16 December 2009

IMAGING IN COHERENT LIGHT $\Rightarrow$ $h[x,y;z_1,f,z_2] \propto P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$

$$H[\xi,\eta;z_1,f,z_2] \propto p\left[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta\right]$$

COHERENT $\Rightarrow$ MONOCHROMATIC $\Rightarrow$ "PREDICTABLE" PHASES
"DETERMINISTIC"

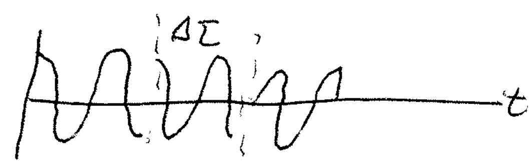
$\Rightarrow$ DESTRUCTIVE INTERFERENCE

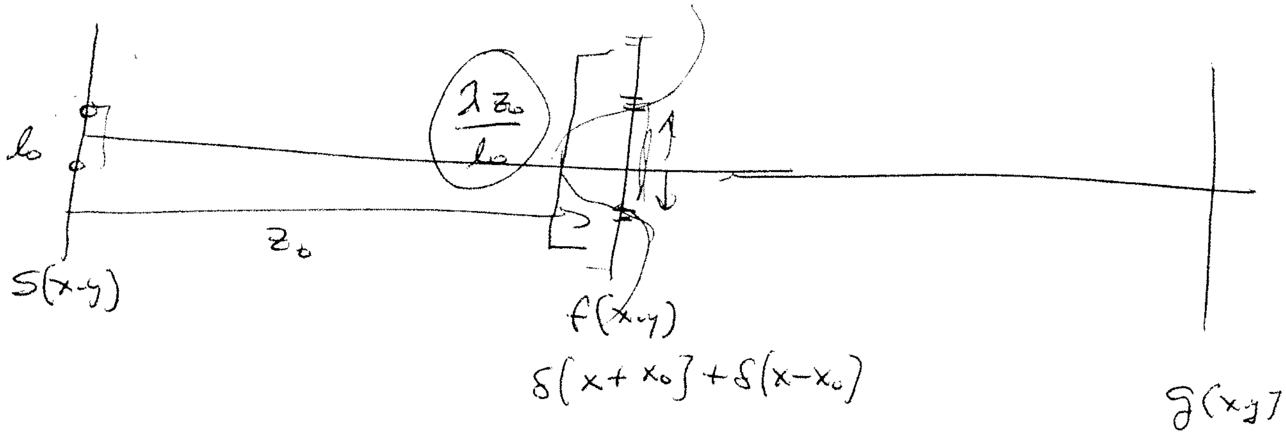
$\Rightarrow$ NEGATIVE AMPLITUDE

$\Rightarrow$ "FRINGE PATTERNS" $\Rightarrow$ AFFECTS ABILITY TO DISTINGUISH ADJACENT POINT OBJECTS

INCOHERENT $\Rightarrow$ UNPREDICTABLE PHASES OVER MEASUREMENT TIME $\Rightarrow$ POLYCHROMATIC LIGHT OR MONOCHROMATIC LIGHT ~~WITH~~ RANDOM PHASE

$$h[x,y;z_1,f,z_2] \propto \left|P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right|^2$$





SYSTEM FOR CHARACTERIZING ~~OBJ~~ SOURCE COHERENCE

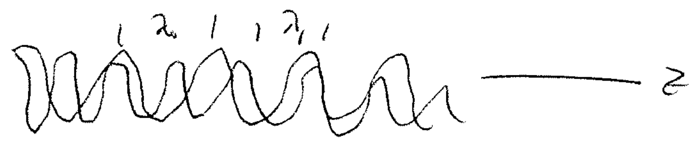
§22

COHERENCE "WIDTH" = REGION AT OBJECT WHERE AMPLITUDE IS COHERENT $= \frac{\lambda z_0}{l_0}$

λ_0

λ_1

$\tau = \frac{1}{\Delta\nu}$



$\nu_0 = \frac{c}{\lambda_0}, \quad \nu_1 = \frac{c}{\lambda_1}$

$\tau = \frac{1}{\Delta\nu}$ = TIME INTERVAL OVER WHICH PHASES ARE CORRELATED = COHERENCE TIME

$\tau = \frac{1}{\Delta \nu}$ $\Rightarrow$ TIME OVER WHICH PHASE IS CORRELATED

$c \cdot \tau = \frac{c}{\Delta \nu}$ $\Rightarrow$ LONGITUDINAL DISTANCE OVER PHASE IS CORRELATED

$\Rightarrow$ COHERENCE LENGTH

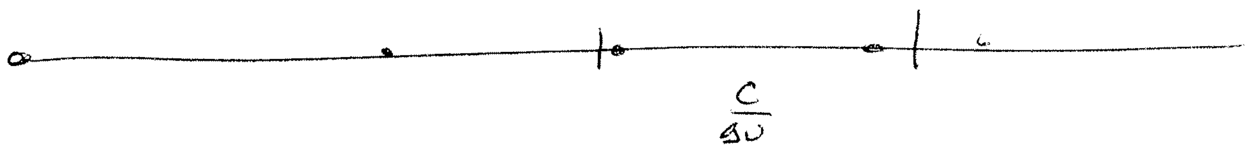


IMAGE IRRADIANCE $\Rightarrow$ TIME AVERAGE OF SQUARED MAGNITUDE

$$f[x,y,t] * h[x,y;z_1,f,z_2] = \iint dx'\,dy'\, f[x',y',t]\, h[x-x',y-y']$$

$$\frac{1}{T_0}\int_{-\frac{T_0}{2}}^{T_0/2} \left| g[x,y,t;z_1,f,z_2]\right|^2 dt = I[x,y;\ldots]$$

T_0 = MEASUREMENT TIME

$$\cancel{I = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt}$$

$$f[x,y;t] = |f(x,y)|\, \cancel{e^{i\Phi[t]}}\, e^{i\Phi[x,y,t]}$$

$$f^*[x,y,t] = |f(x,y)|\, \cancel{e^{-i\Phi[t]}}\, e^{-i\Phi[x,y,t]}$$

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$$f[x,y,t] * h(x,y) = |f[x,y]|\, e^{i\phi[x,y,t]} * h(x,y]$$

$$\left(f[x,y,t] * h(x,y)\right)^* = |f[x,y]|\, e^{-i\phi[x,y,t]} * h^*(x,y]$$

$$\iint_{-\infty}^{+\infty} dx'dy' \; |f[x',y']|\, e^{i\Phi[x',y',t]}\, h[x-x', y-y']$$

$$\iint_{-\infty}^{+\infty} dx''dy'' \; |f[x'',y'']|\, e^{-i\Phi[x'',y'',t]}\, h^*[x-x'', y-y'']$$

$$I[x,y;\cdots T_0] = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} \iint dx'dy' \iint dx''dy'' \; |f(x',y')|\,|f(x'',y'')|\; h(x-x', y-y')\, h^*[x-x'', y-y'']\, e^{+i\phi[x',y',t]}\, e^{-i\phi[x'',y''}$$

$$I[x,y,\ldots T_0] = \iint dx'dy' \iint dx''dy'' \, |f(x',y')| \, |f(x'',y'')| \, h[x-x',y-y'] \, h^*[x-x'',y-y''] \cdot \underbrace{\frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} e^{i\Phi[x',y',t]} \, e^{-i\Phi[x'',y'',t]} \, dt}_{\text{MEASUREMENT OF CORRELATION OF PHASES AT TWO LOCATIONS } [x',y'],\,[x'',y'']}$$

2 CASES (1) PERFECT CORRELATION

$\Longrightarrow$ TEMPORAL INTEGRAL $= 1$

COHERENT LIGHT

(2) "PERFECTLY UNCORRELATION"

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{i\Delta\Phi(x',y',x'',y'',t)} \, dt \propto \delta[x'-x'',y'-y'']$$

INCOHERENT

INCOHERENT CASE

$$\frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} e^{i\left(\Phi[x',y';t] - \Phi(x'',y'';t)\right)}\,dt$$

MUTUAL COHERENCE FUNCTION $\Gamma[x',y';x'',y'';T_0]$

INCOHERENT MCF $\Gamma[x',y';x'',y'';T_0] \propto \delta(x'-x'',y'-y'')$

$$I[x,y;T_0] = \iint dx'dy' \iint dx''dy'' \,|f[x',y']|\,|f[x'',y'']|\, h[x-x',y-y']\, h^*[x-x'',y-y'']\ \delta(x'-x'',y'-y'')$$

$$= \iint dx'dy' \underbrace{|f[x',y']|\,|f[x',y']|}_{|f[x',y']|^2}\ \underbrace{h[x-x',y-y']\, h^*[x-x',y-y']}_{|h[x-x',y-y']|^2}$$

INCOHERENT IRRADIANCE

$$I[x,y;T_0] = \int dx'dy' \, |f[x',y']|^2 \cdot |h[x-x',y-y']|^2$$

$$= \underbrace{|f[x,y]|^2}_{\text{"INPUT"}} * \underbrace{|h[x,y]|^2}_{\text{"IMPULSE RESPONSE"}}$$

$$|h[x,y]|^2 = \mathscr{h}[x,y] = \text{INCOHERENT IMPULSE RESPONSE}$$

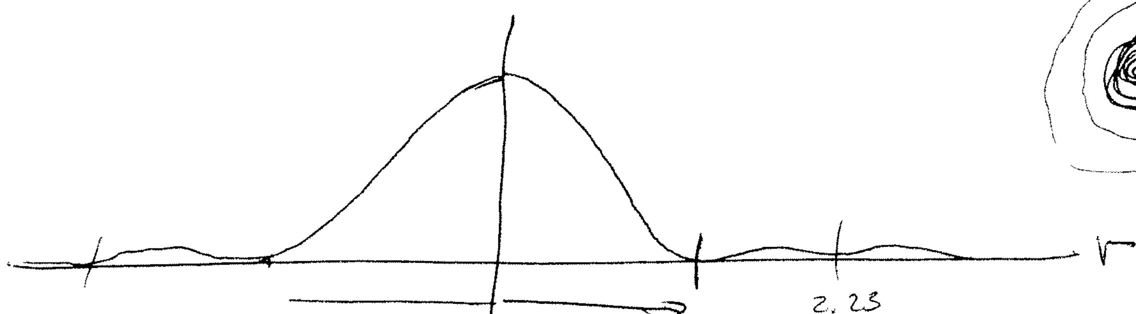
$$\mathscr{h}[x,y] \propto \left| P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right] \right|^2 ; \quad p(r) = \mathrm{CYL}\left(\frac{r}{d_0}\right)$$

$$P(\rho) = \frac{\pi d_0^2}{4} \, \mathrm{SOMB}(d_0 \rho)$$

$$P\left(\frac{r}{\lambda_0 z_2}\right) = \frac{\pi d_0^2}{4} \, \mathrm{SOMB}\left(\frac{r}{\lambda_0 z_2 / d_0}\right)$$

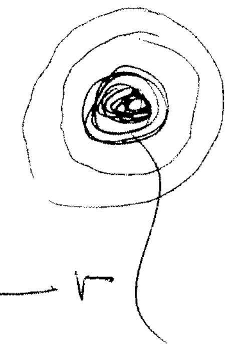
$$P\left(\frac{r}{\lambda_0 z_2}\right) = \frac{\pi d_0^2}{4} \operatorname{somb}\left(\frac{r}{\lambda_0 z_2 / d_0}\right)$$

$$\left|P\left(\frac{r}{\lambda_0 z_2}\right)\right|^2 = \left(\frac{\pi d_0^2}{4}\right)^2 \operatorname{somb}^2\left(\frac{r}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right)$$



$\approx 1.22 \frac{\lambda_0 z_2}{d_0}$ — λ_μ

$2.44 \frac{\lambda_0 z_2}{d_0}$



AIRY DISK

INCOHERENT, BUT QUASI MONOCHROMATIC, LIGHT

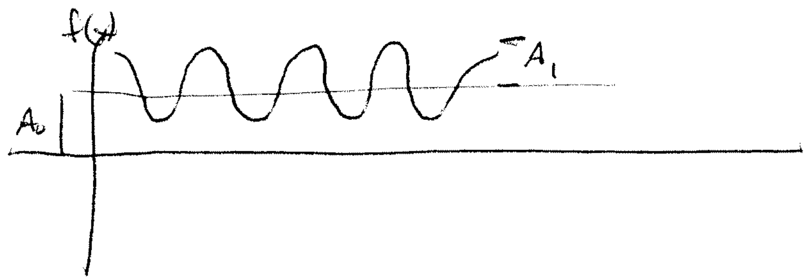
$$h[x,y;\cdots] \propto \left|P\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right]\right|^2$$

$$\mathcal{H}[\xi,\eta;\cdots] \propto (\lambda_0 z_2)^2 \cdot \underbrace{\left(p[-\lambda_0 z_2\xi, -\lambda_0 z_2\eta] \star p[-\lambda_0 z_2\xi, -\lambda_0 z_2\eta]\right)}_{\text{AUTOCORRELATION OF PUPIL}}$$

$$\mathcal{H}_{max}[\xi,\eta] = \mathcal{H}[0,0]$$

NORMALIZE $\mathcal{H}$ BY $\mathcal{H}_{max}$ $\Longrightarrow -1 \le \left(\dfrac{\mathcal{H}[\xi,\eta]}{\mathcal{H}[0,0]}\right) \le 1$

$$f[x] = A_0 + A_1\cos(2\pi\xi_0 x) \ge 0 \qquad A_0 \ge A_1$$



$$m_f = \frac{A_1}{A_0}$$

$$g(x) = A_0 \cdot H[0] + A_1 \cdot H[\xi_0] \cdot \cos(2\pi\xi_0 x)$$

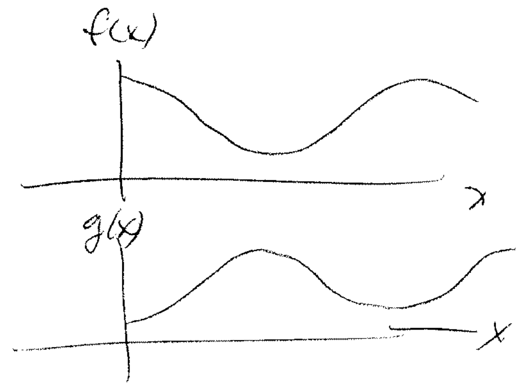
$$= H[0]\left(A_0 + A_1 \frac{H[\xi_0]}{H[0]} \cdot \cos(2\pi\xi_0 x)\right)$$

$$m_g = \frac{A_1 \cdot \frac{H[\xi_0]}{H(0)}}{A_0} = m_f \cdot \underbrace{\left(\frac{H[\xi_0]}{H[0]}\right)}_{MT[\xi_0]}$$

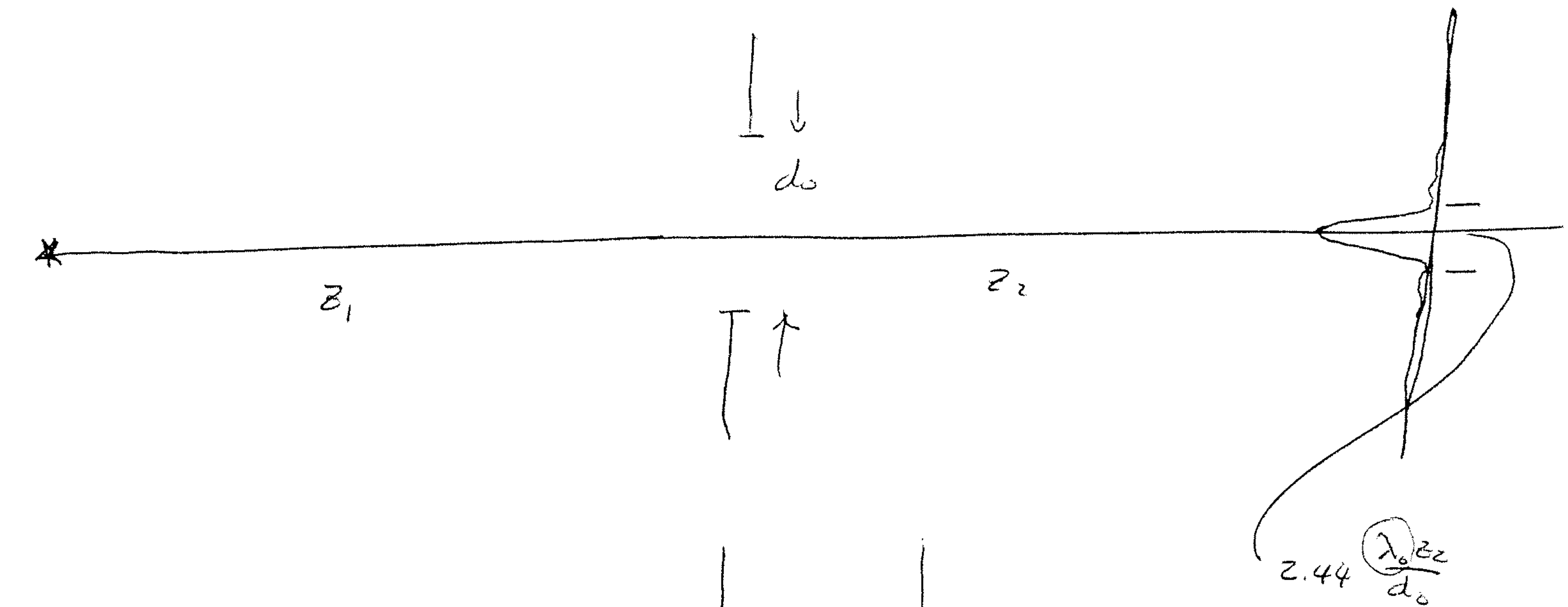
$$\frac{H[\xi]}{H(0]} = MT[\xi] \equiv MTF \Rightarrow$$ HOW MODULATION OF INPUT IS AFFECTED BY SYSTEM

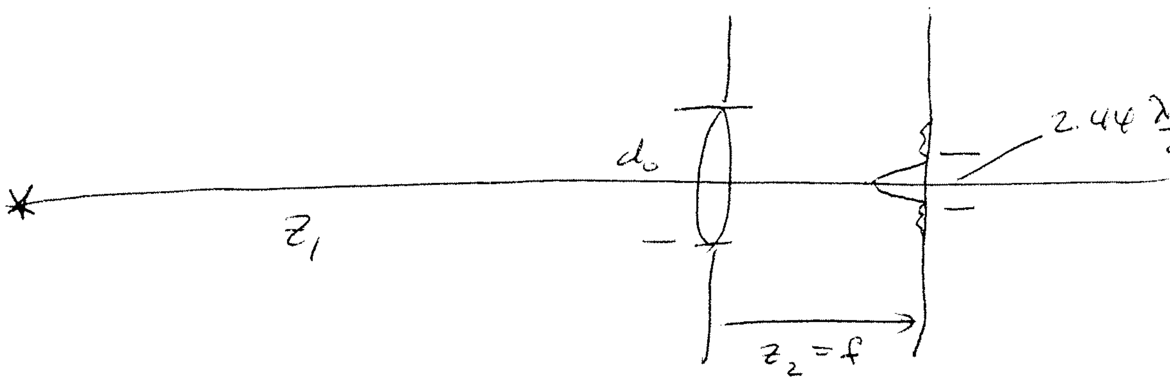
$$-1 \le MT(\xi) \le 1$$

CONTRAST REVERSAL



RESOLUTION OF INCOHERENT IMAGING SYSTEM



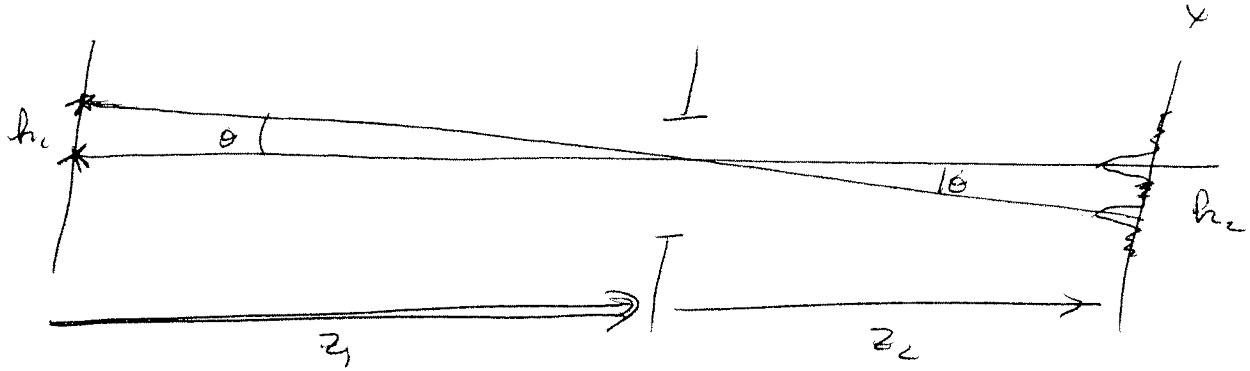


$2.44 \frac{\lambda z_2}{d_o} \rightarrow 2.44 \frac{\lambda f}{d_o}$

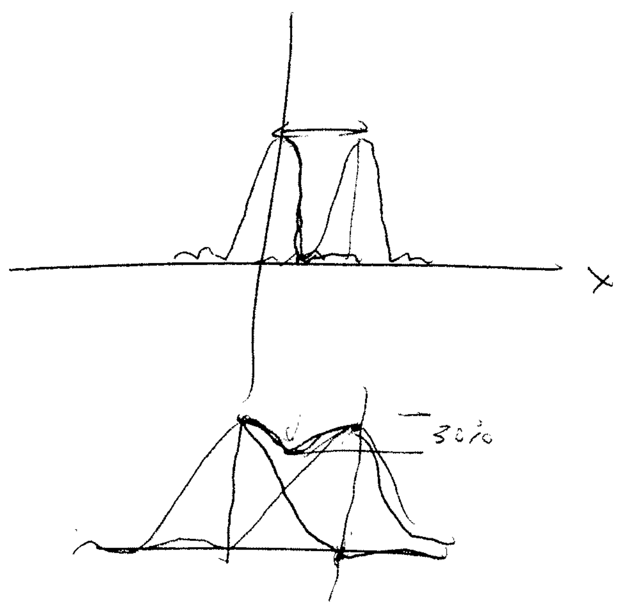
$f/d_o = f/\#$

DIAMETER OF AIRY DISK $\sim 2.44 \lambda \cdot f/\#$

$2 \lambda f/\#$ SQUARE APERTURE



$$\theta = \frac{h_1}{z_1} = \frac{-h_2}{z_2}$$



METRICS FOR RESOLUTION

(1) RAYLEIGH'S LIMIT