the separation between the two lens halves is increased to 0.5 mm.
- (b) Describe in words and/or equations what will happen if the lens and observation screen are moved farther from the point source.

(a) **SOLUTION:**



The lens creates two images of the point source. We know from the imaging equation that:

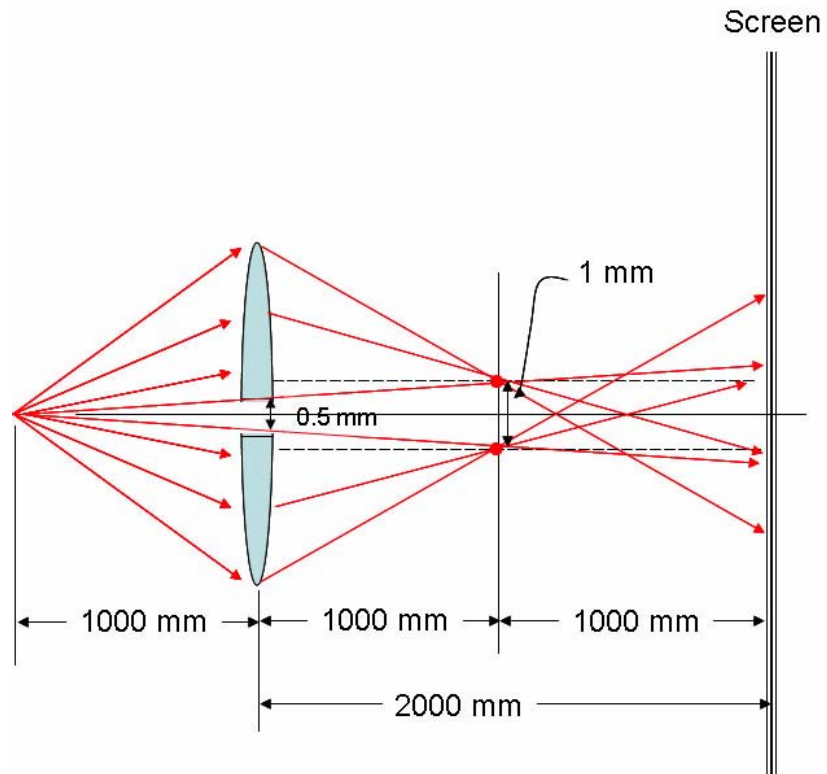
$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \implies z_2 = \frac{z_1 f}{z_1 - f} = \frac{1000 \text{ mm} \cdot 500 \text{ mm}}{1000 \text{ mm} - 500 \text{ mm}} = 1000 \text{ mm}$$

which you probably knew already since this is the distance for equal conjugates

Since the transverse magnification is $M_T = -1$, we can find the separation of the images of the point source by geometric considerations:

$$d_0 = 0.5 \text{ mm} \cdot \frac{2000 \text{ mm}}{1000 \text{ mm}} = 1 \text{ mm}$$

This is the distance between the sources in a Young's two-aperture experiment.



The distance to the observation screen is $L_0 = 1000$ mm, so the period of the irradiance fringe pattern is

$$\begin{aligned}
 D &= \frac{L_0 \lambda_0}{d_0} \\
 &= \frac{1000 \text{ mm} \cdot 500 \text{ nm}}{1 \text{ mm}} = \boxed{0.5 \text{ mm} = D}
 \end{aligned}$$

(b) **SOLUTION:** as the lens is moved away from the source, the magnification changes so the point sources appear closer together, which means that the fringe period will increase.