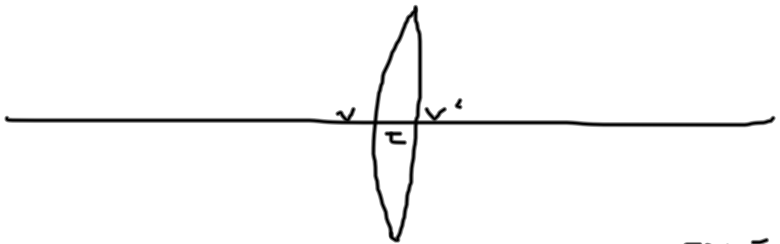
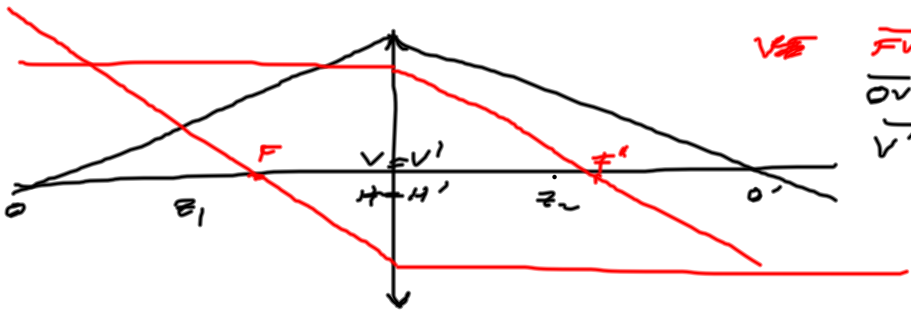
15 FEBRUARY 2010

RAY OPTICS THIN LENS FORMULA



OBJECT SPACE IMAGE SPACE

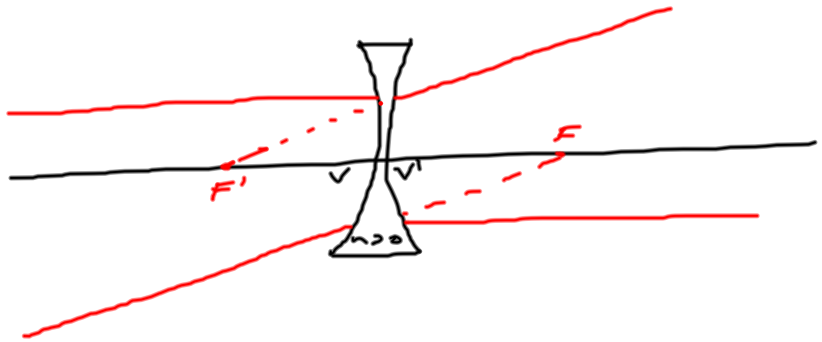


$\overline{FV} = \overline{V'F'} = f = f' > 0$

$\overline{OV} = \overline{OH} = z_1$

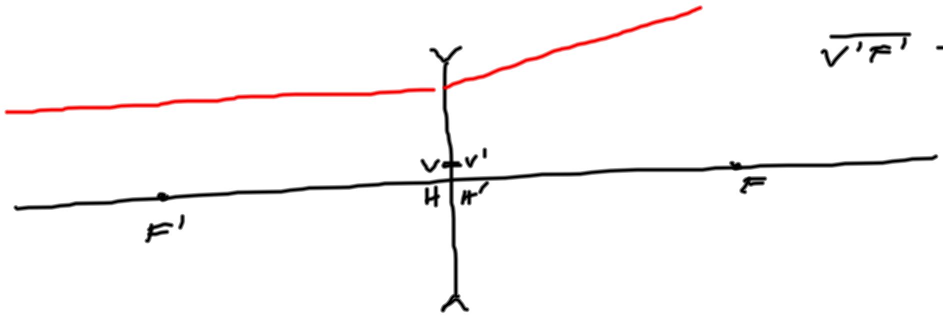
$\overline{V'O'} = \overline{H'O'} = z_2$

(2)



$\overline{V'F'} < 0 \quad f < 0$

$$\frac{1}{f[m]} = \varphi \text{ [Diopters] } [m^{-1}]$$

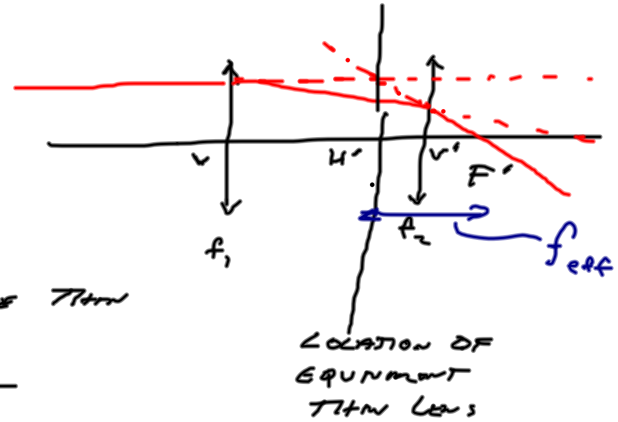


$\overline{V'F'} = \overline{FV} < 0$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

(a) TWO THIN LENSES $\Longleftrightarrow$ (b) THICK LENS

(a) $$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} \right)^{-1}$$



H' = LOCATION OF EQUIVALENT SINGLE THIN LENS ON IMAGE SIDE

IMAGE-SPACE PRINCIPAL POINT

(b) $$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t/n}{f_1 f_2} \right)^{-1}$$

$$\left(\quad \right)$$

2/15 - 4

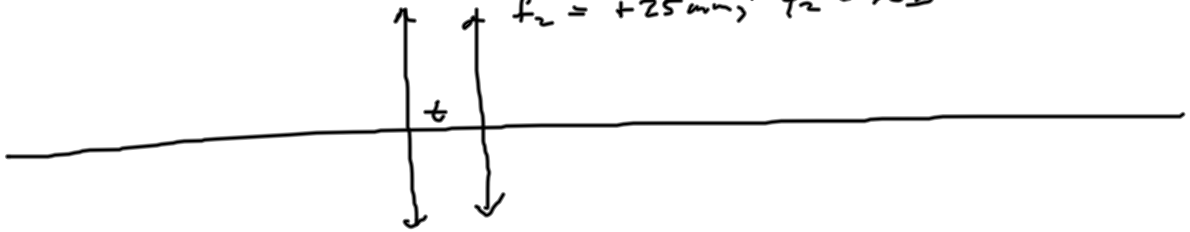
$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} \iff \varphi_{eff} = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 t$$

Diopters

if $f \downarrow$, $\varphi \uparrow$

$f_1 = +100$ mm, $\varphi_1 = 10$ D

$f_2 = +25$ mm; $\varphi_2 = 40$ D



t	φ	f_{eff}
0	50 D	20 mm

2/15 – 5

t	φ	f
0	50	20 mm
10 mm	46	21.74 mm
f_2 = 25 mm	40 D = φ_2	25 mm = f_2
50 mm	30 D	$\frac{100}{3}$ mm = 33 1/3
75 mm	20 D	50 mm
f_1 = 100 mm	10 D	100 mm = f_1
125 mm	0 D	∞
150 mm	–10 D	–100 mm

$$\varphi = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 t$$

$\varphi_1 = 10$ D

$\varphi_2 = 40$ D

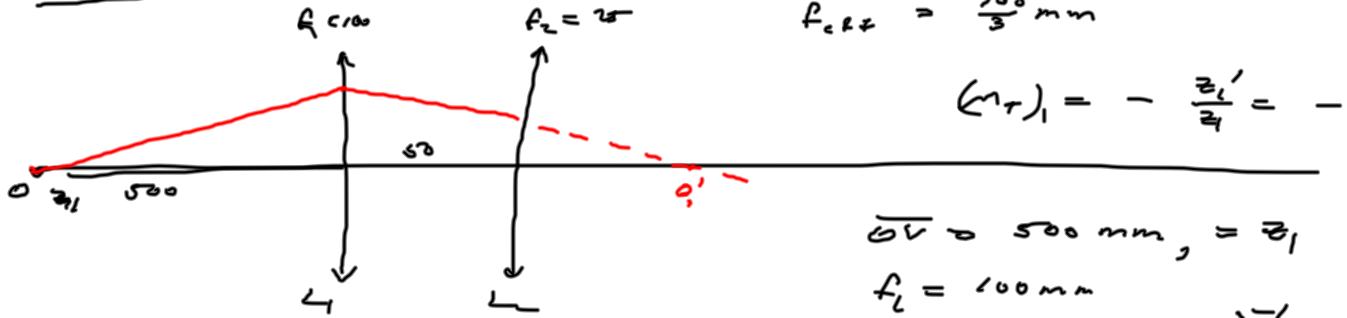
$$\frac{1}{f_{k+2}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{f_1 + f_2}{f_1 f_2} = 0$$

$t = 0$, $f_{eff} = 20m$ $\qquad$ z 15 -

Given z_1 , f_{eff} , $\quad z_2 = \left(\frac{1}{f_{eff}} - \frac{1}{z_1} \right)^{-1}$

$f_{eff} = \frac{100}{3}$ mm



LOCATION + MAGNIFICATION

$(m_T)_1 = - \frac{z_1'}{z_1} = - \frac{125}{500} = - \frac{1}{4}$

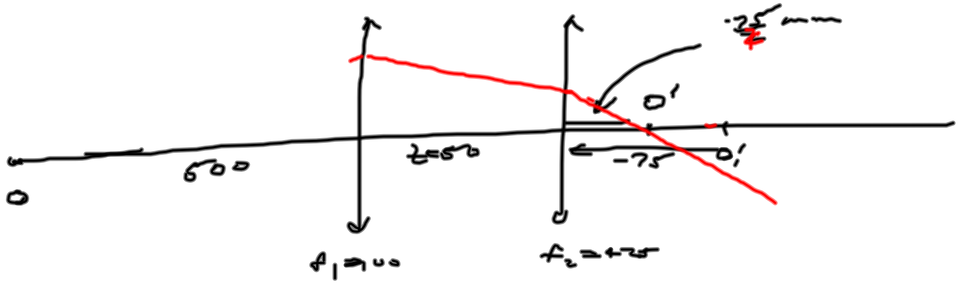
$\overline{OV} = 500$ mm, $= z_1$

$f_1 = 100$ mm

$z_1' = \left(\frac{1}{f_1} - \frac{1}{z_1} \right)^{-1}$

$= \left(\frac{1}{100} - \frac{1}{500} \right)^{-1} = \frac{500}{4} = 125$ mm

2/15 − 7



$\overline{V_1 O_1'} = 125$

$\overline{O_1' V'} = t - \overline{V_1 O_1'} = 50 - 125 = -75 \text{ mm} = z_2$

$$\frac{1}{z_2} + \frac{1}{z_2'} = \frac{1}{f_2}$$

$$\frac{1}{-75} + \frac{1}{z_2'} = \frac{1}{25} \Longrightarrow z_2' = \left(\frac{1}{25} + \frac{1}{75}\right)^{-1} = \frac{75}{4} \text{ mm}$$

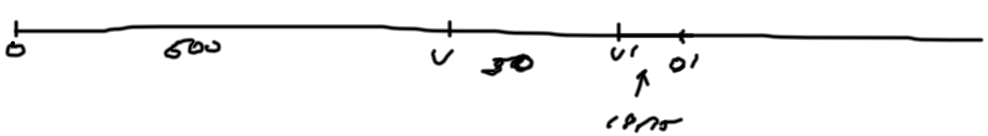
$= \cancel{37.5}$ mm

$= 18.75$ m

$$(M_T)_2 = -\frac{+\frac{75}{4}}{-75} = +\frac{1}{4}$$

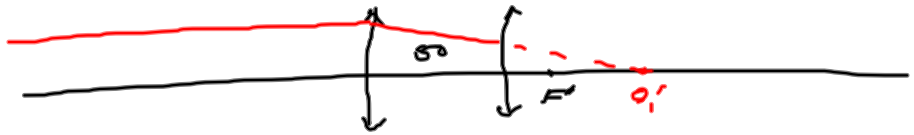
$$M_T = (M_T)_1 (M_T)_2 = -\frac{1}{4} \cdot +\frac{1}{4} = -1/16$$

$$f_{eff} = \frac{200}{3}\,mm$$



$$\frac{1}{z} + \frac{1}{z'} = \frac{1}{f_{eff}}$$

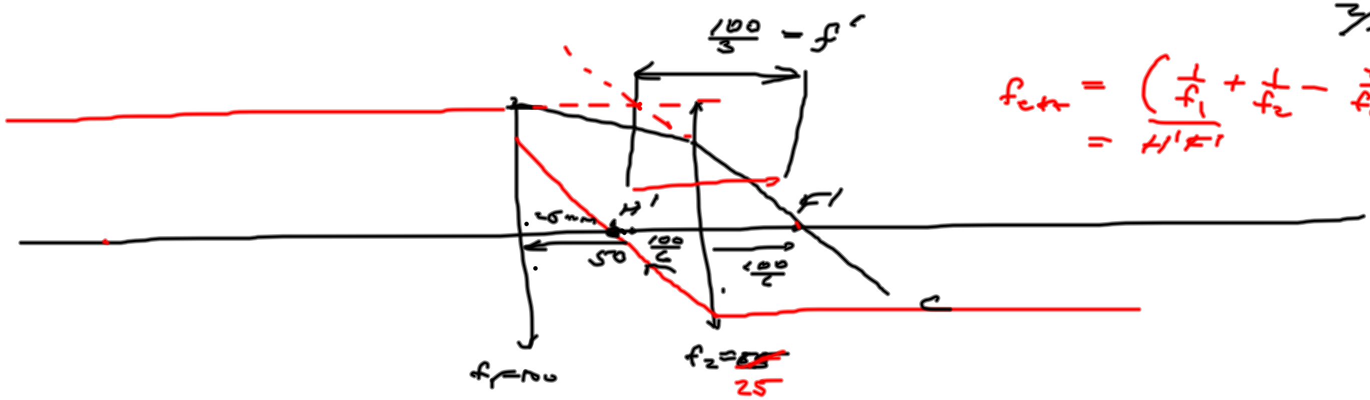
$$z_1 = \infty \Rightarrow z_2' \to \cancel{F'} \to F'$$



$$\frac{1}{z_1} + \frac{1}{z_1'} = \frac{1}{100} \Rightarrow z_1' = 100, \quad z_2 = t - z_1' = -50\ mm$$

$$\frac{1}{-50} + \frac{1}{z_2'} = \frac{1}{25} \Rightarrow \frac{1}{z_2'} = \frac{1}{25} + \frac{1}{50} \Rightarrow z_2' = \frac{50}{3} = \frac{100}{6} = 16.7$$

3/15 9



$$f_{eff} = \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}\right)^{-1}$$
$$= \overline{H'F'}$$

$$\frac{1}{z_1} + \frac{1}{z_1'} = \frac{1}{f_1}$$

$$\frac{1}{z_2} + \frac{1}{z_2'} = \frac{1}{f_2} \qquad z_2 = t - z_1'$$

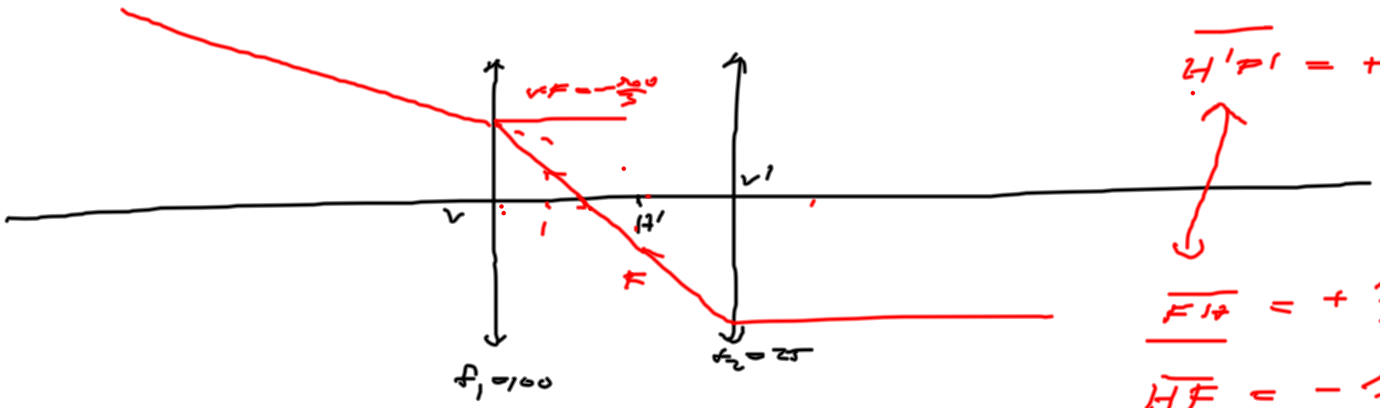
FIND F & H (FROM F) $\qquad \overline{FH} = f = \frac{100}{3}$

$$z_2 = +25$$
$$f_2 = +100$$

$$z_2' = \left(\frac{1}{f_2} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{100} - \frac{1}{25}\right)^{-1}$$

$$\left(\frac{1}{100} - \frac{4}{100}\right)^{-1} = \cdot\frac{\cancel{100}}{3} \quad - \frac{100}{3}$$

2/5 - 10



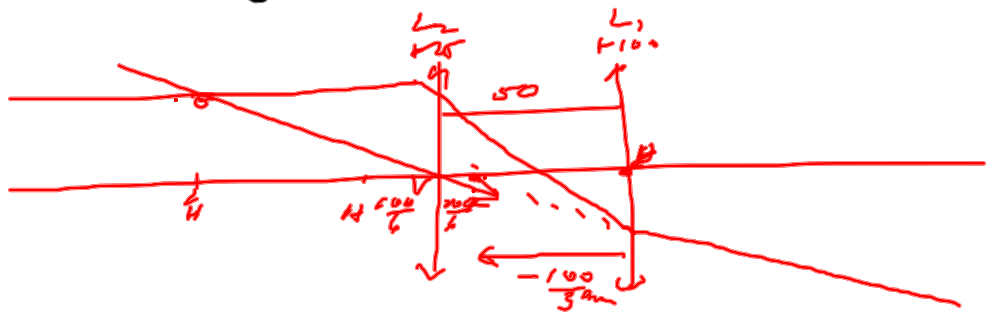
$\overline{H'F'} = +\frac{100}{3}$

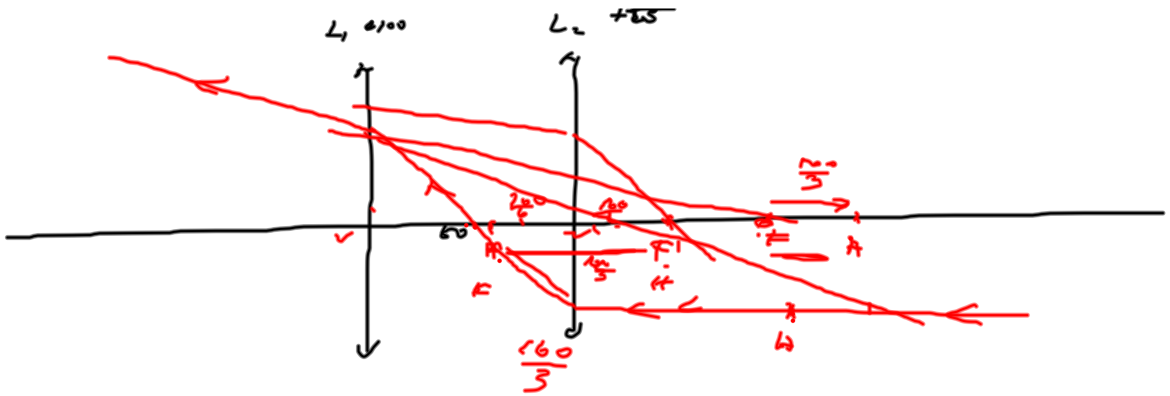
$\overline{FH} = +\frac{100}{3}$

$\overline{HF} = -\frac{100}{3}$

VF

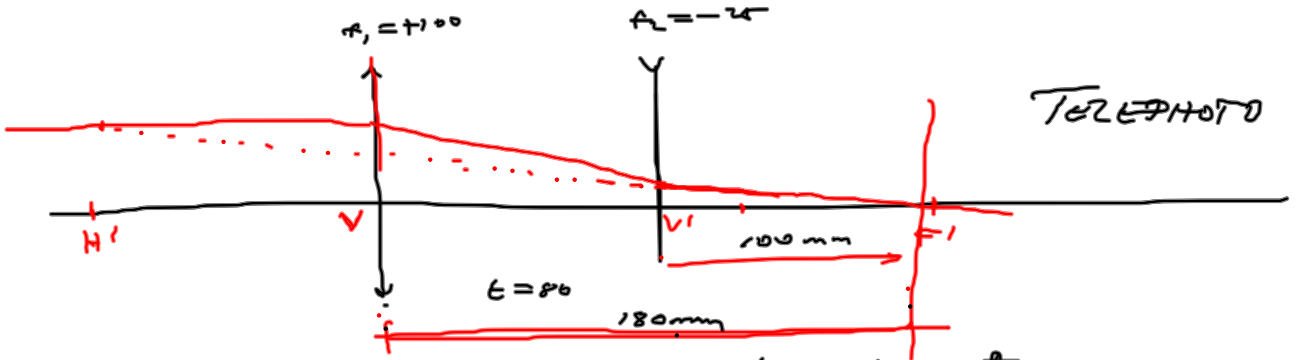
$\cancel{VF}' = -\frac{100}{3}$ mm





$$\frac{1}{25} + \frac{1}{x} = \frac{1}{100}$$

$$\frac{1}{100} - \frac{1}{25} = \frac{1}{100} - \frac{4}{100} = \frac{3}{100} \rightarrow \frac{100}{3}$$



$t \geq f_1 + f_2$

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

$$= \frac{1}{100} - \frac{1}{25} + \frac{80}{100 \cdot 25}$$

$$= -\frac{3}{100} + \frac{8}{250} = -\frac{15}{500} + \frac{16}{500} = \frac{1}{500}$$

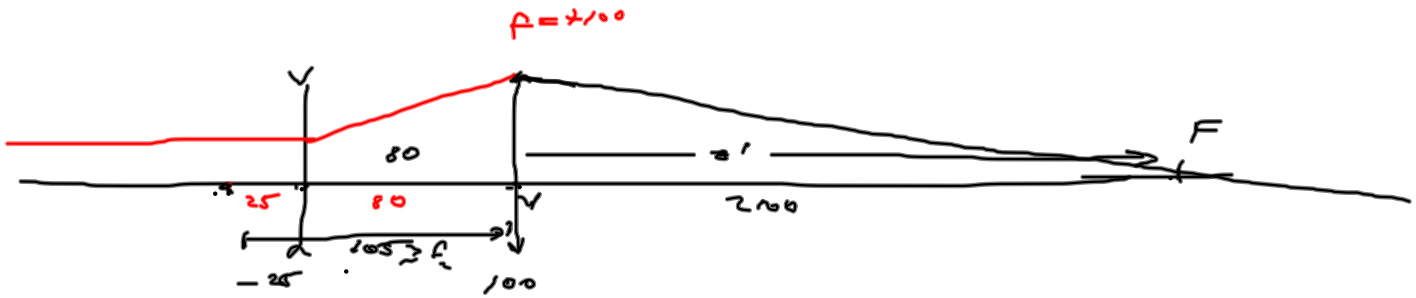
$f_{eff} = 500 \text{ mm} \Rightarrow \overline{H'F'} = \overline{FH}$

For L_2, $z_2 = t - z_1' = 80 - 100 = -20$ mm

$f_2 = -25$ mm $\Rightarrow z_2' = \overline{V'F'} = \left(\frac{1}{f_2} - \frac{1}{z_2}\right)^{-1}$

$$= \left(\frac{1}{-25} - \frac{1}{-20}\right)^{-1} \left(\frac{5}{100} - \frac{4}{100}\right)^{-1} = 100$$

2/15 — F2

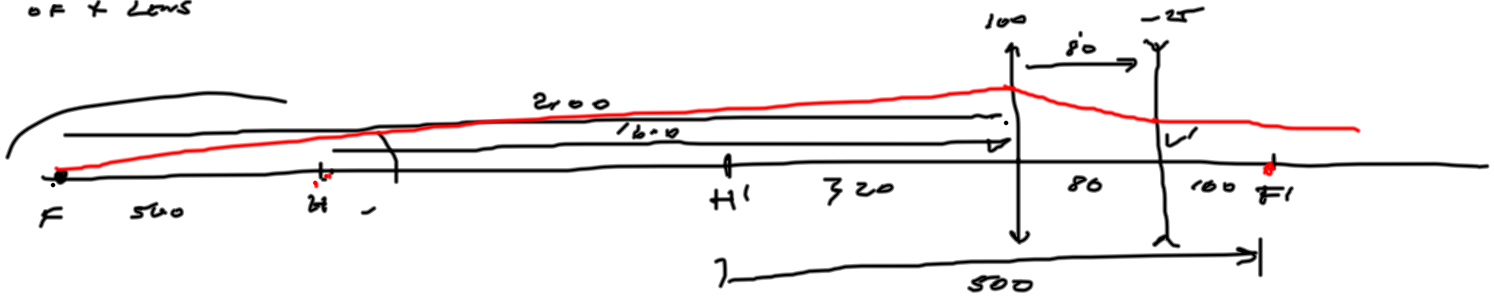


OBJ Dist for + Lens: $z = 105$

Focal Length of + Lens: $f = 100$

$$\left(\frac{1}{f} - \frac{1}{z}\right)^{-1} = z' = \left(\frac{1}{100} - \frac{1}{105}\right)^{-1}$$

$$= 2100 \text{ mm}$$



$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f_{eff}}$$

$z_1 = \overline{OH}$

$z_1 = \overline{OH}$

$z_2 = \overline{H'O'}$

$z_1 = 2f_{eff} = 1000$ mm

$z_2 = 1000$ mm $= 2f_{eff}$ } equal conjugates

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f} \qquad m_T = -\frac{2f}{2f} = -1$$

$z_1 = 2f = z_2$

$\overline{FH} = 500$ $\qquad \overline{HV} = 1600$

$\overline{OV} =$ $\qquad \overline{OH} = 1000$ $\qquad \overline{OV} = 2600$

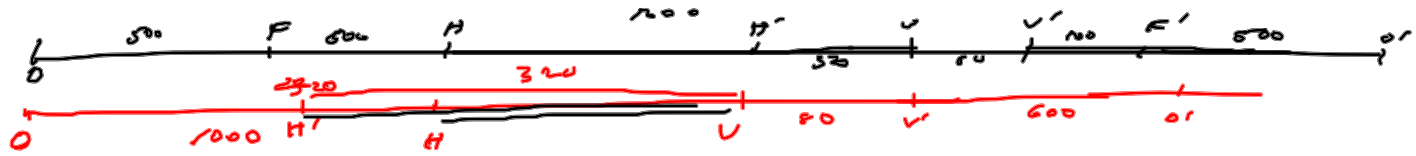
To 1st lens, $z = 2600$, $f_1 = 100$ } $z' = \left(\frac{1}{100} - \frac{1}{2600}\right)^{-1} = 105$

$$\frac{1}{-24} + \frac{1}{z'} = \frac{1}{-25} \qquad z' = +600 = \overline{V'O'}$$

-25

$$-\frac{104}{2600} = M_1 \approx -$$

$$-\frac{600}{-24} = +25 \qquad (M).$$



$$\overline{OH} = 1000 = 2R$$

$$\overline{H'V} + \overline{VV'} + \overline{V'O'}$$

$$320 + 80 + 600 = 1000 = \overline{H'O'} = 2R$$

$$\frac{1}{\overline{OH}} + \frac{1}{\overline{H'O'}} = \frac{1}{f_{eff}} \qquad M_T = -\frac{z'}{z} = -\frac{\overline{H'O'}}{\overline{OH}}$$

$\uparrow$ z $\qquad$ $\uparrow$ z'

SMALL

F, F', H, H', N, N' CARDINAL POINTS

$$\frac{f_{eff}}{D_{EP}} = f/\#$$

0.1 Example of Two-Lens System

$$\begin{aligned}\mathbf{f}_1 &= +100\,\text{mm}\\ \mathbf{f}_2 &= +25\,\text{mm}\\ t &= +50\,\text{mm}\end{aligned}$$

$$\begin{aligned}\frac{1}{\mathbf{f}_{\text{eff}}} &= \frac{1}{\mathbf{f}_1} + \frac{1}{\mathbf{f}_2} - \frac{t}{\mathbf{f}_1\mathbf{f}_2}\\ &= \left(\frac{1}{100\,\text{mm}} + \frac{1}{+25\,\text{mm}} - \frac{50\,\text{mm}}{(+100\,\text{mm})\,(+25\,\text{mm})}\right)^{-1}\\ \mathbf{f}_{\text{eff}} &= \frac{100}{3}\,\text{mm} = 33\frac{1}{3}\,\text{mm}\end{aligned}$$

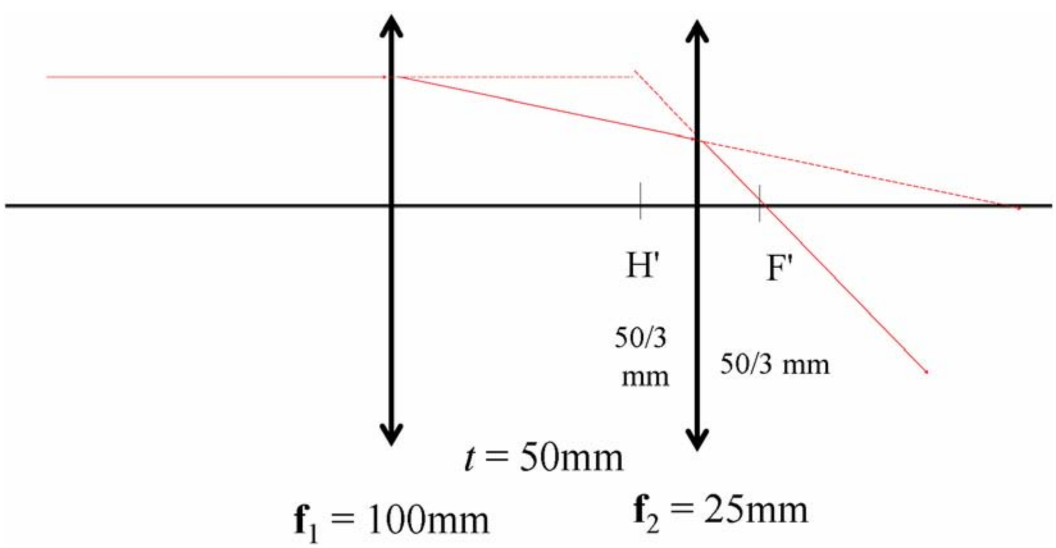
Now find the focal point by cascading through the system. For an object at ∞, the image distance from the first lens is $\mathbf{f}_1 = +100\,\text{mm}$, so the object distance to the second lens is

$$z_2 = t - \mathbf{f}_1 = 50\,\text{mm} - 100\,\text{mm} = -50\,\text{mm}$$

The image distance from the second lens is:

$$z_2' = \left(\frac{1}{\mathbf{f}_2} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{+25\,\text{mm}} - \frac{1}{-50\,\text{mm}}\right)^{-1} = \frac{50}{3}\,\text{mm}$$

which is half the focal length. We can now draw the image space focal and principal points:



Two positive lenses separated by distance $t = 50\,\text{mm}$

To find the image-space focal and principal point, we "turn the system around" and bring in light from the left. The image distance from the "first lens" (actually L_2) is equal to its focal length:

$$z_1' = \mathbf{f}_2 = +25\,\text{mm}$$

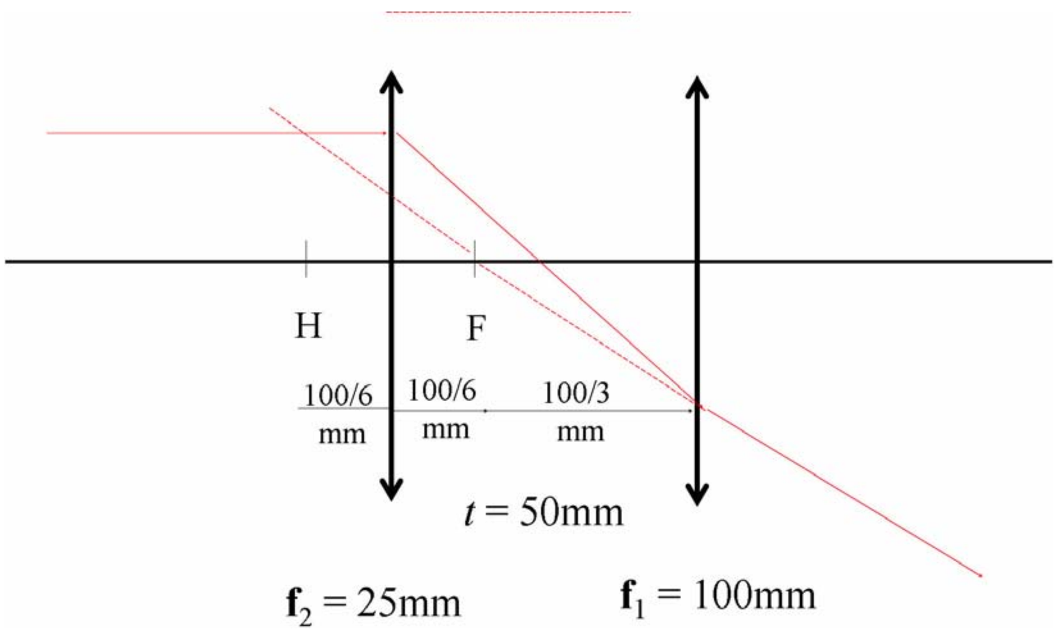
So the object distance to the lens with $\mathbf{f}_1 = +100\,\text{mm}$ is:

$$z_2 = t - z_1' = 50\,\text{mm} - 25\,\text{mm} = +25\,\text{mm}$$

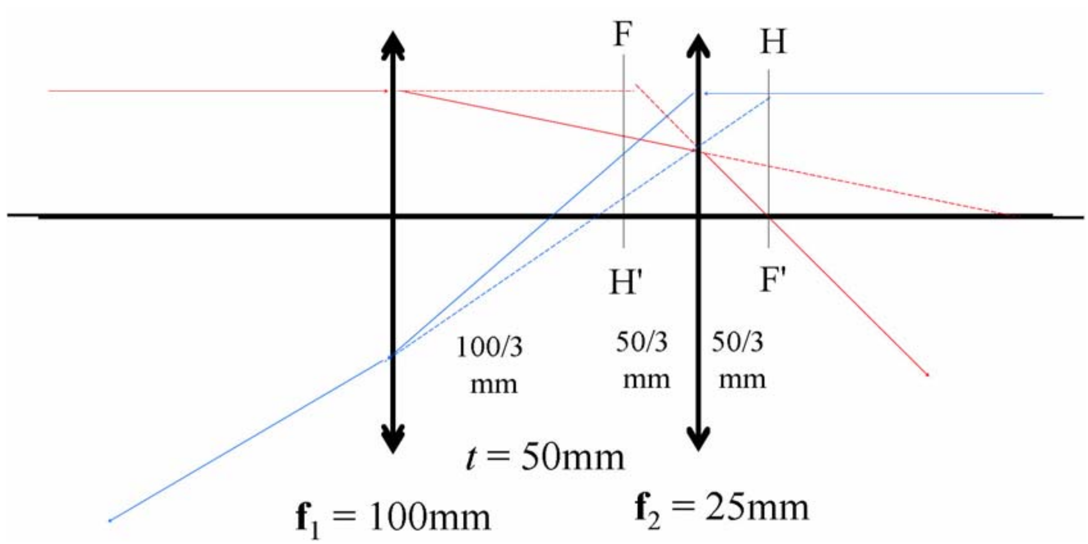
So the distance from this lens to the focal point is:

$$z_2' = \left(\frac{1}{\mathbf{f}_1} - \frac{1}{z_2}\right)^{-1} = \left(\frac{1}{100\,\text{mm}} - \frac{1}{25\,\text{mm}}\right)^{-1} - -\frac{100}{3}\,\text{mm}$$

:The focal point is *virtual* and the object-space principal is located at the distance $\mathbf{f}_{\text{eff}}$ behind it in the reversed system.



We can now reverse the second case and plot F, F', H, H' on the same graph:



In this case, the object-space focal point F just happens to coincide with the image-space principal point H'; the same applies to the object-space principal point H and the image-space focal point F'. This is of no real significance, since the two spaces are independent.