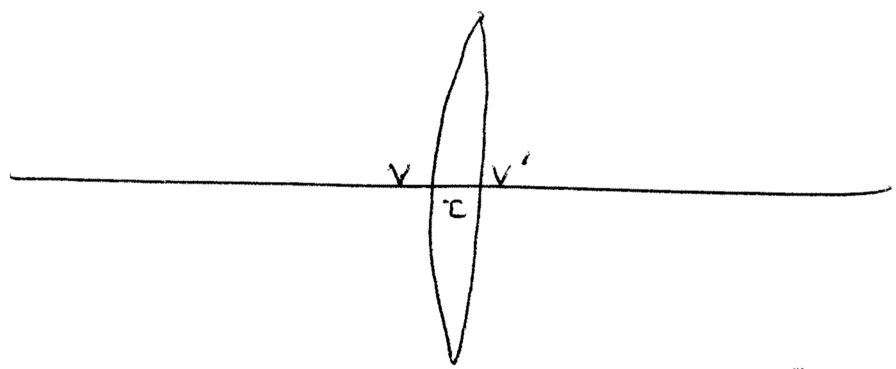
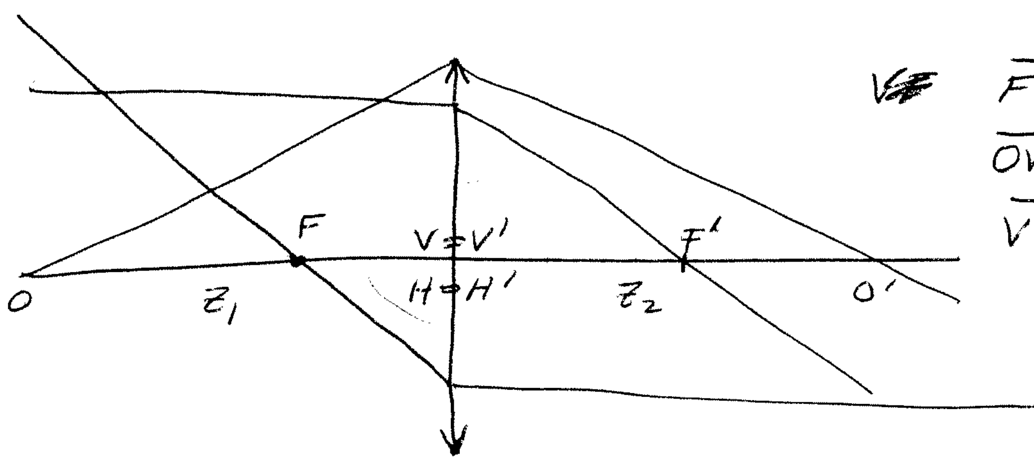
15 FEBRUARY 2010

RAY OPTICS THIN LENS FORMULA



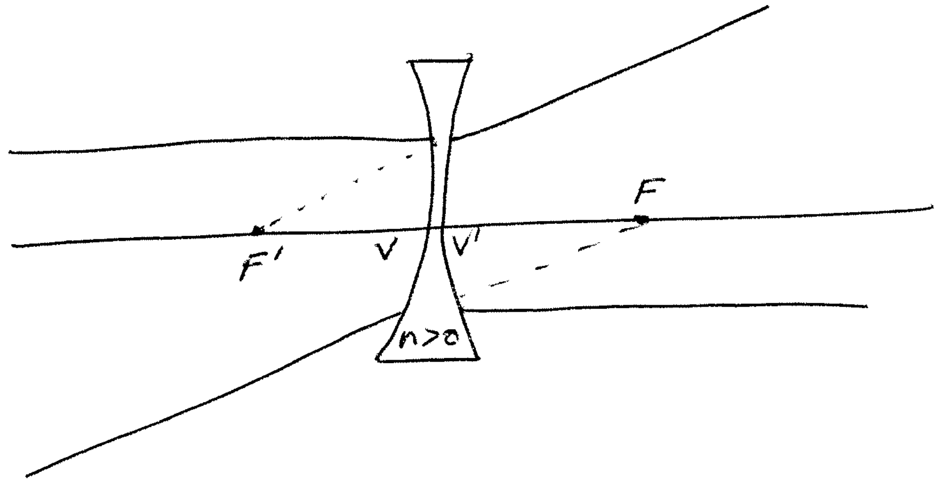
OBJECT SPACE IMAGE SPACE



$$\overline{FV} = \overline{V'F'} = f = f' > 0$$

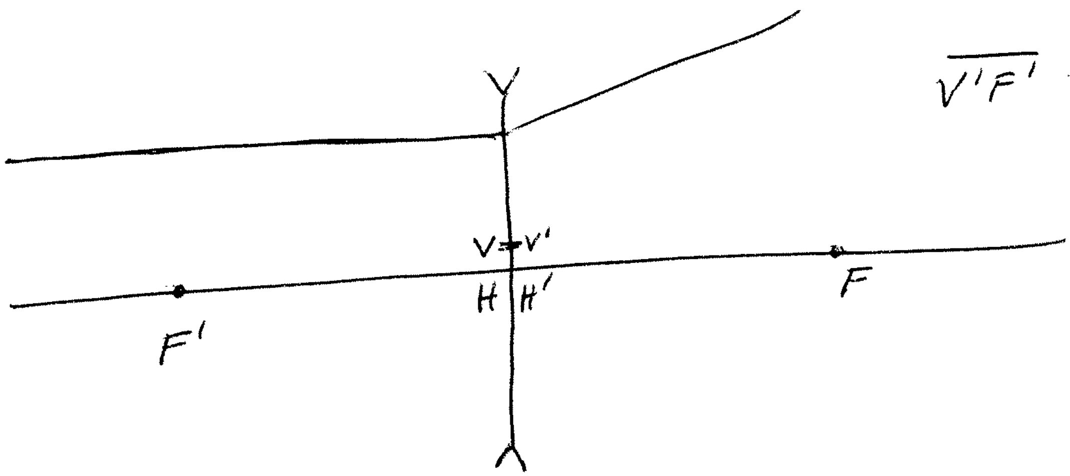
$$\overline{OV} = \overline{OH} = z_1$$

$$\overline{V'O'} = \overline{H'O'} = z_2$$



$\overline{V'F'} < 0 \qquad f < 0$

$$\frac{1}{f[m]} \equiv \varphi \; [\text{DIOPTERS}] \; [m^{-1}]$$

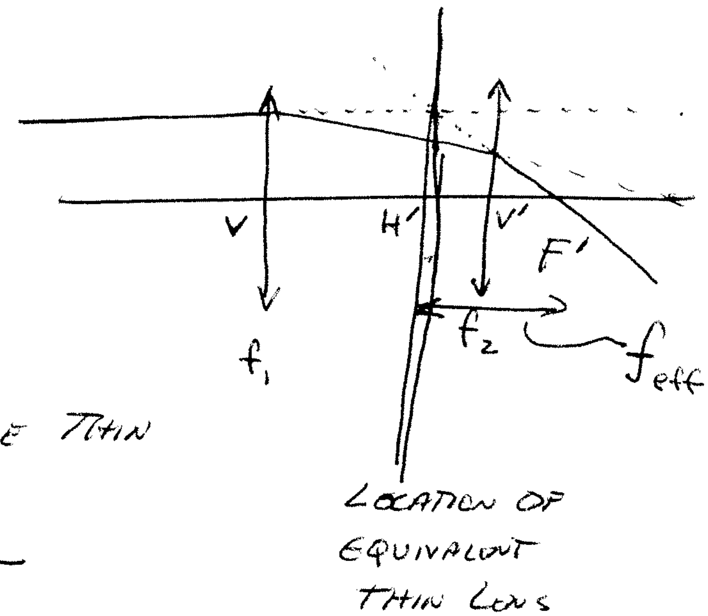


$\overline{V'F'} = \overline{FV} < 0$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

(a) TWO THIN LENSES $\Longleftrightarrow$ (b) THICK LENS

(a) $$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} \right)^{-1}$$



H' = LOCATION OF EQUIVALENT SINGLE THIN LENS ON IMAGE SIDE

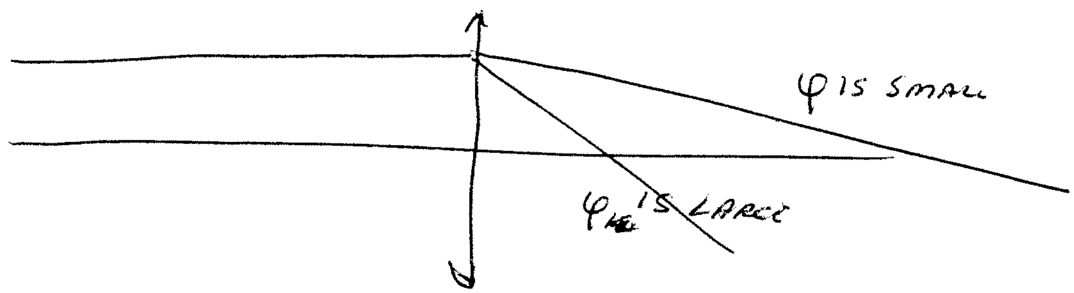
IMAGE-SPACE PRINCIPAL POINT

(b) $$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t/n}{f_1 f_2} \right)^{-1}$$

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} \iff \varphi_{eff} = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 t$$

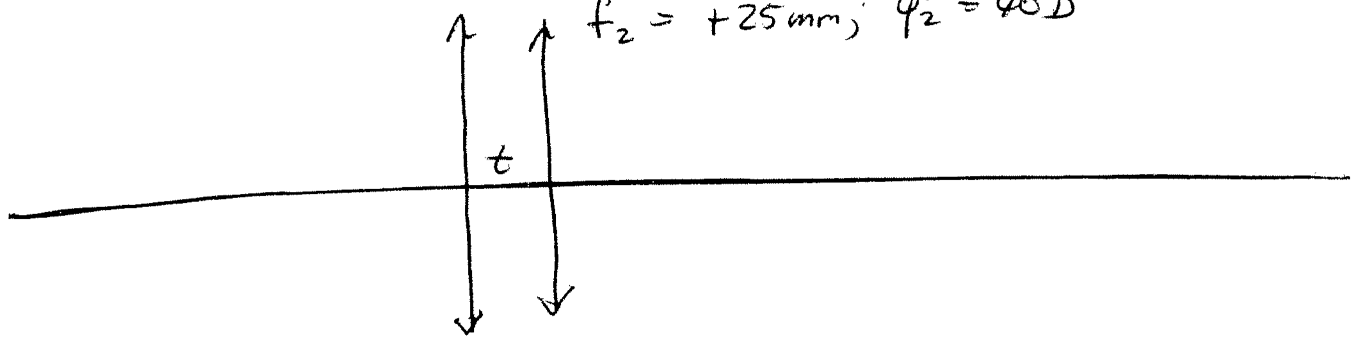
DIOPTERS

IF $f \downarrow$, $\varphi \uparrow$



$f_1 = +100\ mm$, $\varphi_1 = 10D$

$f_2 = +25\ mm$; $\varphi_2 = 40D$



t	φ	f_{eff}
0	50D	20 mm

	t	φ	f
	0	50	20 mm
	10 mm	46	21.74 mm
f_2 -	25 mm	$40D = \varphi_2$	25 mm $= f_2$
	50 mm	30 D	$\frac{100}{3}$ mm $= 33\frac{1}{3}$
	75 mm	20D	50 mm
f_1 =	100 mm	10D	100 mm $= f_1$
	125 mm	0D	∞
	150 mm	$-10D$	-100 mm

$$\varphi = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 t$$

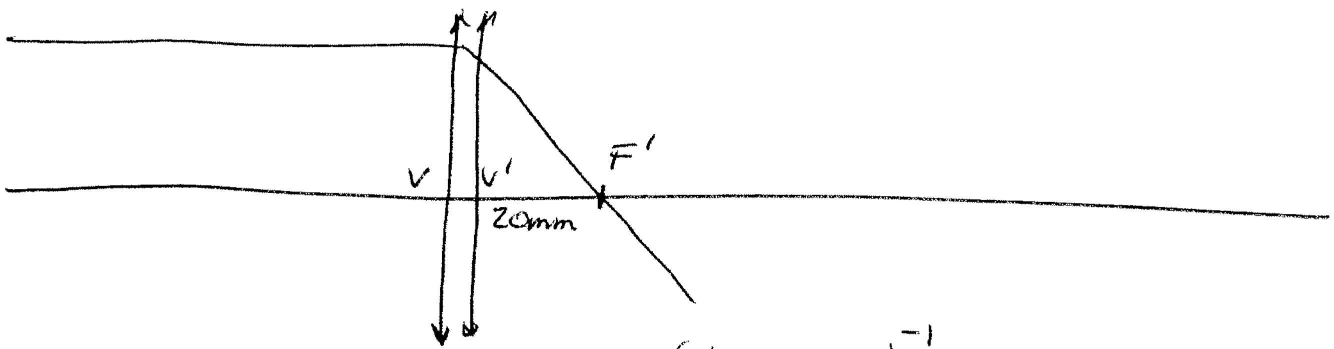
$$\varphi_1 = 10\,D$$

$$\varphi_2 = 40\,D$$

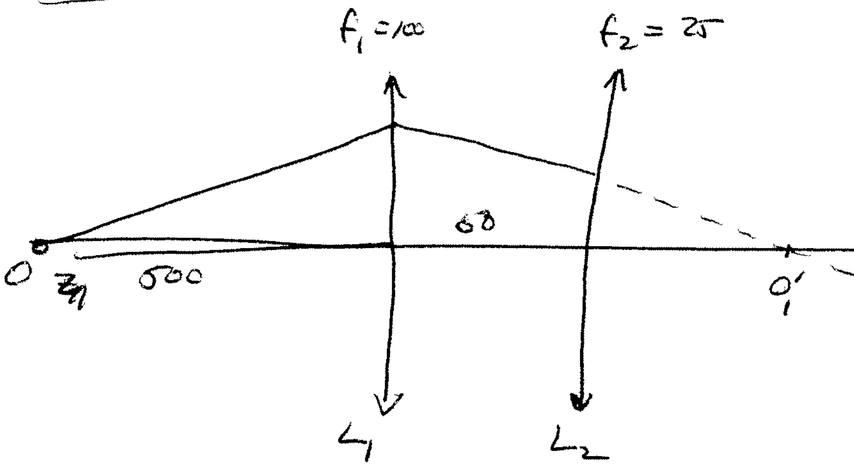
$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{f_1 + f_2}{f_1 f_2} = 0$$

$t=0$, $f_{eff} = 20\,mm$



Given z_1, f_{eff}, $\quad z_2 = \left(\frac{1}{f_{eff}} - \frac{1}{z_1}\right)^{-1}$



$f_{eff} = \frac{100}{3}\,mm$

$(M_T)_1 = -\frac{z_1'}{z_1} = -\frac{125}{500} = -\frac{1}{4}$

LOCATION + MAGNIFICATION

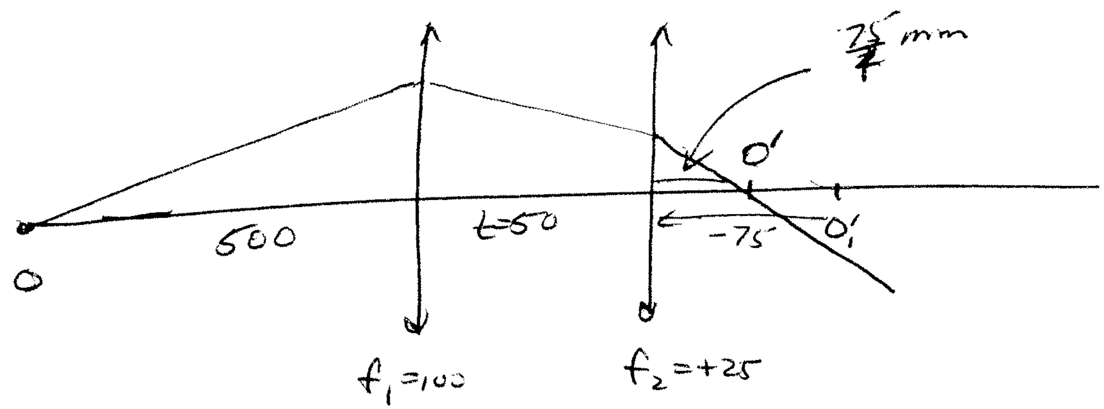
$\overline{OV} = 500\,mm, = z_1$

$f_1 = 100\,mm$

$z_1' = \left(\frac{1}{f_1} - \frac{1}{z_1}\right)^{-1}$

$= \left(\frac{1}{100} - \frac{1}{500}\right)^{-1} = \frac{500}{4} = 125\,m$

2/15 - 7



$$\overline{V_1 O_1'} = 125$$

$$\overline{O_1' V'} = t - \overline{V_1 O_1'} = 50 - 125 = -75 \text{ mm} = z_2$$

$$\frac{1}{z_2} + \frac{1}{z_2'} = \frac{1}{f_2}$$

$$\frac{1}{-75} + \frac{1}{z_2'} = \frac{1}{25} \implies z_2' = \left(\frac{1}{25} + \frac{1}{75}\right)^{-1} = \frac{75}{4} \text{ mm}$$

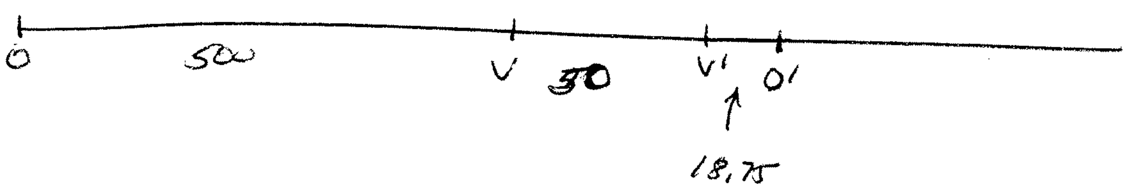
$$= \cancel{37.5 \text{ mm}}$$

$$= 18.75 \text{ m}$$

$$(M_T)_2 = -\frac{+\frac{75}{4}}{-75} = +\frac{1}{4}$$

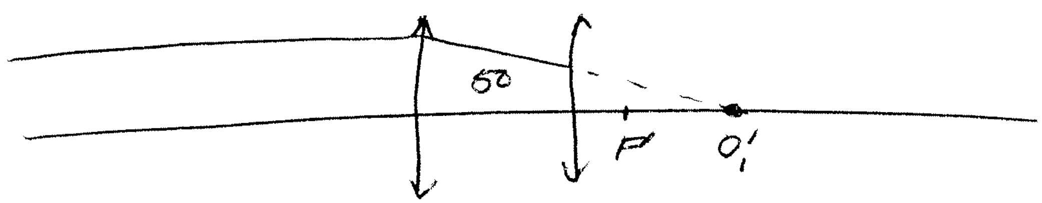
$$M_T = (M_T)_1 (M_T)_2 = -\frac{1}{4} \cdot +\frac{1}{4} = -1/16$$

$$f_{eff} = \frac{100}{3}\,mm$$



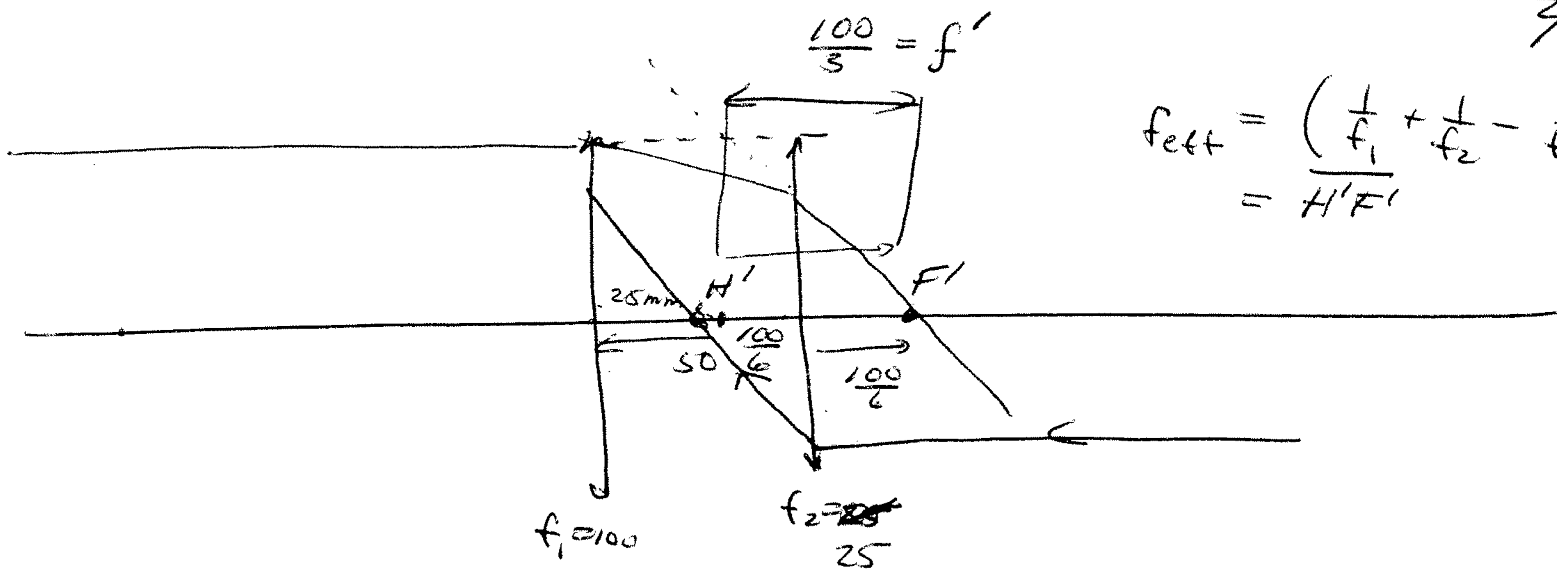
$$\frac{1}{z} + \frac{1}{z'} = \frac{1}{f_{eff}}$$

$z_1 = \infty \Rightarrow z_2' \rightarrow F'$



$$\frac{1}{z_1} + \frac{1}{z_1'} = \frac{1}{100} \Rightarrow z_1' = 100, \quad z_2 = t - z_1' = -50\,mm$$

$$\frac{1}{-50} + \frac{1}{z_2'} = \frac{1}{25} \Rightarrow \frac{1}{z_2'} = \frac{1}{25} + \frac{1}{50} \Rightarrow z_2' = \frac{50}{3} = \frac{100}{6} = 16.\tfrac{2}{3}$$



$$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}\right)^{-1} = \overline{H'F'}$$

$$\frac{1}{z_1} + \frac{1}{z_1'} = \frac{1}{f_1}$$

$$\frac{1}{z_2} + \frac{1}{z_2'} = \frac{1}{f_2} \qquad z_2 = t - z_1'$$

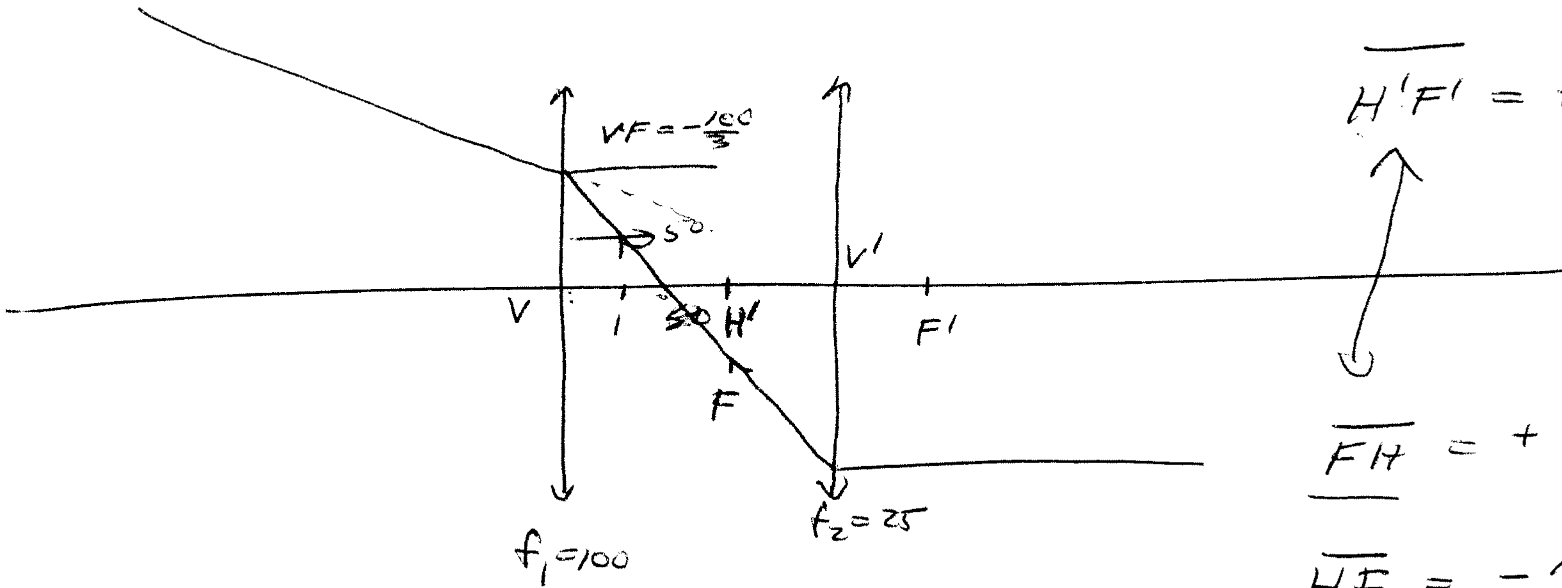
FIND F & H (FROM F) $\qquad \overline{FH} = f = \frac{100}{3}$

$z_2 = +25$

$f_2 = +100$

$$z_2' = \left(\frac{1}{f_2} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{100} - \frac{1}{25}\right)^{-1}$$

$$\left(\frac{1}{100} - \frac{4}{100}\right)^{-1} = -\frac{100}{3} \qquad -\frac{100}{3}$$



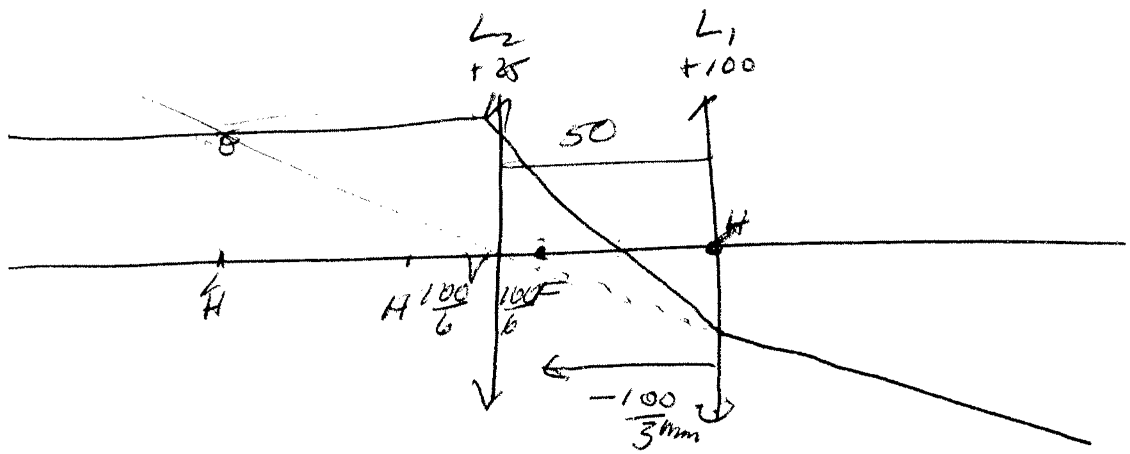
$$\overline{H'F'} = +\frac{100}{3}$$

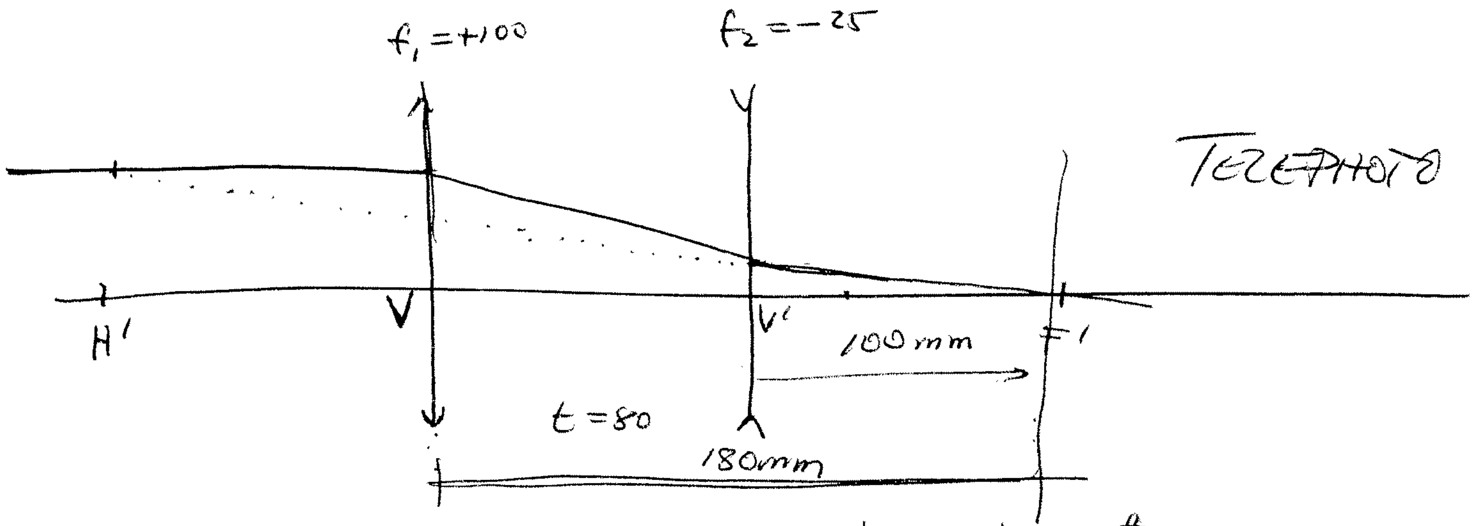
$$\overline{FH} = +\frac{100}{3}$$

$$\overline{HF} = -\frac{100}{3}$$

VF

$\cancel{F}' = -\frac{100}{3}$ mm





TELEPHOTO

$$t \gtrsim f_1 + f_2$$

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

$$= \frac{1}{100} - \frac{1}{25} + \frac{80}{100 \cdot 25}$$

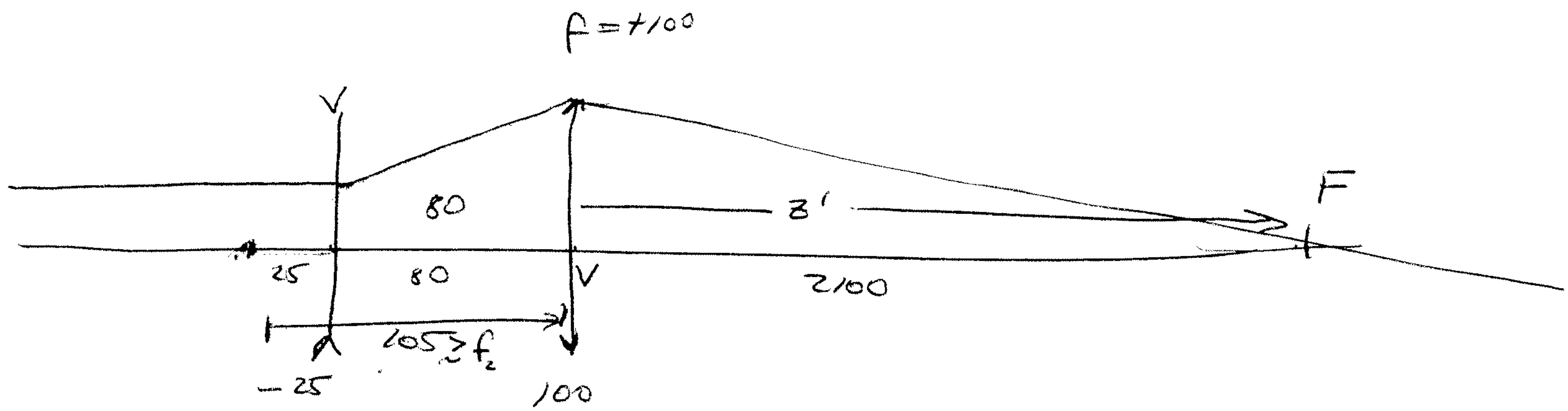
$$= -\frac{3}{100} + \frac{8}{250} = -\frac{15}{500} + \frac{16}{500} = \frac{1}{500}$$

$$f_{eff} = 500\text{ mm} = \underline{\overline{H'F'}} = \underline{\overline{FH}}$$

For L_2, $z_2 = t - z_1' = 80 - 100 = -20$ mm

$f_2 = -25$ mm $\Rightarrow z_2' = \overline{V'F'} = \left(\frac{1}{f_2} - \frac{1}{z_2}\right)^{-1}$

$$= \left(\frac{1}{-25} - \frac{1}{-20}\right)^{-1} = \left(\frac{5}{100} - \frac{4}{100}\right)^{-1} = 100$$

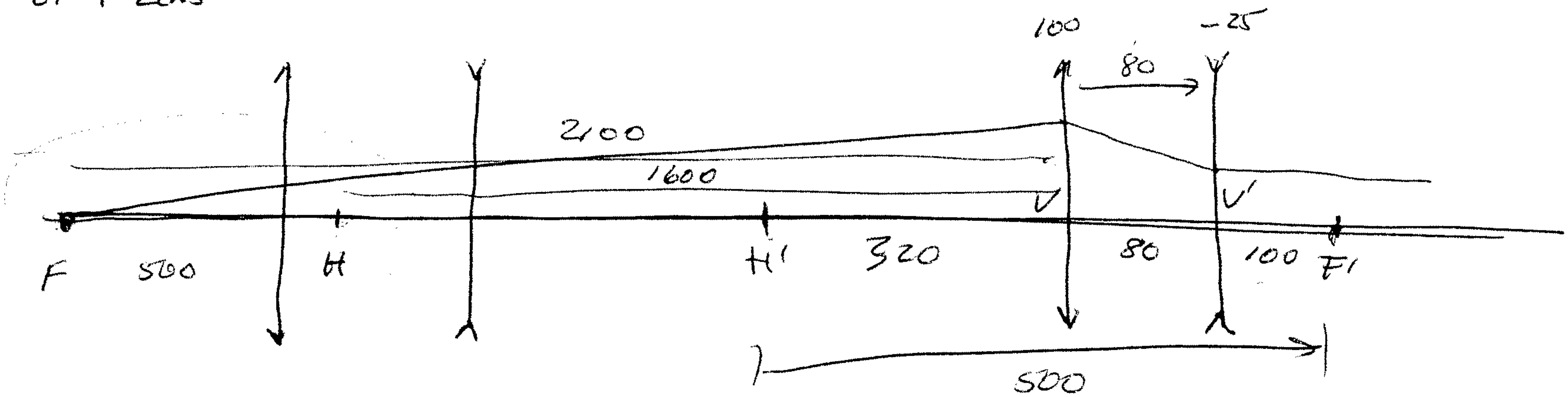


OBJ DIS FOR + LENS $z = 105$

FOCAL LENGTH OF + LENS $f = 100$

$$\left(\frac{1}{f} - \frac{1}{z}\right)^{-1} = z' = \left(\frac{1}{100} - \frac{1}{105}\right)^{-1}$$

$$= \underline{\underline{2100 \text{ mm}}}$$



$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f_{eff}}$$

$z_1 = \overline{OH}$

$z_1 = \overline{OH}$

$z_2 = \overline{H'O'}$

$z_1 = 2f_{eff} = 1000\ mm$ } EQUAL CONJUGATES
$z_2 = 1000\ mm = 2f_{eff}$

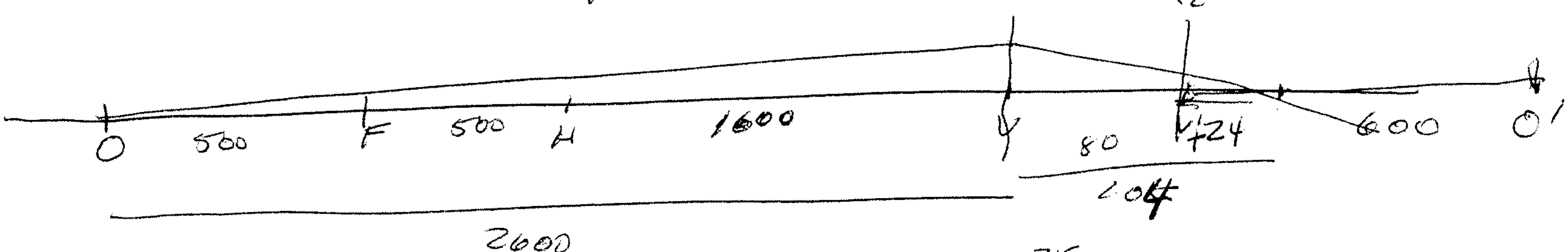
$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f} \qquad m_T = -\frac{2f}{2f} = -1$$

$z_1 = 2f = z_2$

$\overline{FH} = 500 \qquad \overline{HV} = 1600$

$\overline{OV} = \cancel{\overline{OH}} \qquad \overline{OH} = 1000 \qquad \overline{OV} = 2600$

TO 1ST LENS, $z = 2600$, $f_1 = 100$ } $z' = \left(\frac{1}{100} - \frac{1}{2100}\right)^{-1} = 105$

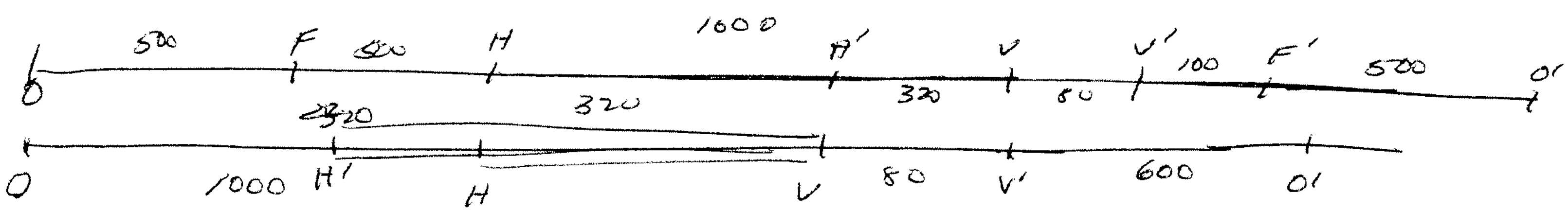


$$\frac{1}{-24} + \frac{1}{z'} = \frac{1}{-25} \qquad z' = +600 = \overline{V'O'} \quad -25$$

$-\frac{104}{2600} = m_1 = -0.04$

$-\frac{600}{-24} = +25 \qquad (m) = -1$

m



$\overline{OH} = 1000 = 2f$

$\overline{H'V} + \overline{VV'} + \overline{V'O'}$

$320 + 80 + 600 = 1000 = \overline{H'O'} = 2f$

$$\frac{1}{\overline{OH}} + \frac{1}{\overline{H'O'}} = \frac{1}{f_{eff}} \qquad M_T = -\frac{z'}{z} = -\frac{\overline{H'O'}}{\overline{OH}}$$

z $\qquad$ z'

IMAGE

F, F', H, H' CARDINAL POINTS

N, N'

$$\frac{f_{eff}}{D_{UP}} = f/\#$$

0.1 Example of Two-Lens System

$$\mathbf{f}_1 = +100\,\text{mm}$$
$$\mathbf{f}_2 = +25\,\text{mm}$$
$$t = +50\,\text{mm}$$

$$\begin{aligned}
\frac{1}{\mathbf{f}_{\text{eff}}} &= \frac{1}{\mathbf{f}_1} + \frac{1}{\mathbf{f}_2} - \frac{t}{\mathbf{f}_1\mathbf{f}_2} \\
&= \left(\frac{1}{100\,\text{mm}} + \frac{1}{+25\,\text{mm}} - \frac{50\,\text{mm}}{(+100\,\text{mm})\,(+25\,\text{mm})}\right)^{-1} \\
\mathbf{f}_{\text{eff}} &= \frac{100}{3}\,\text{mm} = 33\frac{1}{3}\,\text{mm}
\end{aligned}$$

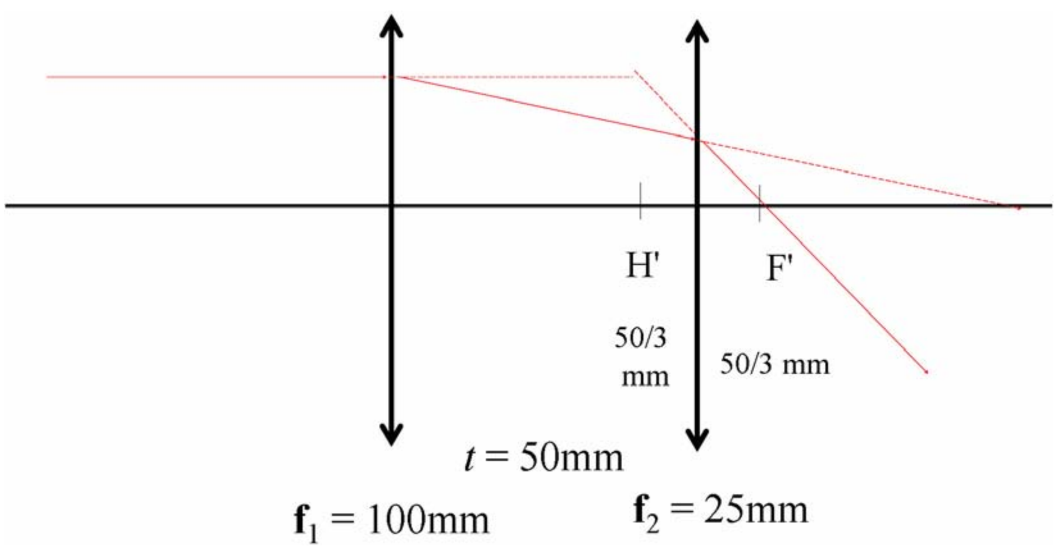
Now find the focal point by cascading through the system. For an object at ∞, the image distance from the first lens is $\mathbf{f}_1 = +100\,\text{mm}$, so the object distance to the second lens is

$$z_2 = t - \mathbf{f}_1 = 50\,\text{mm} - 100\,\text{mm} = -50\,\text{mm}$$

The image distance from the second lens is:

$$z_2' = \left(\frac{1}{\mathbf{f}_2} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{+25\,\text{mm}} - \frac{1}{-50\,\text{mm}}\right)^{-1} = \frac{50}{3}\,\text{mm}$$

which is half the focal length. We can now draw the image space focal and principal points:



Two positive lenses separated by distance $t = 50\,\text{mm}$

To find the image-space focal and principal point, we "turn the system around" and bring in light from the left. The image distance from the "first lens" (actually L_2) is equal to its focal length:

$$z_1' = \mathbf{f}_2 = +25\,\text{mm}$$

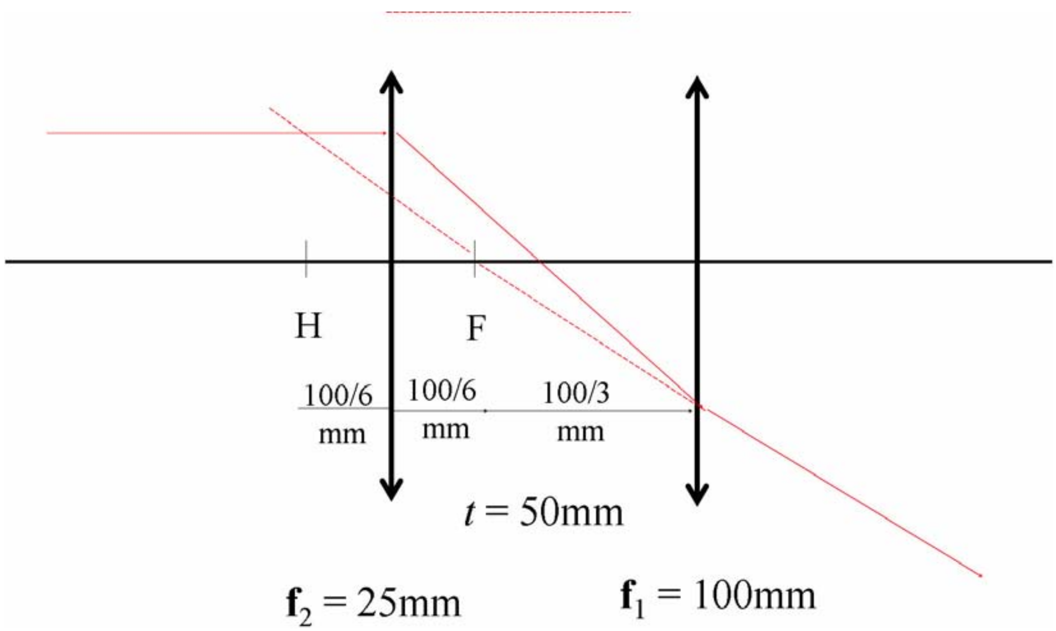
So the object distance to the lens with $\mathbf{f}_1 = +100\,\text{mm}$ is:

$$z_2 = t - z_1' = 50\,\text{mm} - 25\,\text{mm} = +25\,\text{mm}$$

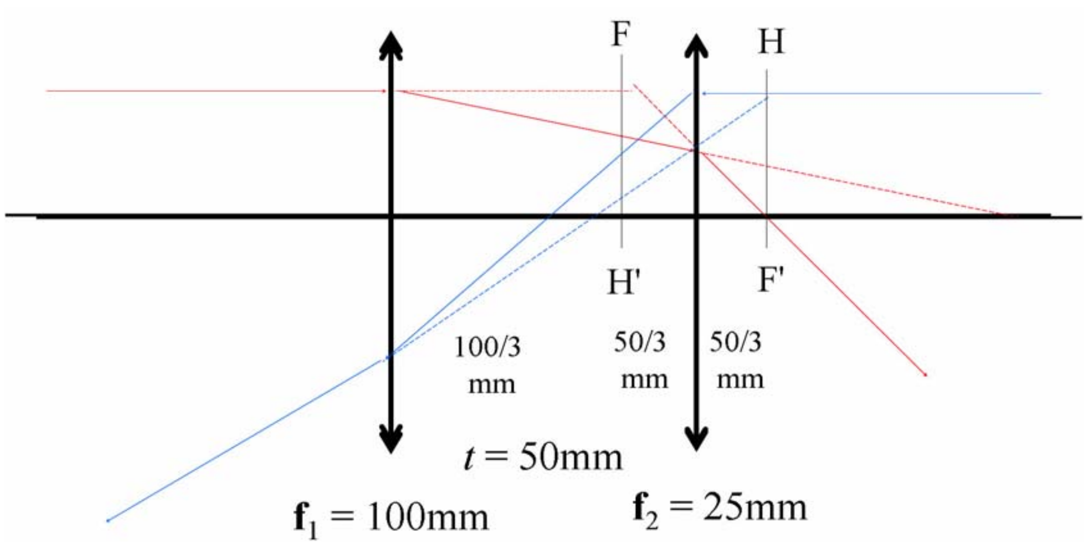
So the distance from this lens to the focal point is:

$$z_2' = \left(\frac{1}{\mathbf{f}_1} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{100\,\text{mm}} - \frac{1}{25\,\text{mm}}\right)^{-1} - -\frac{100}{3}\,\text{mm}$$

:The focal point is *virtual* and the object-space principal is located at the distance $\mathbf{f}_{\text{eff}}$ behind it in the reversed system.



We can now reverse the second case and plot F, F', H, H' on the same graph:



In this case, the object-space focal point F just happens to coincide with the image-space principal point H'; the same applies to the object-space principal point H and the image-space focal point F'. This is of no real significance, since the two spaces are independent.