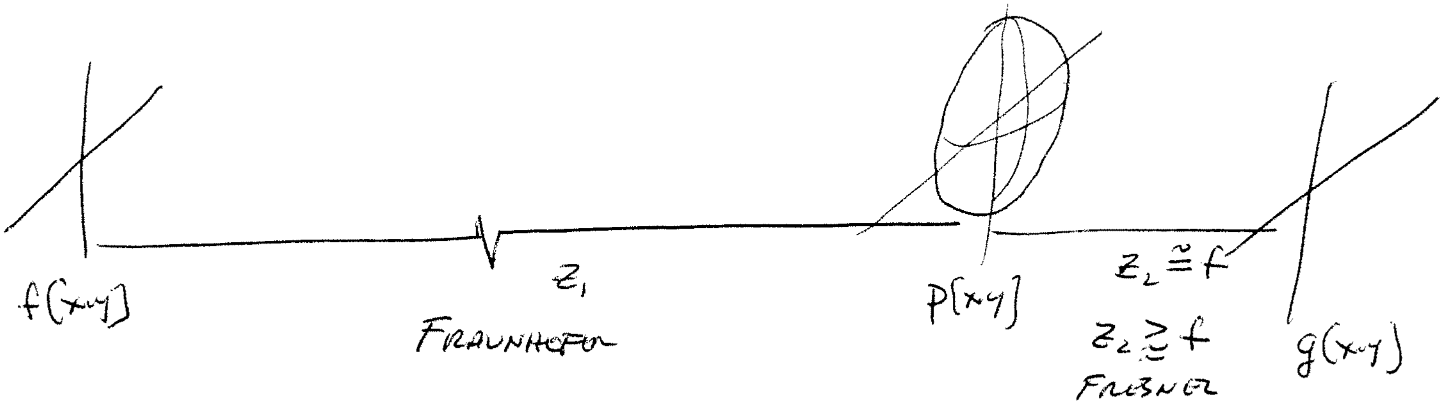
14 DECEMBER 2009

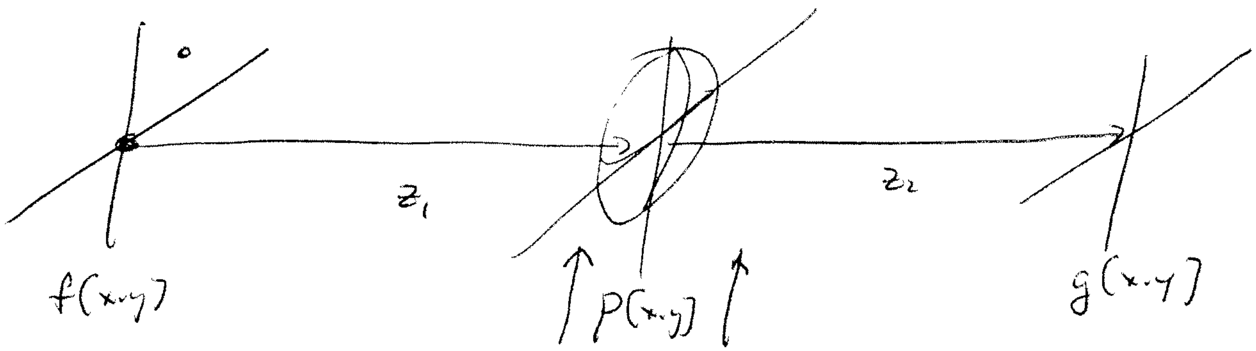
$z_1 >> f$



$$g[x,y] \propto f\left[\frac{x}{(-f/z_1)}, \frac{y}{(-f/z_1)}\right] * P\left[\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right]$$

$$h[x,y] \propto P\left[\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right]$$

FRESNEL + LENS + FRESNEL



$$f[x,y] = \delta[x,y]\,\delta[z]\,\underline{\delta[\lambda-\lambda_0]}$$

AT FRONT OF LENS $f[x,y] * h[x,y;\, z=z_1,\, \lambda_0]$ (FRESNEL ~~[illegible]~~ IMPULSE RESPONSE)

$$h[x,y;\, z=z_1, \lambda_0] = \left(\frac{1}{i\lambda_0 z_1}\, e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)} \right) e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$p[x,y;\, f] \equiv p[x,y]\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

APODIZED PUPIL ≠ BINARY

AT BACK OF LENS

$$(K_0)\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}\, p[x,y]\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}} = K_0\, p[x,y]\, e^{+i\pi \frac{x^2+y^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)}$$

PROPAGATE z_2 TO OBSERVATION PLANE

$$\left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t_1\right)} \frac{1}{i\lambda_0 z_2} e^{+2\pi i\left(\frac{z_2}{\lambda_0} - \nu_0 t_2\right)}\right)\left[p[x,y]\, e^{+i\pi \frac{x^2+y^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} * e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}}\right]$$

EVALUATE CONVOLUTION IN SPACE DOMAIN

1-D: $$p[x]\, e^{+i\pi \frac{x^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} * e^{+i\pi \frac{x^2}{\lambda_0 z_2}} = \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} \cdot e^{+i\pi \frac{(x-u)^2}{\lambda_0 z_2}}\, du$$

$$= \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)}\, e^{+i\pi \frac{x^2}{\lambda_0 z_2}}\, e^{+i\pi \frac{u^2}{\lambda_0 z_2}}\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0} \underbrace{\left(\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2}\right)}_{=0}}\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

IF $\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2} = 0$, THEN

$$e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \int_{-\infty}^{+\infty} p[u]\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

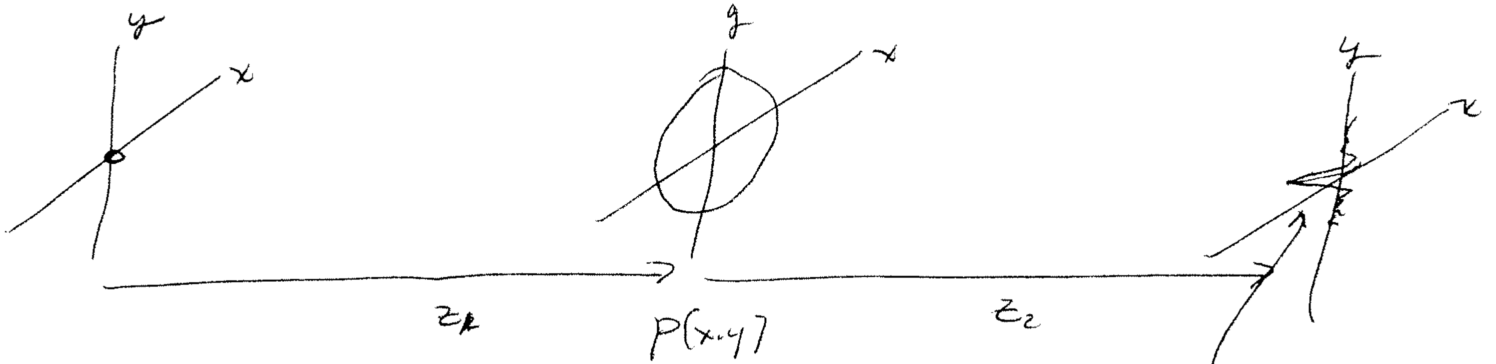
$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}}\, \mathcal{F}_1 \left\{ p[u] \right\} \Big|_{\xi = \frac{x}{\lambda_0 z_2}}$$

$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \cdot P\left[\frac{x}{\lambda_0 z_2}\right]$$

GENERALIZE TO 2-D CASE

$$e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}}\, P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

IF $\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2} = 0 \Longrightarrow \boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}}$



$$g(x,y) = P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

$$|g(x,y)|^2 = \left|P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda z_2}\right]\right|^2$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$z_1 = 2f = z_2$$

$$e^{+i\pi \frac{x^2+y^2}{\lambda_0 (2f)}} \text{ - PROPAGATION}$$

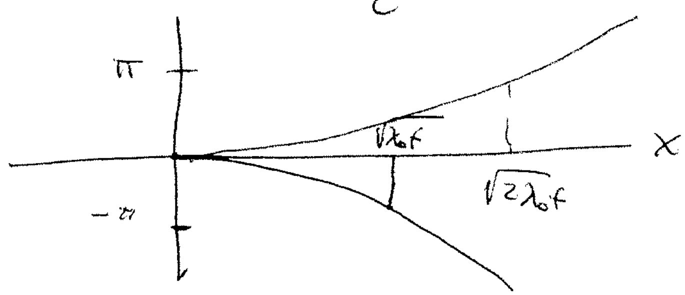
$$-\frac{z_2}{z_1} = M_T$$

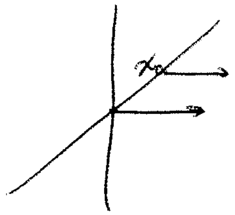
$$M_T = -1$$

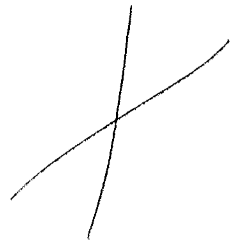
$$e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}} \text{ - LENS}$$

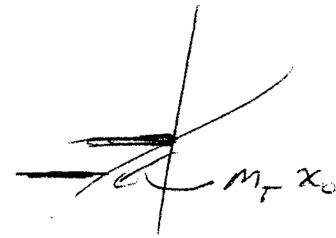
$$\alpha_1 = \sqrt{2\lambda_0 f}$$

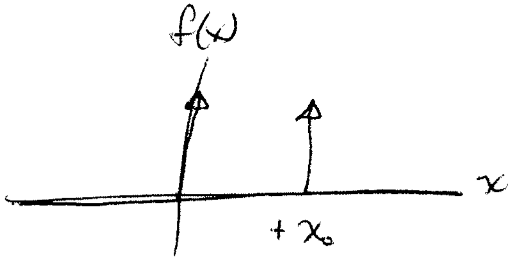
$$\alpha_{LENS} = +\sqrt{\lambda_0 f}$$

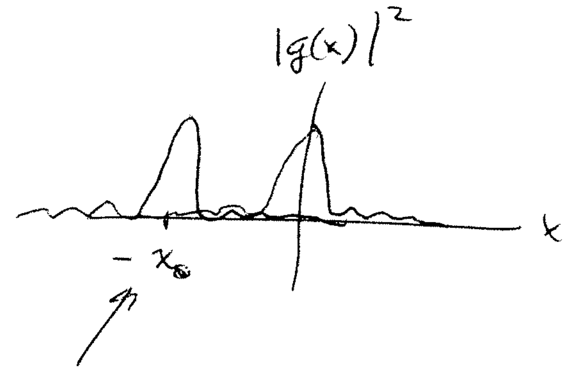












$$z_1 = z_2 \Rightarrow M_T = -1$$

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$$\cancel{\left|g(x,y)\right|^2 = \left| f\left(\frac{x}{-m_T}, \frac{y}{-m_T}\right) * h[x,y;\lambda_0, z_1, z_2]\right|^2}$$

$$h(x,y;\lambda_0;z_1,z_2) = e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}} P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

LSI IF CORRECTLY INTERPRETED

$$\left| f\left[\frac{x}{+m_T}, \frac{y}{+m_T}\right] * h(x,y; \ldots]\right|^2 = g(x,y)$$

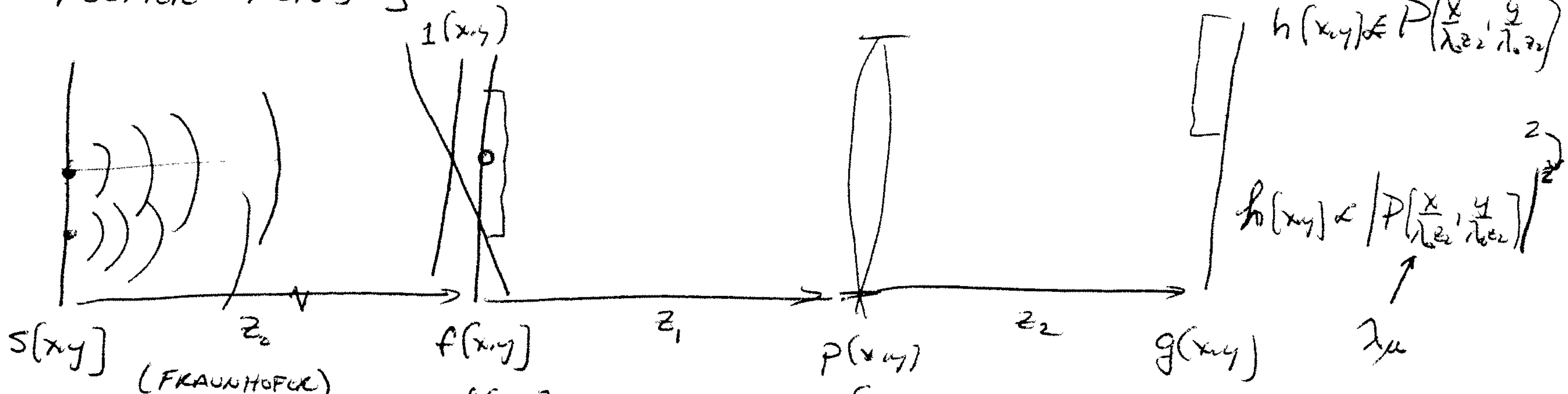
$$m_T = -\frac{z_2}{z_1}$$

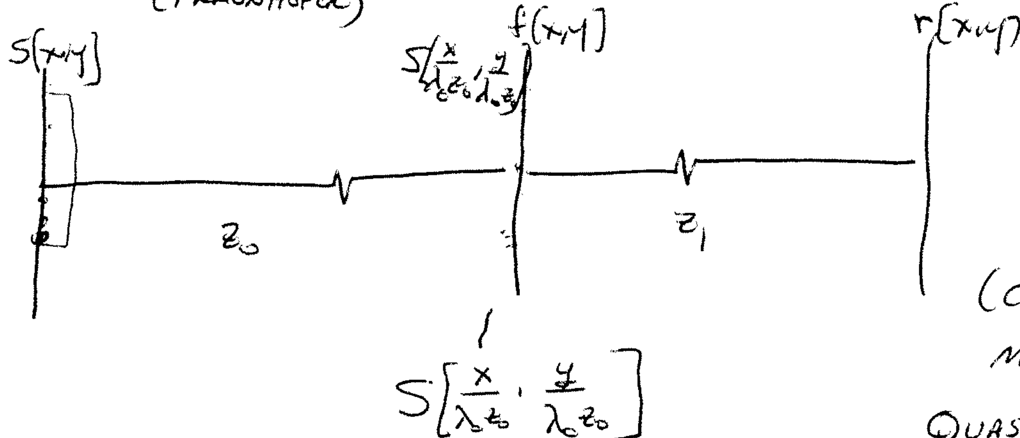
WHAT IF MULTIPLE WAVELENGTHS?

WHAT IF PHASE IS NOT DETERMINISTIC?

OPTICS NOTES § 3.8

FOURIER NOTES § 22





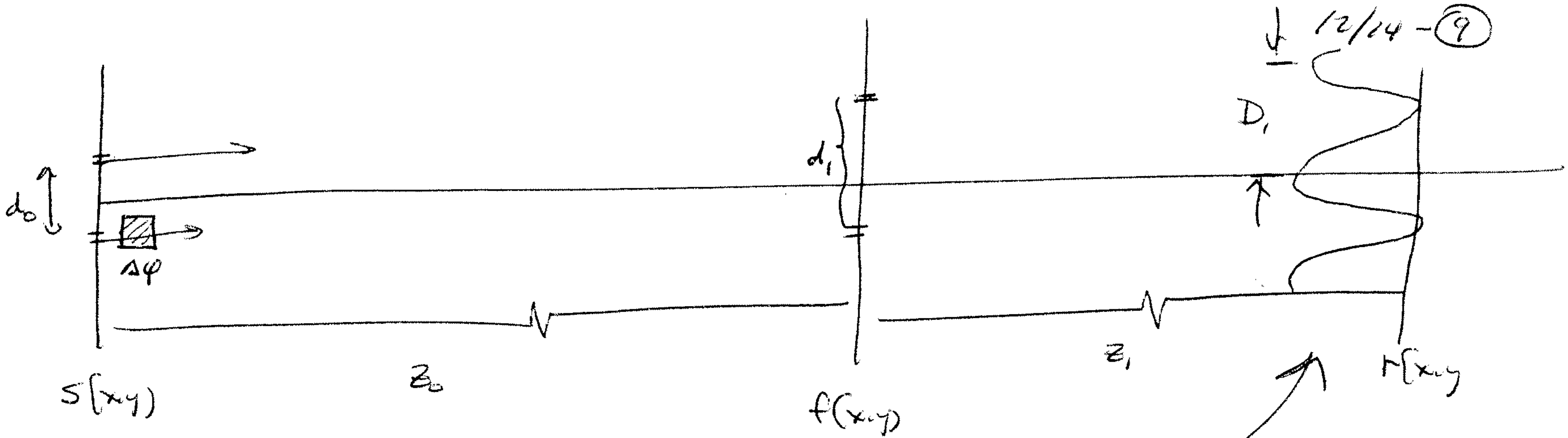
COHERENCE OF SOURCE

TEMPORAL

SPATIAL

(CORRELATION OF PHASES MEASURED AT OBJECT)

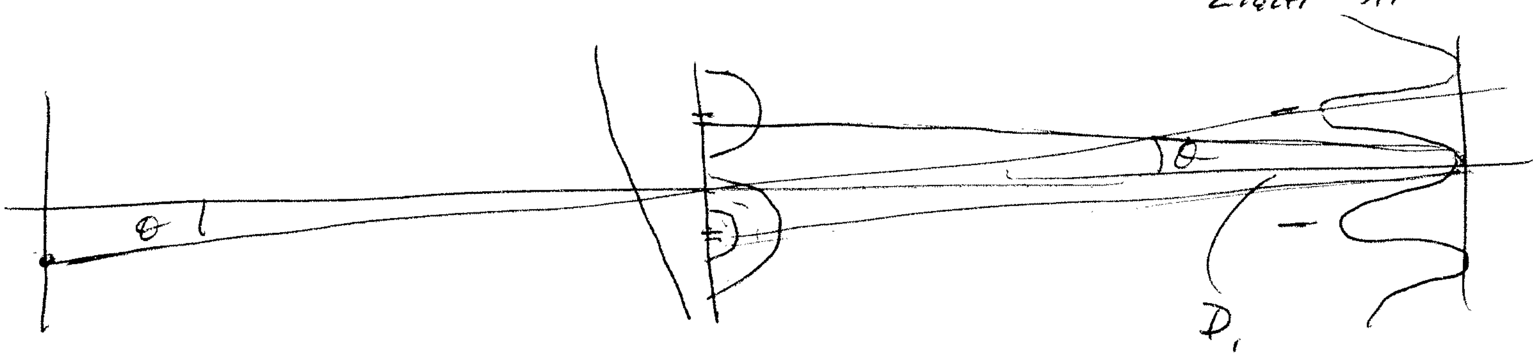
QUASIMONOCHROMATIC LIGHT

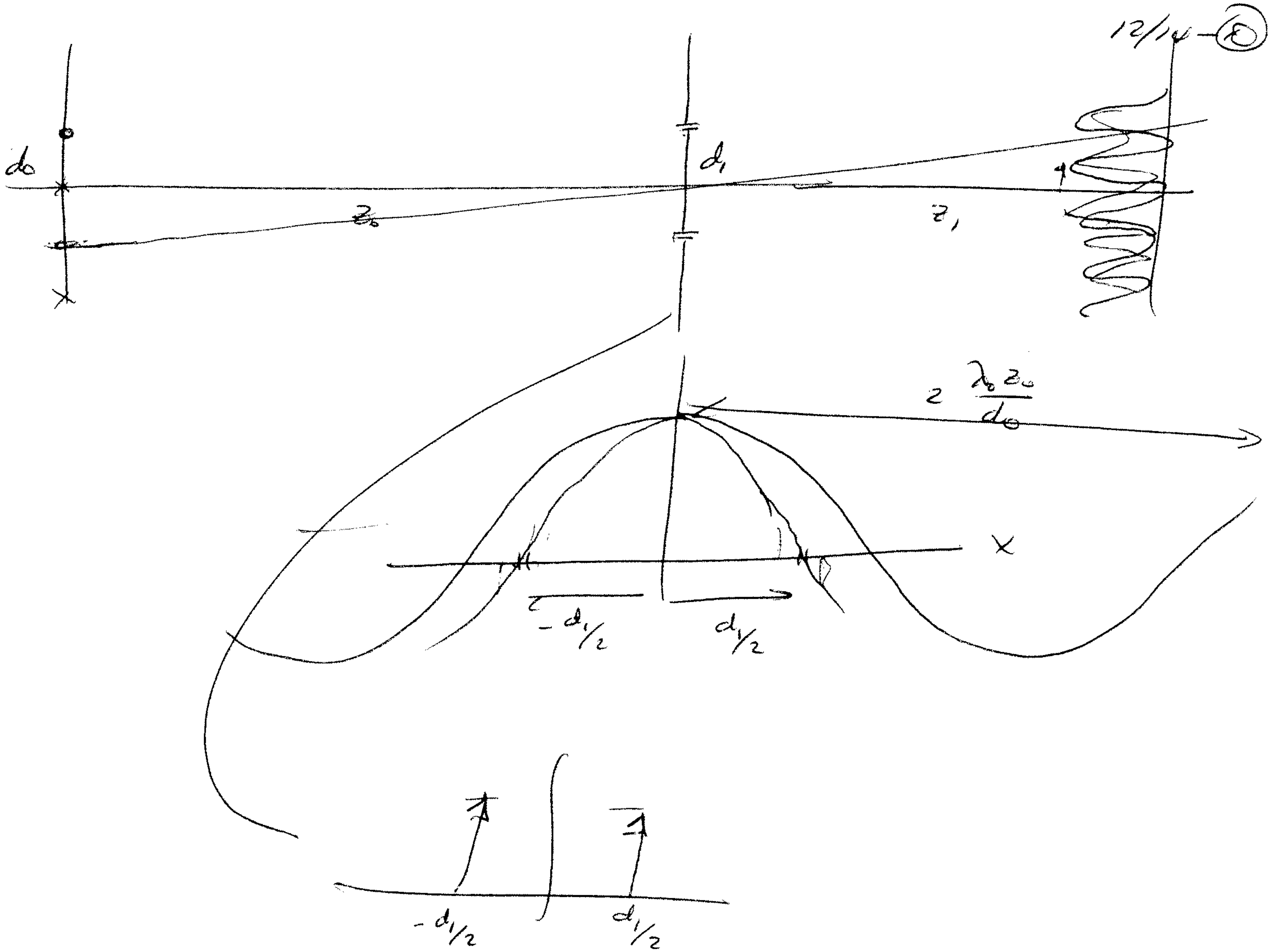


$$D_1 = \frac{\lambda_0 z_1}{d_1}$$

INTERFERENCE

IF INTERFERENCE FRINGES ARE VISIBLE $\Rightarrow$ "COHERENCE" OF LIGHT AT OBJECT





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d_0
d_1
z_0
z_1
$2\frac{\lambda_0 z_0}{d_0}$
x
$-d_1/2$
$d_1/2$
$-d_1/2$
$d_1/2$