11 JANUARY 2010

HW 1, 2 RETURNED TO FOLDERS, SOLUTIONS POSTED

HW 3 DUE W 1/13

MIDTERM W 1/20

NO CLASS M 1/18

PROBLEM SESSION F 1/15?

psf & MTF OF OPTICS

PHYSICAL PROPERTIES THAT AFFECT IMAGING SYSTEMS

e.g. REFRACTIVE INDEX
POLARIZATION
REFLECTANCE & TRANSMITTANCE @ INTERFACE (FRESNEL EQUATIONS)

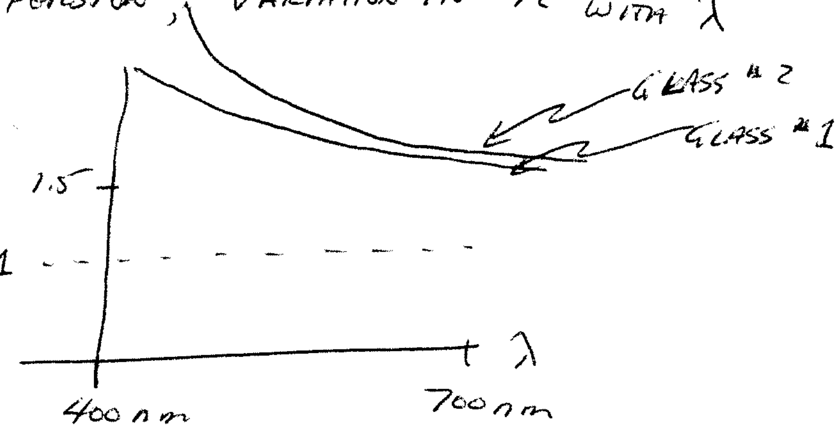
→ INTERFERENCE
GEOMETRICAL OPTICS

1/11/10 - 2

INDEX OF REFRACTION → DISPERSION, VARIATION IN n WITH λ

$$n = \frac{c}{v} \geq 1$$

$$n(\lambda) = \frac{c}{v(\lambda)} \longrightarrow$$



$n(\lambda)\downarrow$ AS $\lambda\uparrow$ NORMAL DISPERSION

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow f\uparrow \text{ AS } \lambda\uparrow$$

$\Rightarrow z_2\uparrow$ (IMAGE DISTANCE) AS $\lambda\uparrow$

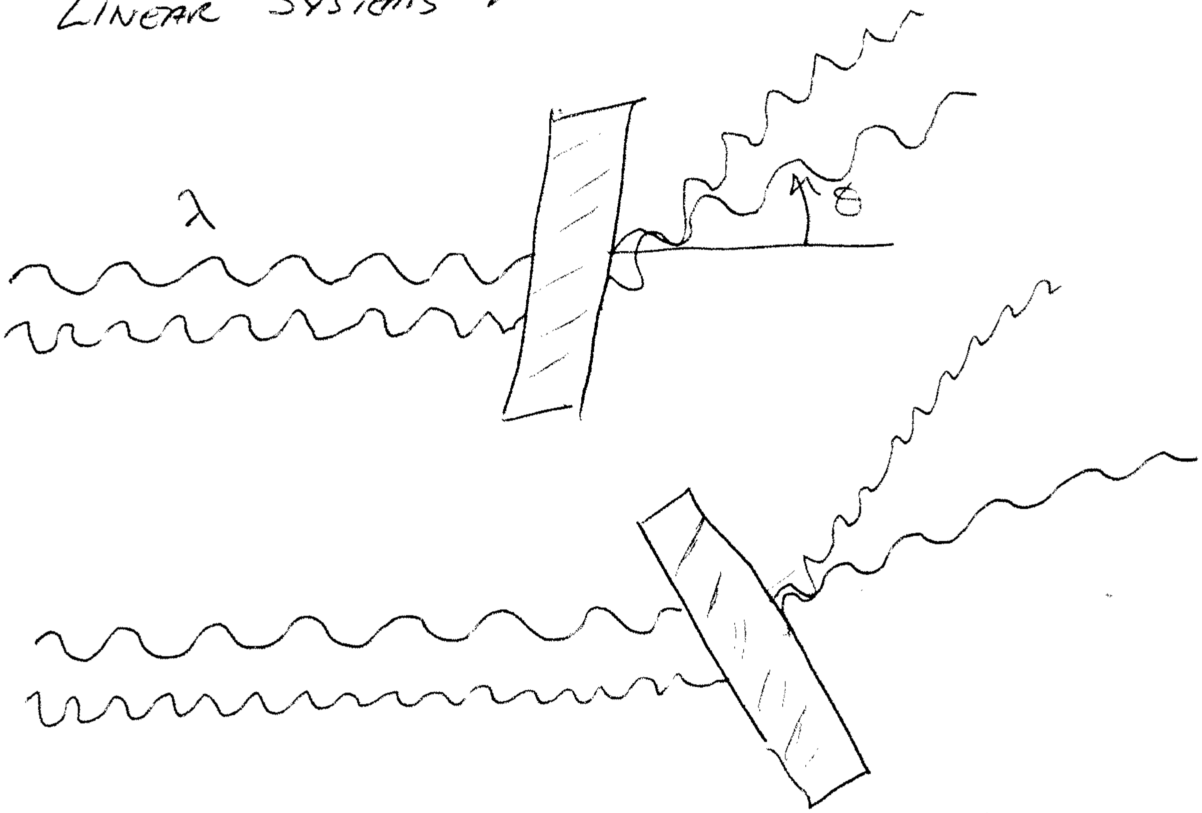
IMAGES FOR DIFFERENT λ AT DIFFERENT $z_2 \Rightarrow$ CHROMATIC ABERRATION

$\Rightarrow$ DEGRADATION OF RECORDED IMAGE OF COLORED OBJECTS

MATCH FOCAL LENGTH @ DIFFERENT λ VIA MULTIPLE ELEMENTS $\Rightarrow$ ACHROMATIC LENS (2λ), APOCHROMATIC LENS (3λ)

(1) PHYSICAL "PROPERTIES" $\Longrightarrow n(\lambda)$ 1/11/10

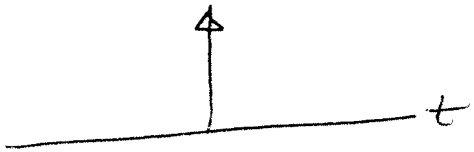
(2) LINEAR SYSTEMS MODEL

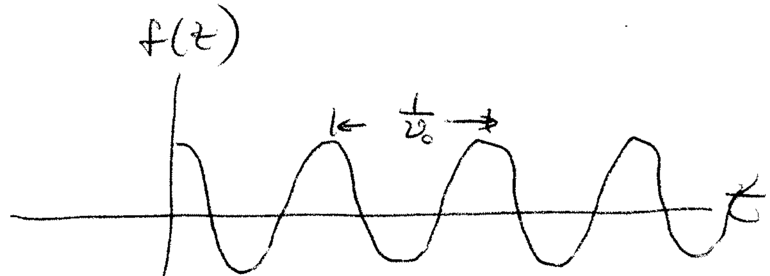


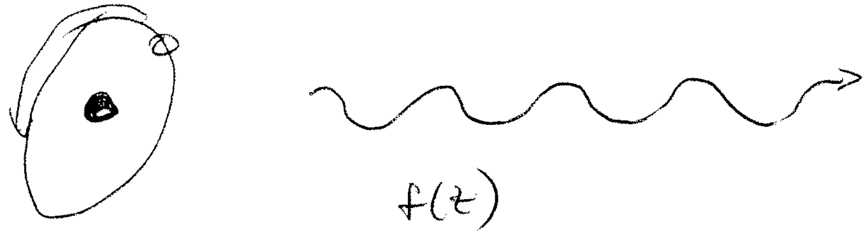
LIGHT INTERACTS W/ ATOMS (ELECTRONS) IN MATERIAL

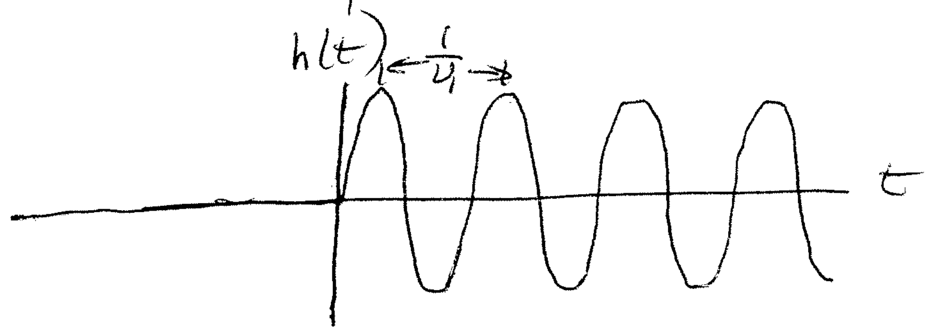
ELECTRONS ABSORB & RE-EMIT LIGHT (SCATTERING)

1/11/10 (4)









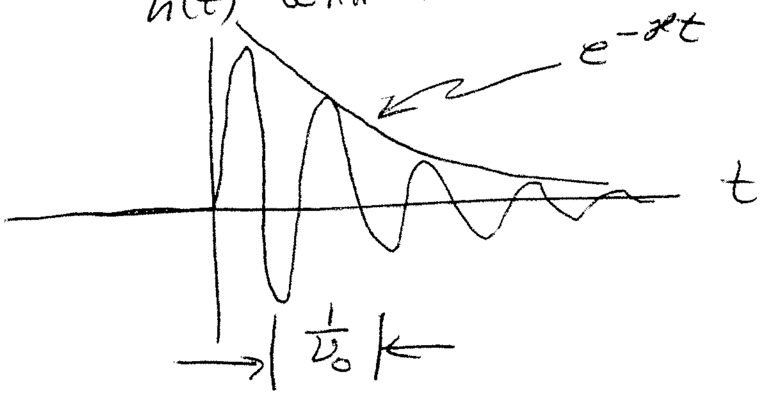
$h(t) \propto \sin(2\pi \nu_0 t) \cdot \mathrm{STEP}(t) \Longrightarrow$ CAUSAL SYSTEM

$h(t) = 0$ FOR $t < 0$

$$H(\nu) \propto i\left(\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right) * \left(\tfrac{1}{2}\delta(\nu) + \frac{1}{2\pi i \nu}\right)$$

1/11/10 5

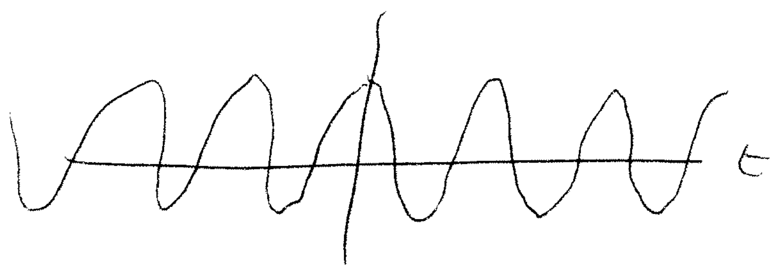
$h(t)$ WITH "DAMPING"



$\frac{1}{\gamma} = t_0 \Rightarrow$ TIME TO DECAY TO $\frac{1}{e}$

WHAT IS A FORM OF $f(t)$? (DRIVING FORCE - INCOMING LIGHT)

$f(t) = \cos(2\pi \nu t)$



NOT PHYSICAL

$f(t) = \text{STEP}(t) \cos(2\pi \nu t - \frac{\pi}{2}) = \text{STEP}(t) \sin(2\pi \nu t)$

$h(t) = \text{STEP}(t) \cos(2\pi \nu_0 t - \frac{\pi}{2}) = \text{STEP}(t) \sin(2\pi \nu_0 t)$

$g(t) = f(t) \ast h(t)$

$$n = \frac{c}{V} \qquad c = \sqrt{\frac{1}{\mu_0 \epsilon_0}} \qquad V = \sqrt{\frac{1}{\mu \epsilon}}$$

μ_0 = PERMEABILITY – MAGNETIC

ϵ_0 = PERMITTIVITY – ELECTRIC

$\mu = \mu_0$ IN MOST MATERIALS OF INTEREST

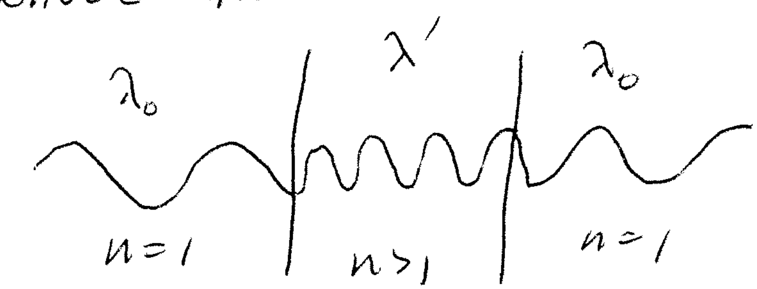
$$n = \sqrt{\frac{\cancel{\mu} \epsilon}{\cancel{\mu_0} \epsilon_0}}$$

$$n^2 = \frac{\epsilon}{\epsilon_0}$$

$E = h\nu = h\frac{c}{\lambda} \propto \frac{1}{\lambda} \Rightarrow$ ν IN VACUUM = ν IN MEDIUM

$\Rightarrow$ WAVELENGTH IN MEDIUM $\neq$ WAVELENGTH IN VACUUM

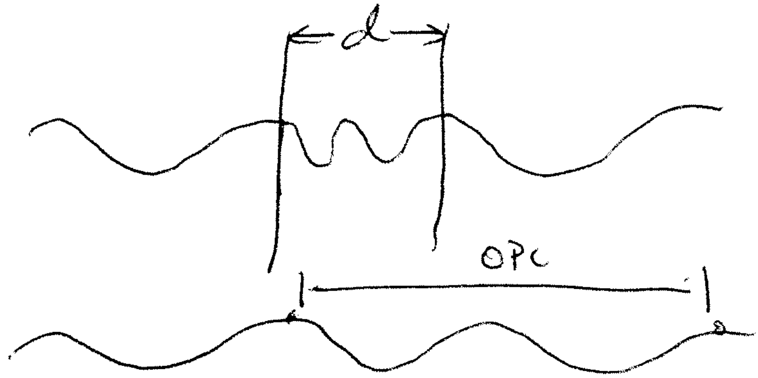
$$\boxed{\lambda' = \frac{\lambda_0}{n}} \qquad \lambda' \neq \lambda_0$$



$$n = \frac{c}{V} = \frac{\lambda_0 \cancel{\nu_0}}{\lambda' \cancel{\nu_0}} = \frac{\lambda_0}{\lambda'}$$

OPTICAL PATH LENGTH OPL

LENGTH IN VACUUM THAT LIGHT TRAVERSES IN SAME TIME AS WOULD TRAVERSE THICKNESS d IN GLASS



$\boxed{OPL = nd} > d$

OPL > PHYSICAL PATH LENGTH

$n + iK = \tilde{n}$ COMPLEX REFRACTIVE INDEX

$\tilde{n}^2 = \frac{\epsilon}{\epsilon_0}$

k_0 – PROPAGATION CONSTANT

$$|k_0| = \frac{2\pi}{\lambda_0} \longrightarrow \frac{2\pi}{\lambda'} = \frac{2\pi}{\lambda_0/n} = \frac{2\pi n}{\lambda_0}$$

IN MEDIUM

COMPLEX n & PHASE

$$\frac{2\pi n}{\lambda_0} \longrightarrow \frac{2\pi \tilde{n}}{\lambda_0} = \frac{2\pi}{\lambda_0}(n + ik)$$

$$e^{+i \frac{2\pi n}{\lambda_0} \cdot z} = e^{+i \frac{2\pi}{\lambda_0} z (n+ik)} = e^{+i \frac{2\pi}{\lambda_0} n z} \cdot e^{+i \frac{2\pi}{\lambda_0} z\, ik}$$

$$= \underbrace{e^{+i \frac{2\pi}{\lambda_0} n z}}_{\text{PROPAGATING (OSCILLATION)}} \cdot \underbrace{e^{-\frac{2\pi}{\lambda_0} z K}}_{\text{ATTENUATION}}$$

IMAGINARY PART OF $\tilde{n}$ $\Rightarrow$ ABSORPTION OF LIGHT BY MEDIUM

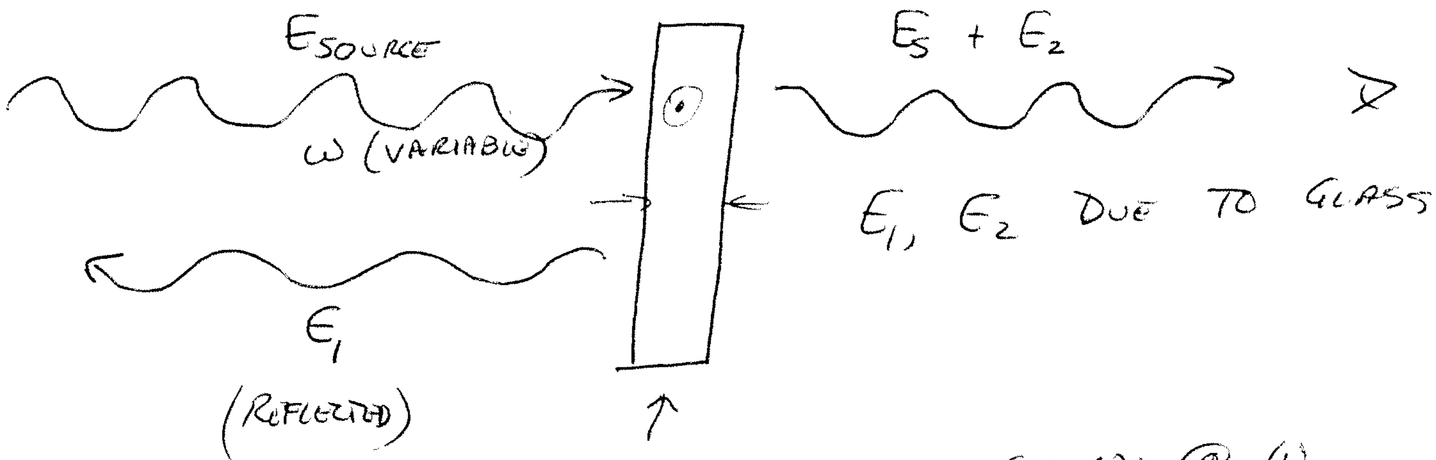
$n(\lambda)\downarrow$ AS $\lambda\uparrow$ NORMAL DISPERSION

$n(\lambda)\uparrow$ AS $\lambda\uparrow$ ANOMALOUS DISPERSION

PHASE VELOCITY & GROUP VELOCITY (MODULATION VELOCITY)

PHYSICAL MODEL FOR n (FEYNMAN LECTURES ON PHYSICS) 1/11/10 - 9

VOL I §31, §32   II §32



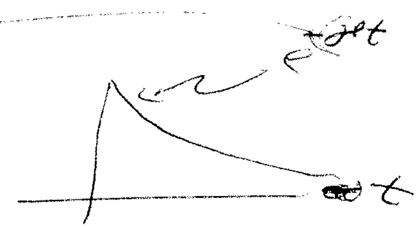
MEDIUM, NATURAL OSCILLATION @ ω_0

(1) IN TERMS OF PHASE
(2) IN TERMS OF FORCES

$\left.\begin{array}{l} (1) \\ (2) \end{array}\right\} n$

(A) UNDAMPED

(B) DAMPED $\longrightarrow$ RESONANCE "WIDTH" IN FREQ

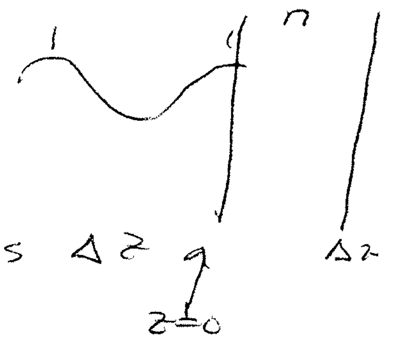


1/11/10 – 10

$$E_S = E_0\, e^{+i(k z - \omega t)} = E_0\, e^{i\omega\left(\frac{k}{\omega} z - t\right)}$$

$\uparrow$ VARIABLE

$$= E_0\, e^{i\omega\left(\frac{z}{c} - t\right)}$$



ASSUME MEDIUM "STARTS" AT $z=0$, THICKNESS IS Δz

FRONT OF GLASS

@ $z=0 \implies E_S = E_0\, e^{-i\omega t}$

@ $z = \Delta z$ IF NO GLASS $\implies E_0\, e^{i\omega\left(\frac{\Delta z}{c} - t\right)} = E_0\, e^{i\frac{\omega \Delta z}{c}}\, e^{-i\omega t}$ PHASE @ $z=\Delta z$ IF NO GLASS

$$\Phi_{NO}[\Delta z, t] = \frac{\omega \Delta z}{c} - \omega t$$

@ $z = \Delta z$ WITH GLASS

$$E_0\, e^{i\omega\left(\frac{\Delta z}{c} \cdot n - t\right)}$$

$\uparrow$ INDEX

$$\Phi_{GLASS}[\Delta z, t] = n\omega \frac{\Delta z}{c} - \omega t$$

PHASE CHANGE DUE TO GLASS

$$\Delta\Phi = \Phi[\Delta z, t, \text{VACUUM}] - \Phi[\Delta z, t, \text{GLASS}]$$

$$= \omega\left(\frac{\Delta z}{c} - t\right) - \omega\left(\frac{\Delta z}{c}n - t\right)$$

$$\Delta\Phi = \omega\frac{\Delta z}{c} - \omega n\frac{\Delta z}{c} = \boxed{-\omega(n-1)\frac{\Delta z}{c} = \Delta\Phi}$$

$$E[\Delta z, t] = \overbrace{\boxed{E_0\, e^{i\omega\left(\frac{\Delta z}{c} - t\right)}}}^{E_S} \cdot \underbrace{e^{-i\omega(n-1)\frac{\Delta z}{c}}}_{\Delta z \text{ IS SMALL}} \Longrightarrow E_S + E_2$$

FIELD @ BACK OF GLASS $= E_S + E_2$

$$e^A = \frac{1}{0!} + \frac{A^1}{1!} + \frac{A^2}{2!} + \cdots$$

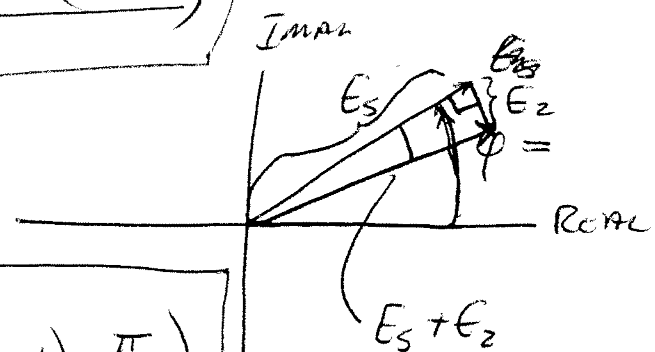
$$e^{-i\omega(n-1)\frac{\Delta z}{c}} \cong 1 - i\omega(n-1)\frac{\Delta z}{C} \cancel{+ \cdots}$$

1/11/10 — 12

$$E_s + E_2 \cong E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)} \left(1 - i\omega(n-1)\frac{\Delta z}{c}\right)$$

$$= \underbrace{E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}_{E_s} \underbrace{- i\omega(n-1)\frac{\Delta z}{c} E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}_{E_2}$$

$$\boxed{E_2 = \left(-i\omega(n-1)\frac{\Delta z}{c} E_0\right) e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}$$



PHASE → AMPLITUDE

$$\Rightarrow -i = e^{-i\frac{\pi}{2}}$$

$$\boxed{E_2 = \underbrace{\omega(n-1)\frac{\Delta z}{c} E_0}\; e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right) - \frac{\pi}{2}\right)}}$$

1/11/10 – 13

FORCES

$$F = ma = m\frac{d^2x}{dt^2} = m\ddot{x} \; ; \quad \ddot{x} \equiv \frac{d^2x}{dt^2}$$

$$\dot{x} = \frac{dx}{dt}$$

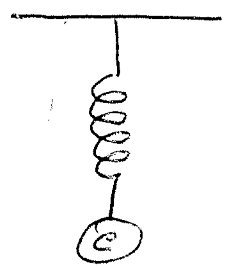
FOR FIELD E_s DRIVING ELECTRON

$$E_s[z=0,t] \cdot e = F$$

$$F = eE_0 e^{+i\omega\left(\frac{0}{c} - t\right)} = eE_0 e^{-i\omega t}$$ DRIVING FORCE

HOOKE'S LAW

RESTORING FORCE



$$F = k(x - x_0) \cdot (-1)$$

↑ SPRING CONSTANT

$$F = -k(x - x_0)$$

$$\frac{k}{m_e} = \left(\frac{1}{\sec}\right)^2$$

$\sqrt{\frac{k}{m_e}}$ IS A FREQUENCY $\Rightarrow \omega_0$

$$\boxed{\omega_0 = \sqrt{\frac{k}{m_e}}}$$