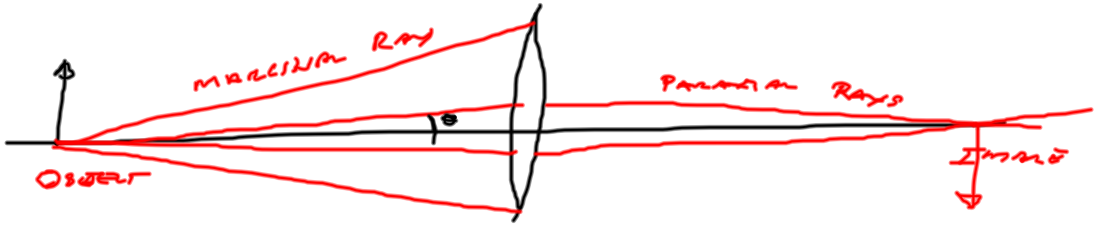
10 FEBRUARY 2010



RAY OPTICS MODEL OF IMAGING

FIND (LOCATE) IMAGES, PARAXIAL APPROXIMATION

MAGNIFICATIONS → QUALITY (ABERRATIONS)




LOGARITHMS

1970s → COMPUTERS, OSLO, CODE V

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HERO OF ALEXANDRIA   LIGHT RAY TRAVELS SHORTEST PATH

FERMAT – LIGHT RAY TRAVELS PATH W/ THE TAKES SHORTEST

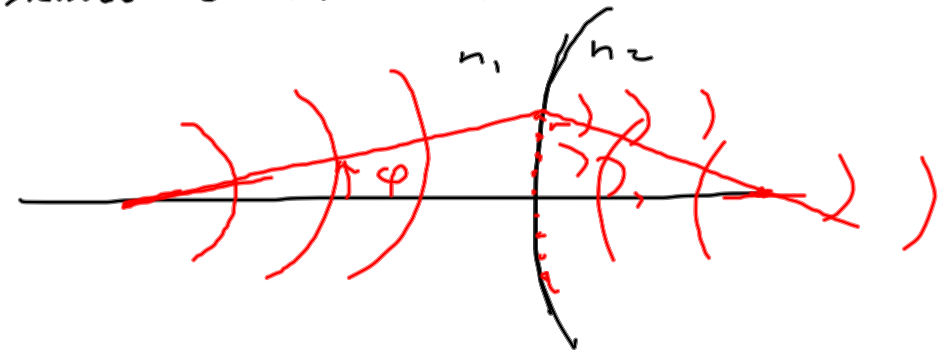
OPTICAL PATH LENGTH $\phi = \frac{2\pi}{\lambda'} z = \frac{2\pi}{\lambda/n} z$

$$\phi = \frac{2\pi\,(n z)}{\lambda_0}$$

OPTICAL PATH LENGTH = $n z$

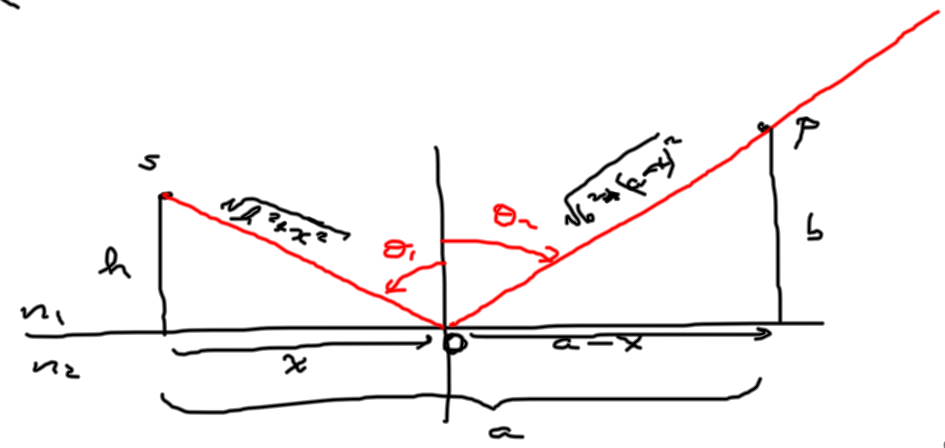
OPL = DISTANCE LIGHT TRAVELS IN VACUUM IN SAME TIME AS WOULD TRAVEL $z$ IN INDEX $n$

OPL > $z$




$$\frac{d\,\mathrm{OPL}}{d\varphi} = 0$$

# FERMAT'S PRINCIPLE AND REFLECTION
(SAME INDEX)

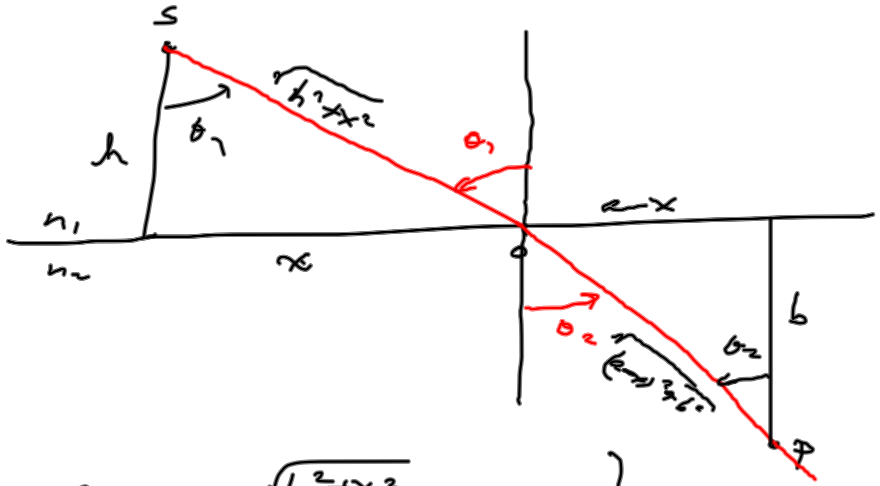



$$\left.\begin{array}{l}\theta_1 > 0 \\ \theta_2 < 0\end{array}\right\}\text{ AS DRAWN}$$

$$\left.\begin{array}{l} l_1 = n_1\sqrt{h^2+x^2} \\ l_2 = n_1\sqrt{b^2+(a-x)^2} \end{array}\right\} \quad OPL = l_1 + l_2$$

$$\frac{d\,OPL}{d\theta_1} = 0 = \frac{d\,OPL}{dx}\cdot\frac{dx}{d\theta}$$

$$\Rightarrow \sin\theta_1 = \sin(-\theta_2) \Leftrightarrow \theta_2 = -\theta_1$$




$$l_1 = n_1\sqrt{h^2+x^2}$$

$$l_2 = n_2\sqrt{b^2+(a-x)^2}$$

$$OPL = l_1 + l_2$$

$$= n_1\sqrt{h^2+x^2} + n_2\sqrt{b^2+(a-x)^2}$$

$$\frac{d}{dx}OPL = n_1\frac{1}{2\sqrt{h^2+x^2}}\cdot 2x + n_2\frac{1\cdot 2\cdot(a-x)\cdot(-1)}{2\sqrt{b^2+(a-x)^2}}$$

$$= n_1\frac{x}{\sqrt{h^2+x^2}} - n_2\frac{a-x}{\sqrt{b^2+(a-x)^2}} =$$

$$\sin\theta_1 - n_2\sin\theta_2 = 0$$

$$\frac{d}{dx}(OPL) = 0 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

SNELL'S LAW FOR REFRACTION

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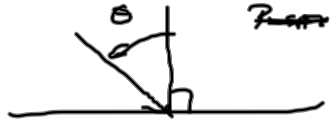
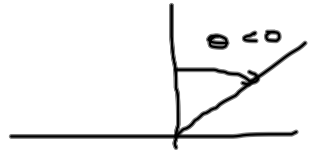
Re SIGN CONVENTIONS




~~Refl.~~ POSITIVE



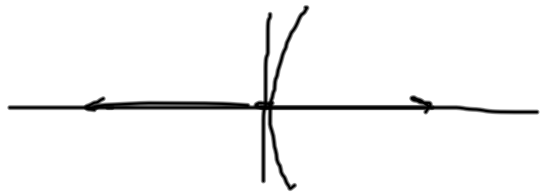

DISTANCE

(DIRECTED DISTANCE)

$d > 0$ $\longrightarrow$

$d < 0$ $\longleftarrow$

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}}$$



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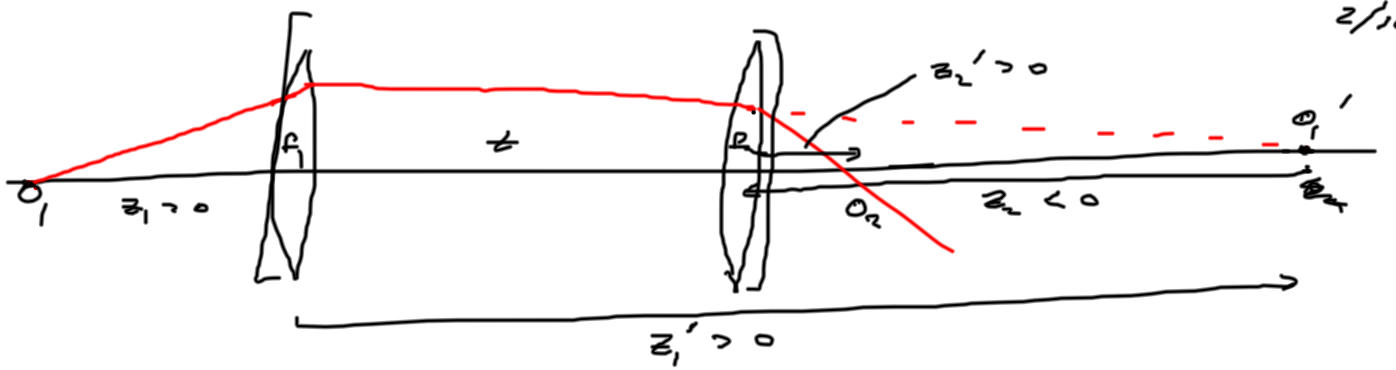


$$\boxed{\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f_{eff}}}$$

effective object Dist — From object to object-space principal pt

eff image Dist — From $H_2$ to $O_2$

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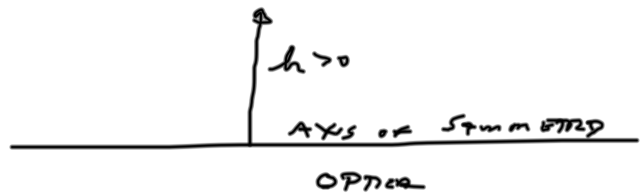
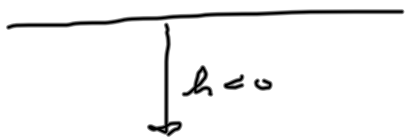
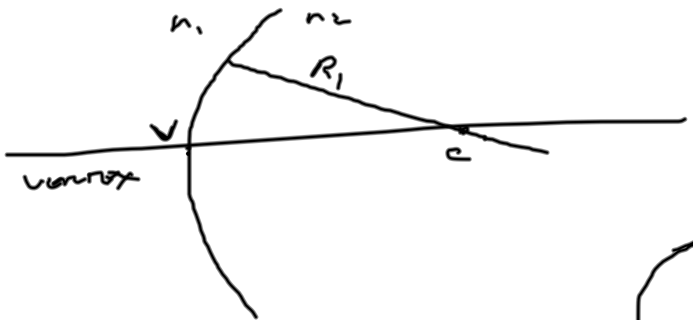
## HEIGHT

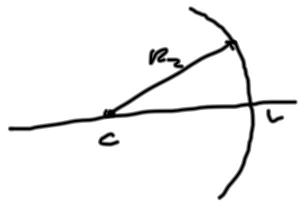



## RADII OF CURVATURE




$\overline{VC} = R_1 > 0$




$R_2 < 0$

"QUALITY" OF OBJECT & IMAGE

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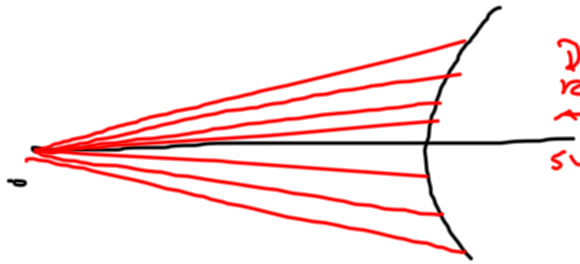
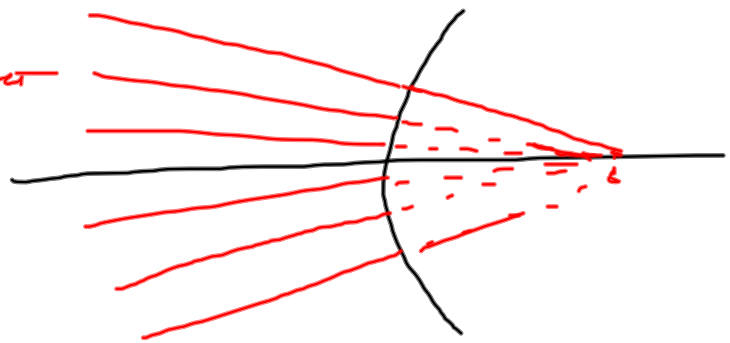
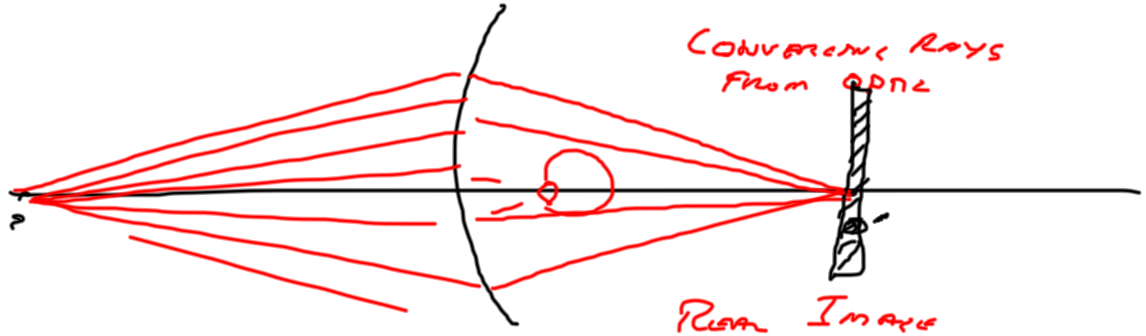
REAL, & VIRTUAL

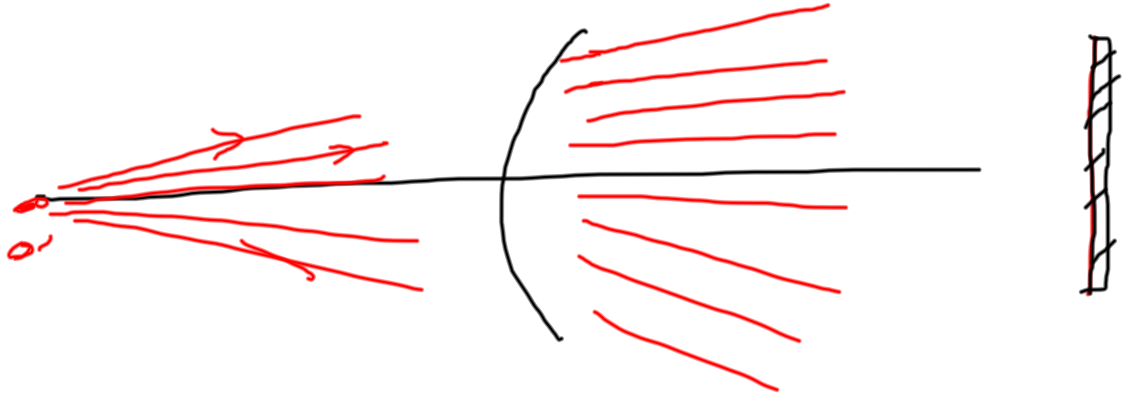
REAL OBJECT



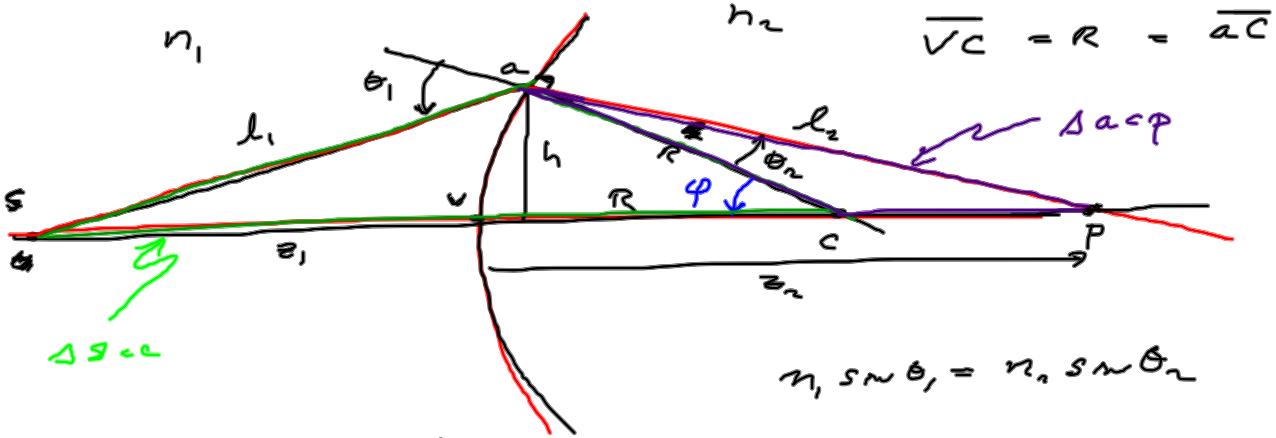

VIRTUAL OBJECT




CONVERGING RAYS
FROM OPTIC
O
O'
REAL IMAGE


O'
VIRTUAL
IMAGE

# REFRACTION FROM SINGLE SURFACE




$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$OPL = n_1 l_1 + n_2 l_2$$

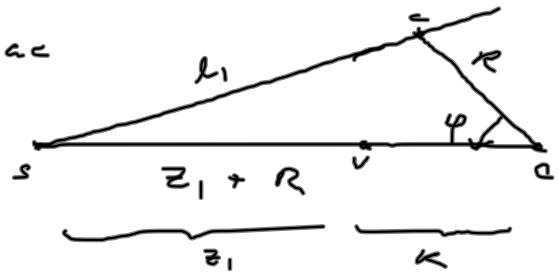
$$\frac{d}{d\varphi}(OPL) \Rightarrow (1)\ f\left[z_1, l_1, n_1, z_2, l_2, n_2\right]$$

(2) IF $\underline{\varphi \gtrsim 0}$, $z_1 \simeq l_1$, $z_2 \simeq l_2 \Rightarrow$ PARAXIAL

— $\Delta sac$

$\sin\varphi \simeq \tan\varphi \simeq \varphi \Rightarrow$ LINEAR EXPRESSION

$\Delta$ sac




$$l_1^{2} = R^2 + (z_1+R)^2 - 2R(z_1+R)\cos\varphi$$

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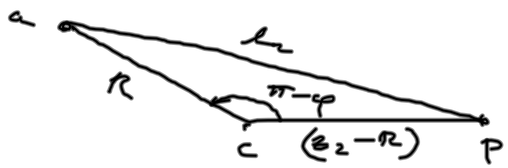
$\Delta$ acp




$$l_2^{2} = R^2 + (z_2-R)^2 - 2R(z_2-R)\cos(\pi-\varphi)$$

$$\cos(\pi-\varphi) = \cos\pi\cos\varphi + \cancelto{0}{\sin\pi}\sin\varphi$$
$$= -\cos\varphi$$

$$l_2^{2} = R^2 + (z_2-R)^2 + 2R(z_2-R)\cos\varphi \Rightarrow l_2 = \sqrt{R^2+(z_2-R)^2+2R(z_2-R)\cos\varphi}$$

$$l_1^{2} = R^2 + (z_1+R)^2 - 2R(z_1+R)\cos\varphi \qquad l_1 = \sqrt{(\qquad)}$$

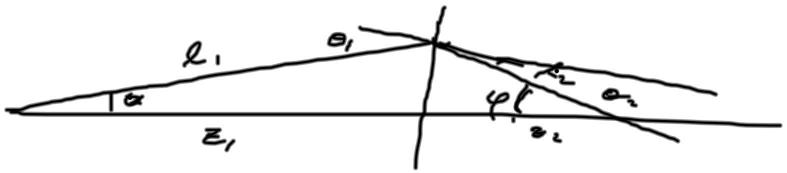
$$OPL = n_1 l_1 + n_2 l_2$$

$$\frac{d\,OPL}{d\varphi} = n_1 \frac{1}{2\sqrt{\quad}} \, +2R(z_1+R)\sin\varphi \; + n_2 \frac{1}{\sqrt{(\quad)}} \, -2R(z_2-R)\sin\varphi$$

2/11 – ③

$$\frac{n_1 (R + z_1)}{l_1} + \frac{n_2 (R - z_2)}{l_2} = 0$$

$$\frac{n_1}{l_1} + \frac{n_2}{l_2} = \frac{1}{R}\left( \frac{n_2 z_2}{l_2} - \frac{n_1 z_1}{l_1} \right)$$




$$\frac{l_2}{z_2} = \frac{1}{\cos\varphi}$$

IF $\varphi \gtrsim 0$, $\cos\varphi \simeq 1 - \frac{\varphi^2}{2} \ldots$

$\sin\varphi \sim \varphi$

$$\left.\begin{aligned} l_2 &\sim z_2 \\ l_1 &\sim z_1 \end{aligned}\right\} \Longrightarrow \boxed{\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{1}{R}(n_2 - n_1)}$$

PARAXIAL APPROX FOR REFRACTION FROM SINGLE SURF

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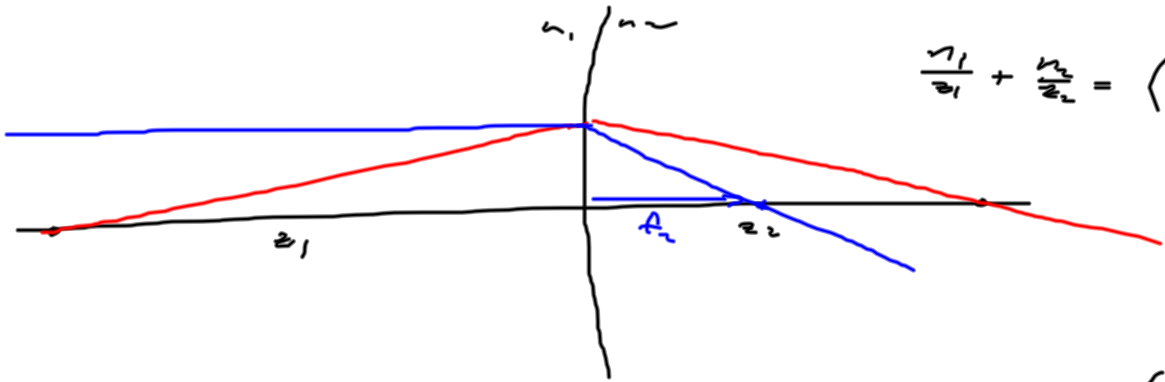
$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \left(\frac{n_2 - n_1}{R}\right)$$




(1) $z_1 = \infty$ $\quad \frac{n_1}{\infty} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R_1} \Rightarrow z_2 = R_1 \cdot \left(\frac{n_2}{n_2 - n_1}\right)$

$$f_2 = \left(\frac{n_2}{n_2 - n_1}\right) R$$

(2) $z_2 = \infty$ $\quad \frac{n_1}{z_1} + 0 = \frac{n_2 - n_1}{R_1} \Rightarrow z_f = f_1 = R_1 \left(\frac{n_1}{n_2 - n_1}\right)$

$$\boxed{\frac{f_1}{f_2} = \frac{n_1}{n_2}}$$

TWO SURFACES

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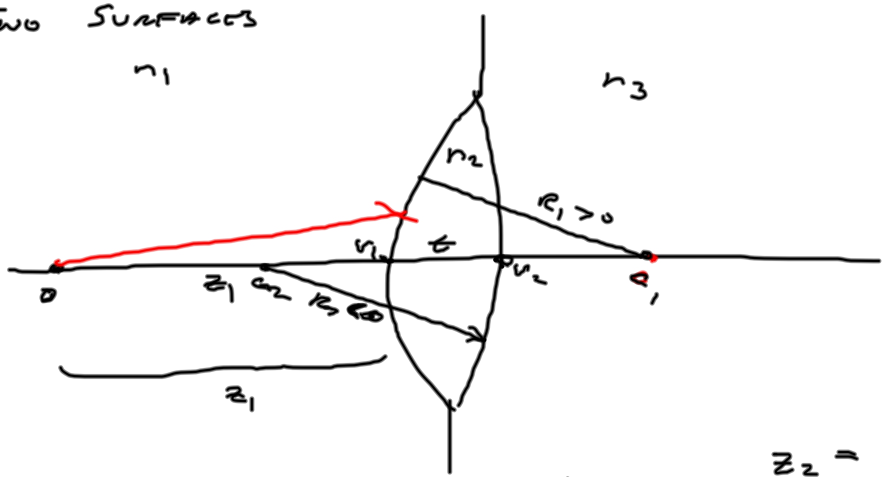


(1) $$\frac{n_1}{z_1} + \frac{n_2}{z_1'} = \frac{n_2 - n_1}{R_1}$$

(2) $$\frac{n_2}{z_2} + \frac{n_3}{z_2'} = \frac{n_3 - n_2}{R_2}$$

$$t = 0 \Rightarrow z_1' = -z_2$$

$$z_2 = t - z_1'$$

$z_2 = -z_1'$ IF $t = 0$ (THIN LENS)

$$z_1' = t - z_2$$

# THIN LENS

$$\frac{n_1}{z_1} + \frac{n_2}{-z_2} = \frac{n_2 - n_1}{R_1}$$

$$\frac{n_2}{z_2} + \frac{n_3}{z_2'} = \frac{n_3 - n_2}{R_2}$$

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$$\frac{n_1}{\underset{\substack{\uparrow \\ \text{OBJECT}}}{z_1}} + \frac{n_3}{\underset{\substack{\uparrow \\ \text{IMAGE}}}{z_2'}} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\frac{n_1}{z_1} + \frac{n_3}{z_2'} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

IN AIR $n_1 = n_3 = 1$

$$\frac{1}{z_1} + \frac{1}{z_2'} = -\frac{1}{R_1} + \frac{1}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= -\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

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THIN LENS, 3 MEDIA

$$\frac{n_1}{z_1} + \frac{n_3}{z_2'} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

IN AIR

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2'} = (n_2-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

LENSMAKER'S EQUATION (FOR THIN LENS)

$z_2' \equiv z_2$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

FROM DIFFRACTION

$$\boxed{\frac{1}{f} = (n_2-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

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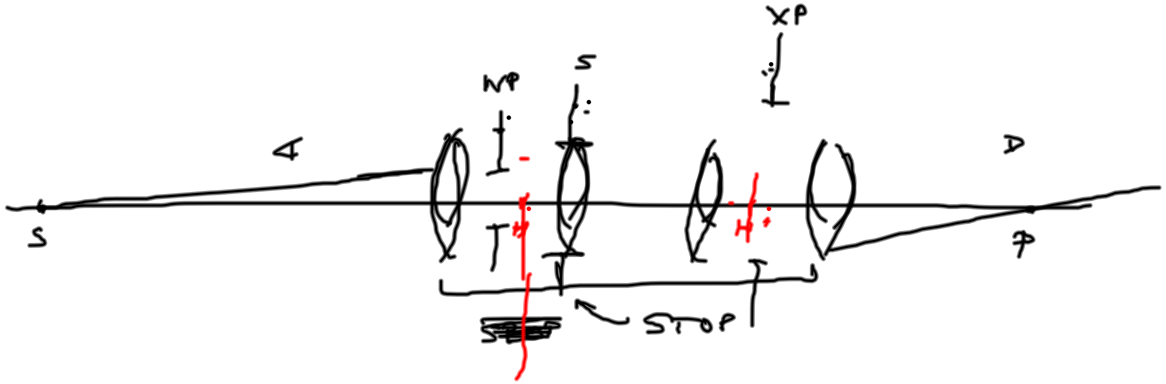
$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$


CHIEF RAY — EDGE OF OBJECT → CENTER OF LENS (PUPIL) → EDGE OF IMAGE

PARAXIAL RAY — CENTER OF OBJECT → INTERSECT LENS → CENTER OF IMAGE

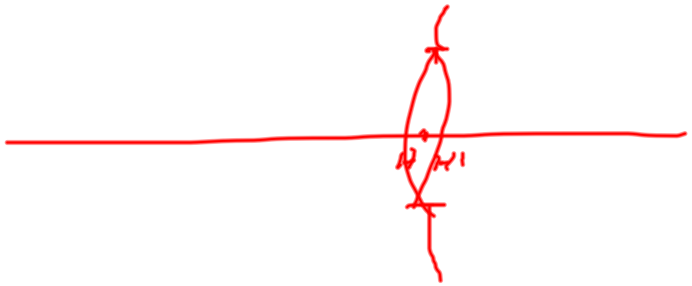
MARGINAL RAY — SCALED REPLICA OF PARAXIAL
EDGE OF APERTURE STOP ⟺ ENTRANCE PUPIL
⟺ EXIT PUPIL

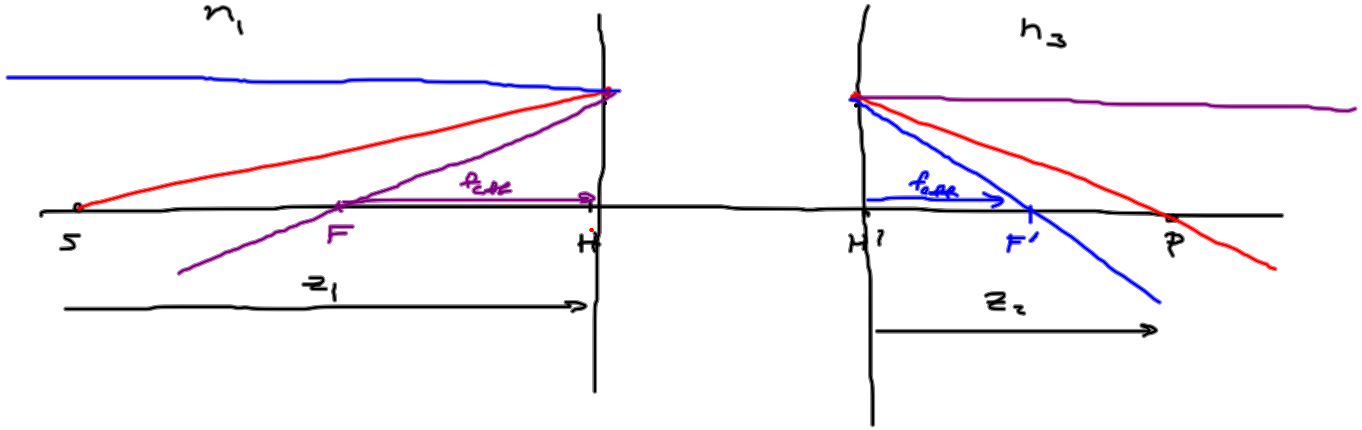
$$\boxed{\frac{h'}{h} = -\frac{z_2}{z_1}} = M_T \qquad M_\theta$$




OBJECT SPACE PRINCIPAL PLANE

H' IS IMAGE OF H WITH $m_T = +1$






$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f_{eff}}$$