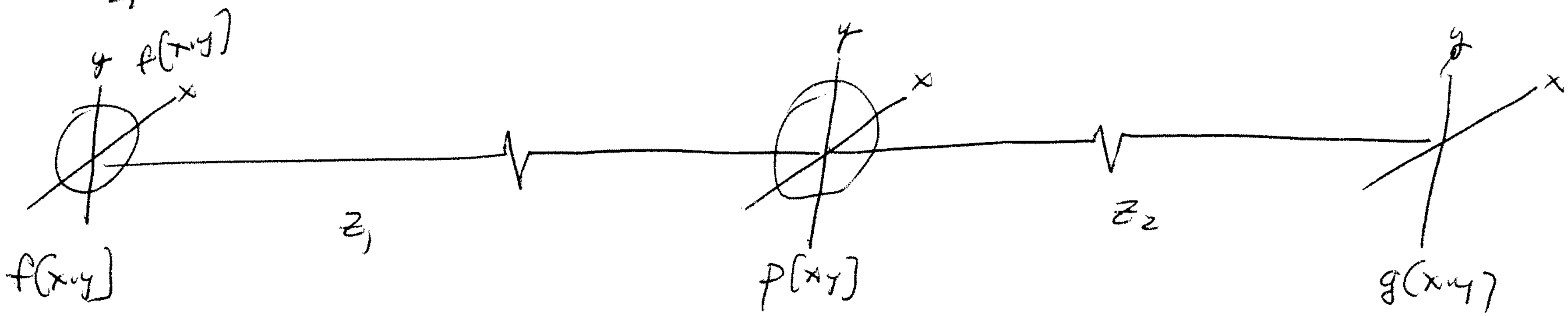
~~9 December 2009~~

9 December 2009

①



$$\left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)} F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]\right) p[x,y]$$ AFTER APERTURE

$$\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t_1\right)} \frac{1}{i\lambda_0 z_2} e^{+2\pi i\left(\frac{z_2}{\lambda_0} - \nu_0 t_2\right)} \left[f\left[\frac{x}{\left(-z_2/z_1\right)}, \frac{y}{\left(-z_2/z_1\right)}\right] * P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right]$$

$$g[x,y] \propto f\left[\frac{x}{M_T}, \frac{y}{M_T}\right] * \underbrace{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]}_{h[x,y]}$$

12/9 – 2

12/9 – 2

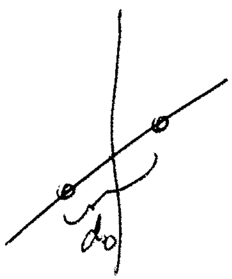
$$h[x,y] \propto P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

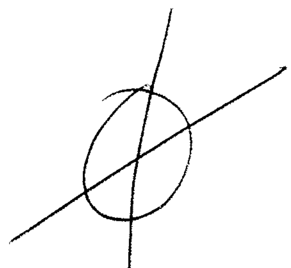
$$H[\xi,\eta] \propto \mathcal{F}\left\{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right\} \quad (\lambda_0 z_2)^2\, p\left[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta\right]$$

SCALED & INVERTED REPLICA OF PUPIL

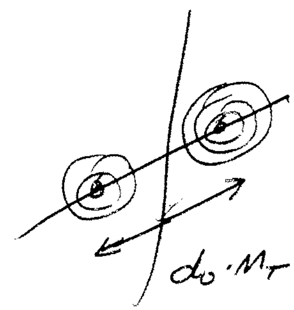
$$H[\xi,\eta] \propto p\left[\frac{\xi}{\left(-\frac{1}{\lambda_0 z_2}\right)}, \frac{\eta}{\left(-\frac{1}{\lambda_0 z_2}\right)}\right]$$

FOR SINGLE λ_0
(CANCELLATION IF OUT OF PHASE BY π)





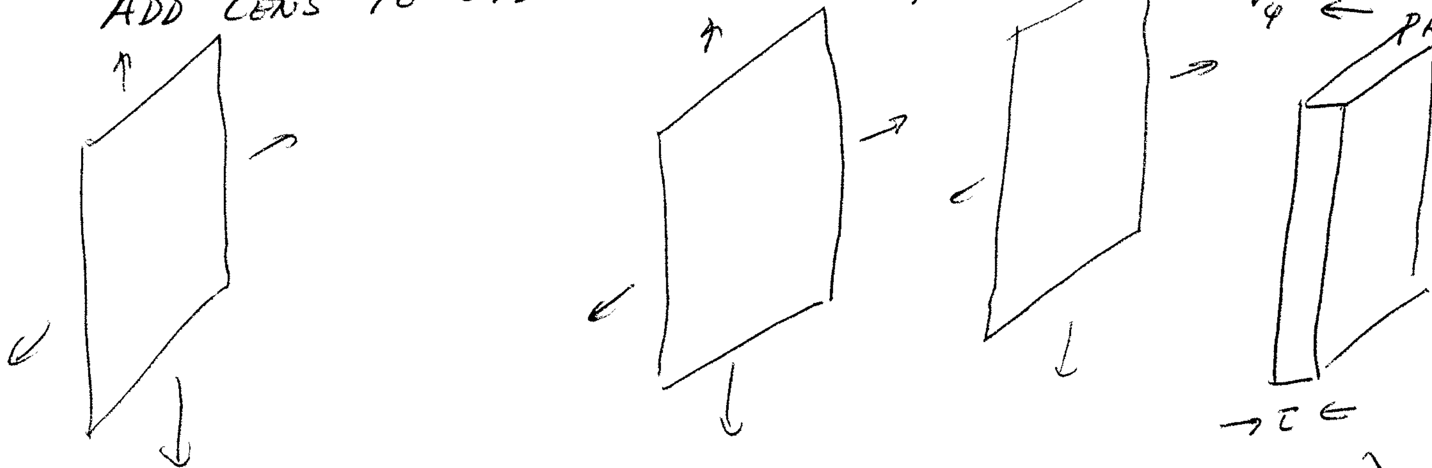
$p(xy)$



$$M_T = -\frac{z_2}{z_1}$$

RESOLUTION - OVERLAPPING IMPULSE RESPONSES

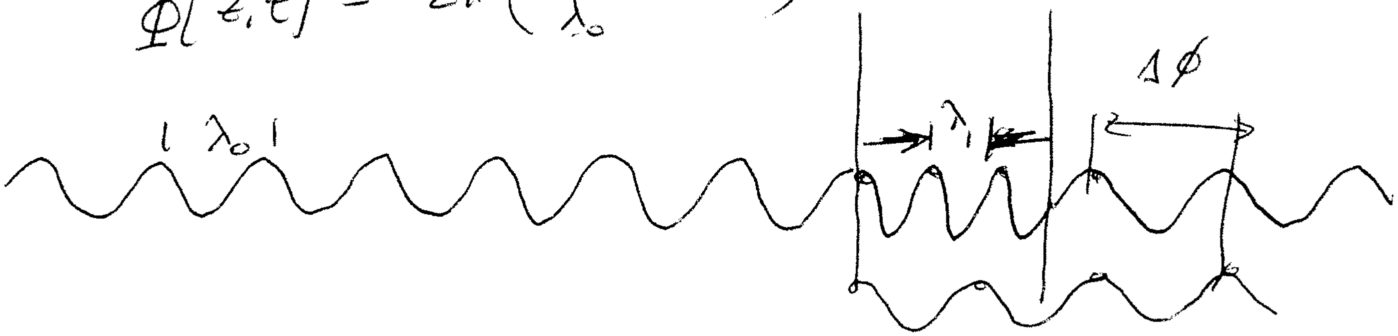
ADD LENS TO SYSTEM $\Longrightarrow$ GLASS, INDEX $n = \frac{c}{V_\varphi}$ ← PHASE VELOCITY IN GLASS



$\Delta\phi$ = CONSTANT

$$\lambda_1 = \frac{\lambda_0}{n}$$

$$\Phi[z,t] = 2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)$$



12/9 - 4

12/9 - 4

$$\Delta\phi = \phi(\text{GLASS}) - \phi(\text{VACUUM})$$

$$\Delta t = \frac{\tau}{V} \implies \Delta t = \frac{\tau}{c/n} - \frac{\tau}{c}$$

$$\Delta t = (n-1)\cdot\frac{\tau}{c}$$

$$\Phi = \frac{2\pi}{\lambda_0}\cdot c\cdot\Delta t$$

$$\Delta\phi = \frac{2\pi}{\lambda_0}\cdot\cancel{c}\,(n-1)\cdot\frac{\tau}{\cancel{c}} = \frac{2\pi}{\lambda_0}\cdot(n-1)\cdot\tau$$

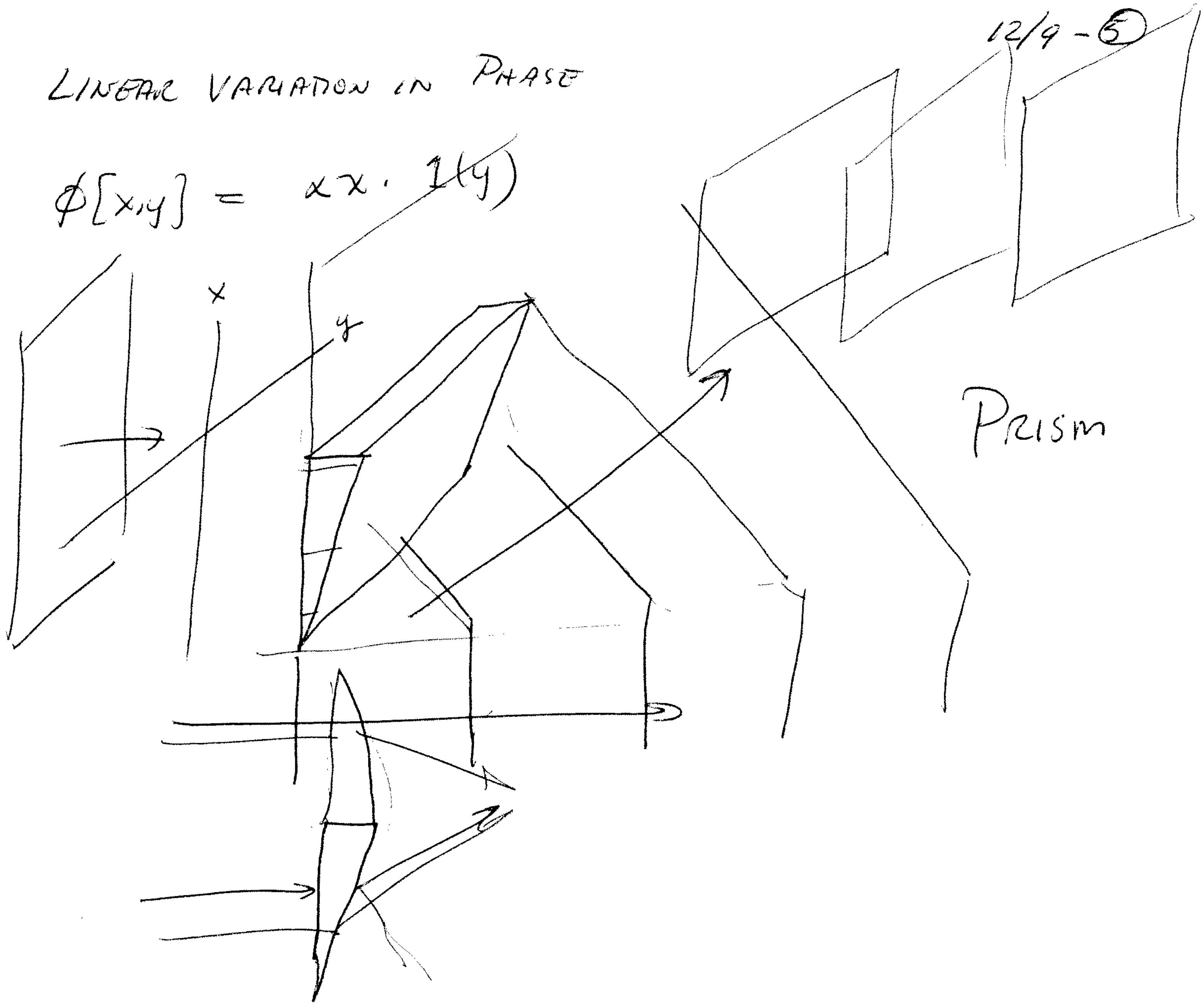
(1) UNIFORM THICKNESS $\rightarrow$ ~~$\Delta\phi = 2\pi$~~

$$\Delta\phi[x,y] = \frac{2\pi}{\lambda_0}\cdot(n-1)\cdot\tau_0\cdot 1[x,y]$$

$$e^{i\Delta\phi} = e^{i\frac{2\pi}{\lambda_0}(n-1)\cdot\tau_0\cdot 1(x,y)}\;;\;|e^{i\Delta\phi}| = 1$$

2 LINEAR VARIATION IN PHASE

$$\phi[x,y] = \alpha x \cdot 1(y)$$

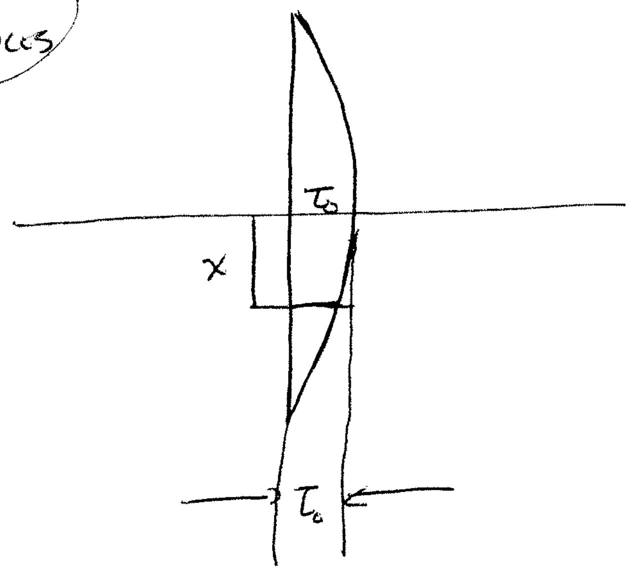


PRISM

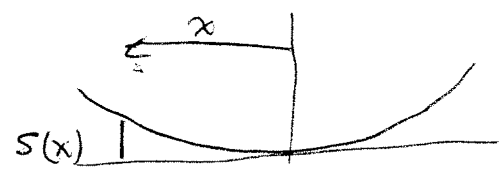
~~③ SPHER~~

③ SPHERICAL SURFACES

12/9 - ⑥



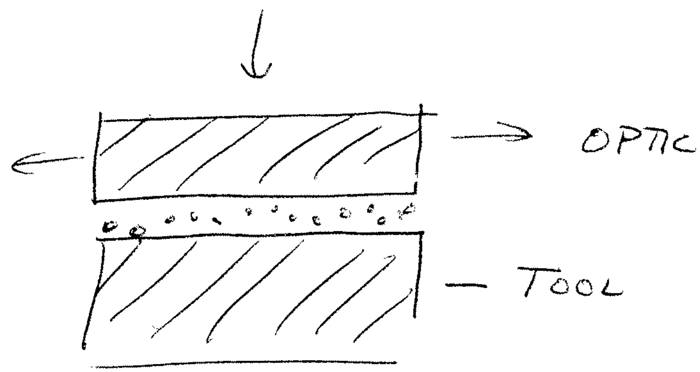
$$\tau_0 = \underset{\text{IN GLASS}}{\tau(x)} + \underset{\text{IN AIR "SAG"}}{s(x)} \Rightarrow s(x) = \tau_0 - \tau(x)$$

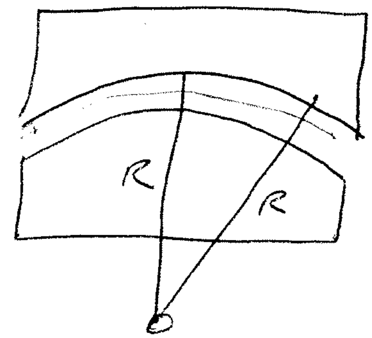


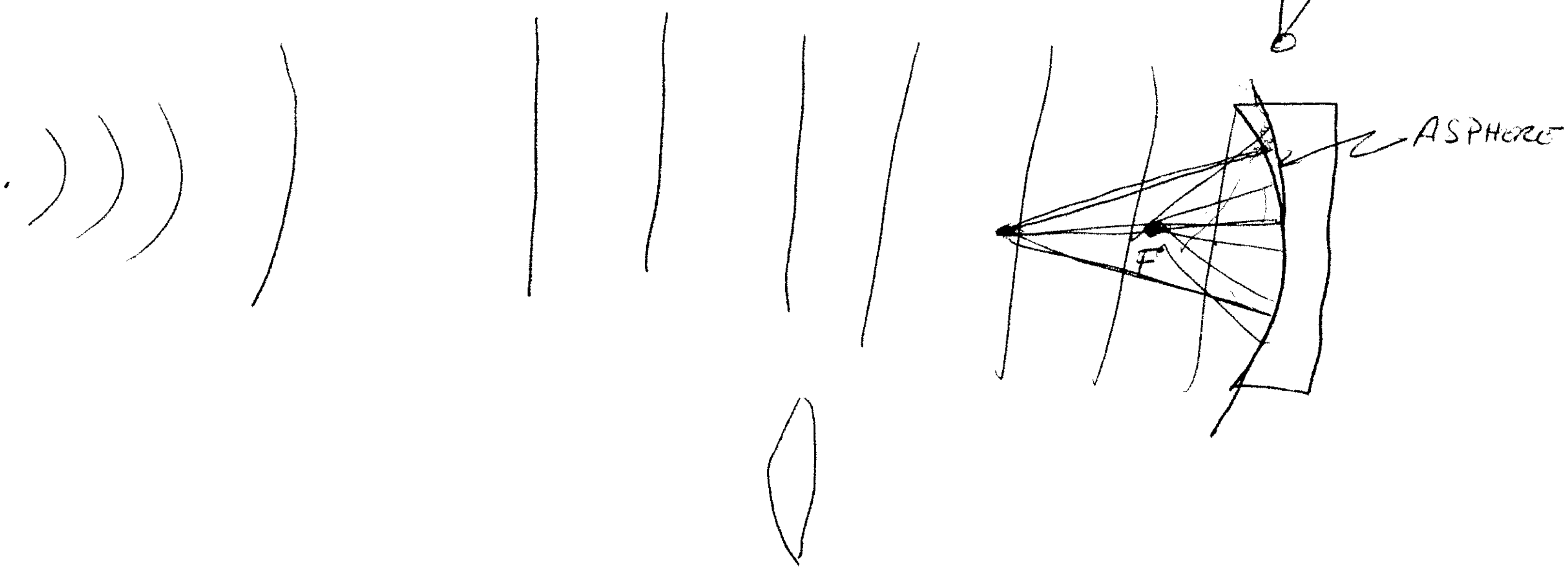
$$\tau(x,y) \rightarrow \tau(x)$$

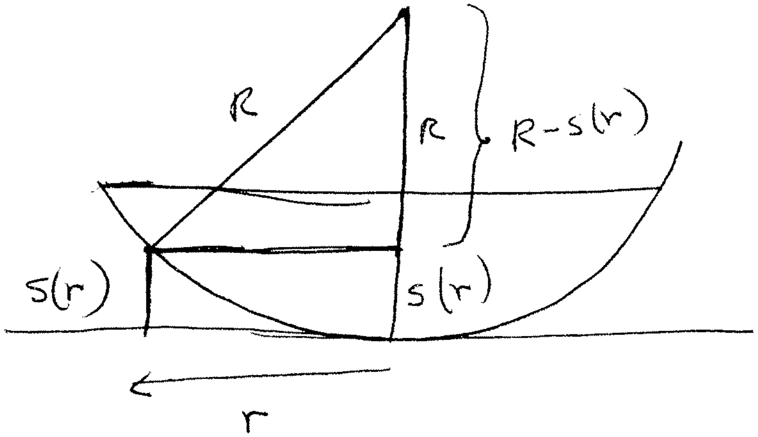
$$s(x) = \tau_0 - \tau(x)$$

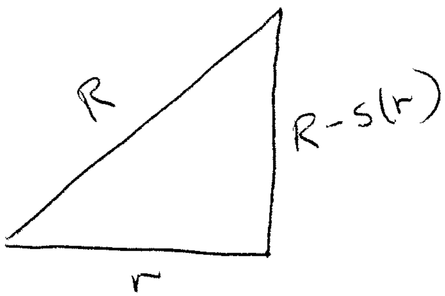
SPHERICAL SURFACES











$$R^2 = r^2 + (R - s(r))^2$$

$$\cancel{R^2} = r^2 + \cancel{R^2} + s^2(r) - 2Rs(r)$$

$$2Rs(r) = r^2 + \cancel{(s(r))^2}$$

IF $R \gg s(r) \Rightarrow R \gg r \Rightarrow (s(r))^2 \ll r^2$

$$\boxed{s(r) \cong \frac{r^2}{2R}}$$ SAG FORMULA

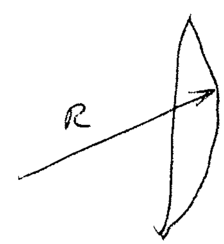
$$s(r) = \frac{r^2}{2R}$$

$$\Delta\phi \propto \tau(r) = \tau_0 - s(r) = \tau_0 - \frac{r^2}{2R}$$

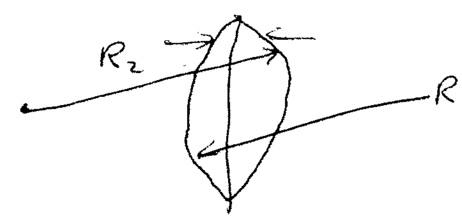
$$\Delta\phi = \frac{2\pi}{\lambda_0}(n-1)\cdot\tau(r) = \frac{2\pi}{\lambda_0}(n-1)\tau_0 - \frac{2\pi}{\lambda_0}(n-1)\frac{r^2}{2R}$$

$$= \underbrace{\frac{2\pi}{\lambda_0}(n-1)\tau_0}_{\text{CONSTANT}} - \underbrace{\frac{\pi}{\lambda_0 R}(n-1)r^2}_{\text{QUADRATIC}}$$

$$e^{i\Delta\phi} = \underbrace{e^{+\frac{2\pi i}{\lambda_0}(n-1)\tau_0}}_{\text{CONSTANT}} \; \underbrace{e^{-\frac{i\pi}{\lambda_0 R}(n-1)r^2}}_{\text{QUADRATIC}}$$



$$\Delta\phi = \pi(n-1)\frac{r^2}{\lambda_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



$$\frac{1}{f}$$

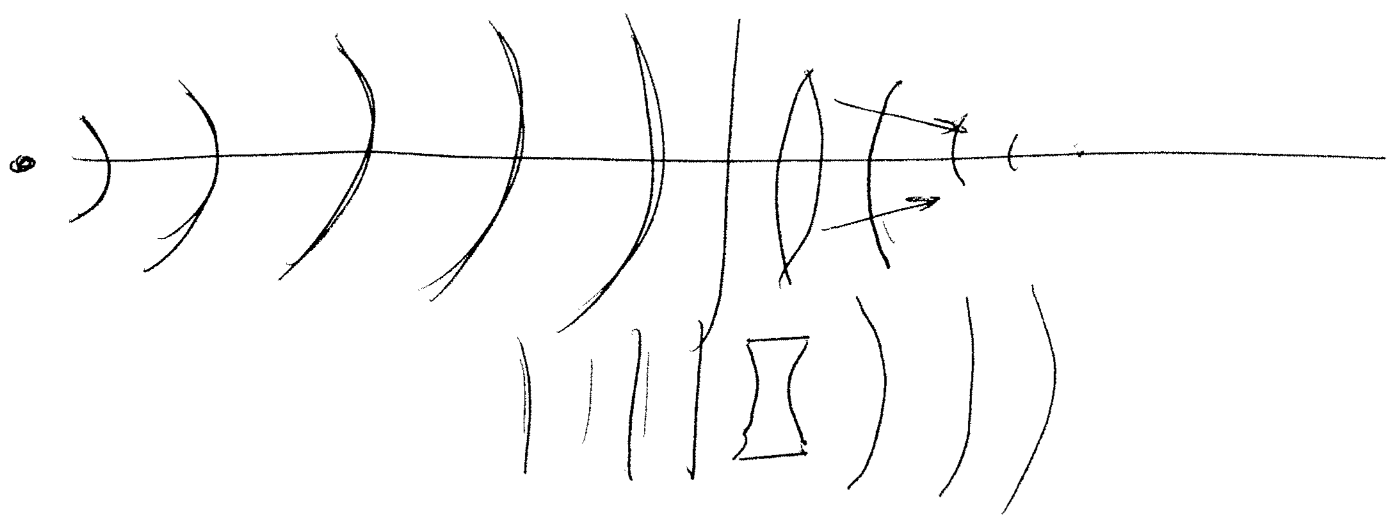
12/9 - 10

$$(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \equiv \frac{1}{f} \quad ; \quad f = \text{"FOCAL LENGTH"}$$

$$\cancel{\Delta\phi} \quad \Delta\phi(r) = -\frac{\pi r^2}{\lambda_0 f}$$

(APPROXIMATE)

$e^{-i\pi \frac{r^2}{\lambda f}}$ — CHANGE IN PHASE AS FUNCTION r FOR LENS WITH FOCAL LENGTH f



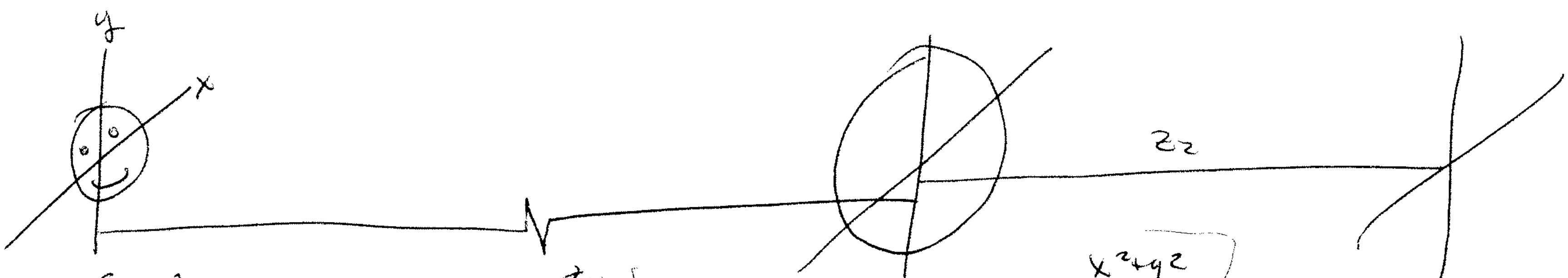
APERTURE $p(r)$

PHASE $e^{-i\pi \frac{r^2}{\lambda_0 f}}$

$$\left.\begin{array}{l} p(r) \\ e^{-i\pi \frac{r^2}{\lambda_0 f}} \end{array}\right\} \quad t(r) = \underline{\underline{p(r)}}\,\underline{\underline{e^{-i\pi \frac{r^2}{\lambda_0 f}}}}$$

$$= p[x,y]\; e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

RETURN TO SYSTEM



$f[x,y]$ z_1 $g(x,y)$

$$\boxed{F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] \cdot p(x,y)\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}}$$

AT BACK OF LENS $\longrightarrow$ PROPAGATE TO FRESNEL DIFFRACTION z_2

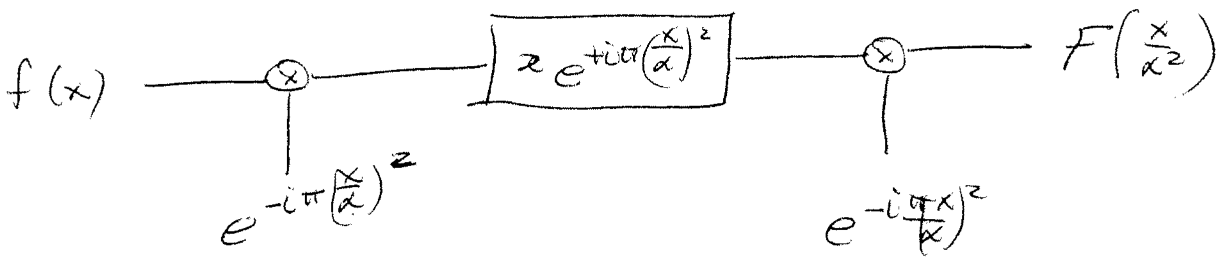
$$(\ )\left(F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]\cdot p[x,y]\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 \cancel{z_1} f}}\right) * (\ )\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}} = g(x,y)$$

$$(\ )\left(\left(F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right)\cdot p(x,y)\right)\cdot e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}\right) * e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}} = g(x,y)$$

$z_2 = f$

M-C-M CHIRP FOURIER TRANSFORM

$\xi \to \frac{x}{\alpha^2}$



1-D Fourier Transform

$$F(\xi) = \int_{-\infty}^{+\infty} f(x)\, e^{-2\pi i \xi x}\, dx$$

$$\boxed{-2\xi x = \left(\alpha\xi - \frac{x}{\alpha}\right)^2 - (\alpha\xi)^2 - \left(\frac{x}{\alpha}\right)^2}$$

$$= \cancel{(\alpha\xi)^2} + \cancel{\left(\frac{x}{\alpha}\right)^2} - 2\cancel{\alpha}\xi\cdot\frac{x}{\cancel{\alpha}} - \cancel{(\alpha\xi)^2} - \cancel{\left(\frac{x}{\alpha}\right)^2}$$

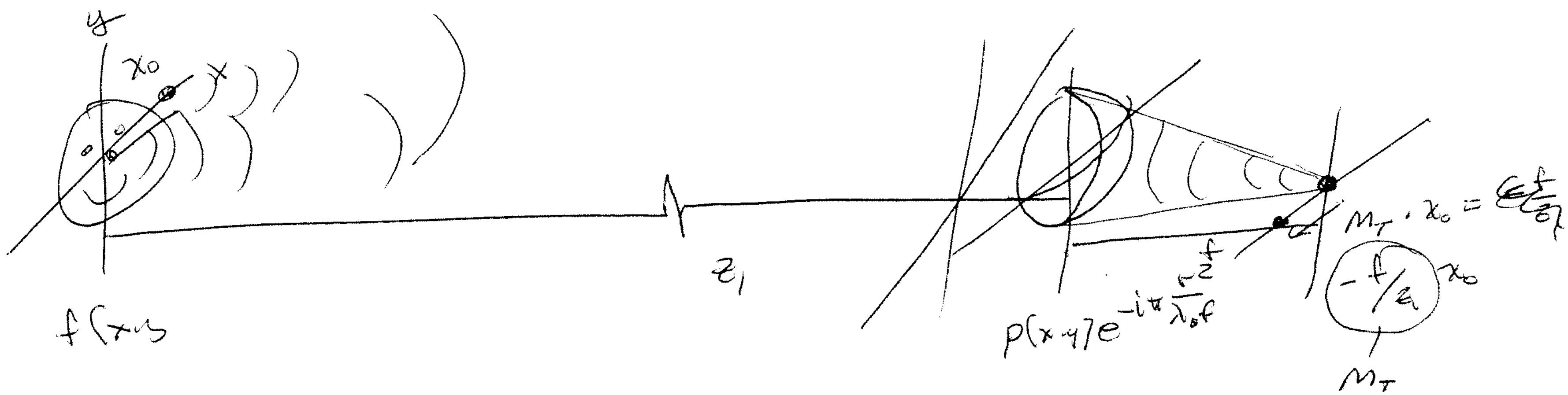
$$F(\xi) = \int_{-\infty}^{+\infty} \left(\underset{m \cdot e^{-i\pi\left(\frac{x}{\alpha}\right)^2}}{f(x)\, e^{-i\pi\left(\frac{x}{\alpha}\right)^2}}\right) \underbrace{e^{+i\pi\left(\alpha\xi - \frac{x}{\alpha}\right)^2}}_{h\left(\alpha\xi - \frac{x}{\alpha}\right)} e^{-i\pi(\alpha\xi)^2}\, dx$$

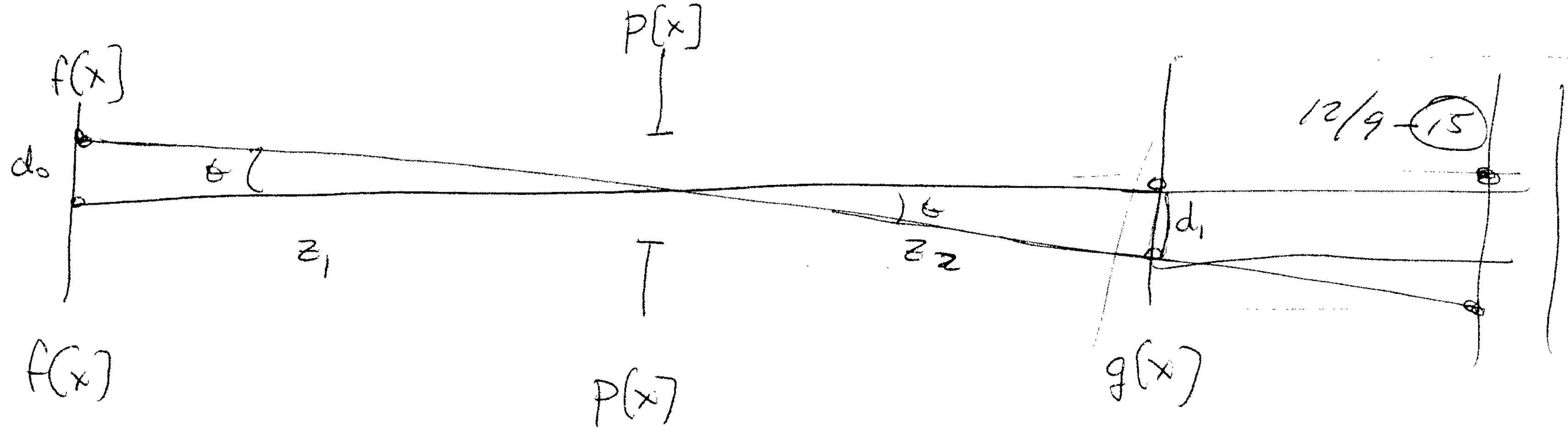
$$F(\xi) = \left(\left(f(x)\, e^{-i\pi\left(\frac{x}{\alpha}\right)^2}\right) * e^{+i\pi\left(\frac{x}{\alpha}\right)^2}\right)\Big|_{\frac{x}{\alpha} \to \alpha\xi} \cdot e^{-i\pi(\alpha\xi)^2}$$

$$g(x,y) = (\ )\left(\left(F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] p(x,y)\right) \cdot e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}\right) * e^{+i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

$$= (\ )\, \mathcal{F}_2\left\{ F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] p(x,y)\right\} \cdot \underline{e^{+i\pi \frac{x^2+y^2}{\lambda_0 f}}}$$

$$|g(x,y)|^2 = |(\ )|^2 \quad (\lambda_0 z_1)^2 \left|\left(f\left[\frac{x}{-f/z_1}, \frac{y}{-f/z_1}\right] * P\left(\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right)\right)\right|^2 \cdot 1$$

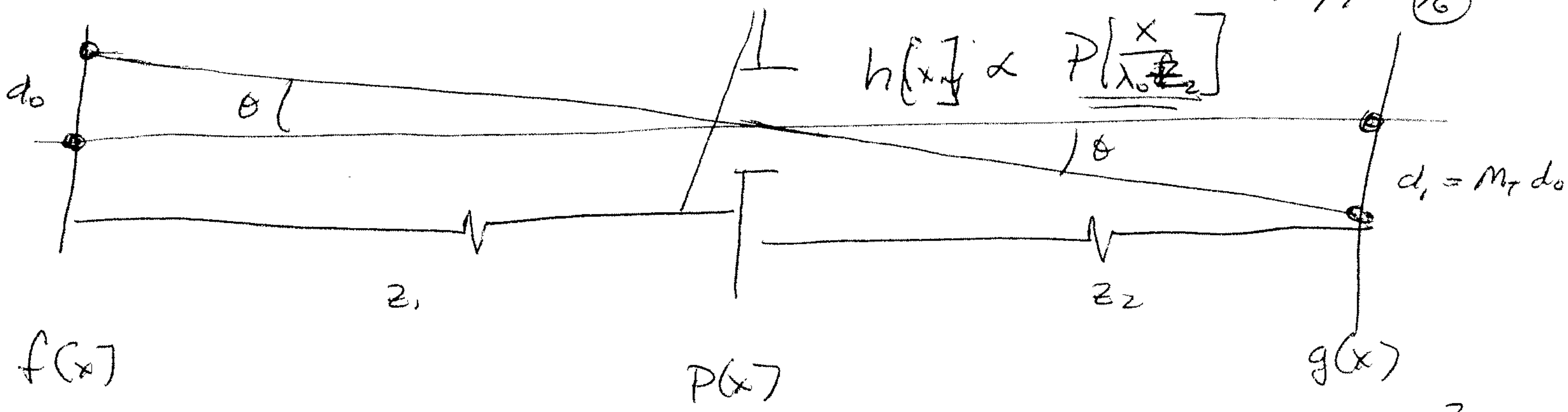




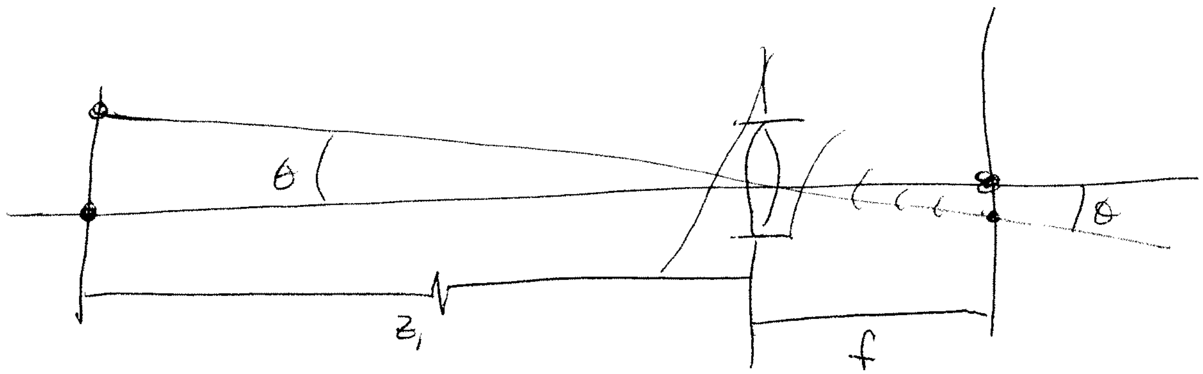
$$g(x] = \cancel{(\quad)\, f\left(\frac{x}{-\frac{z_2}{z_1}}\right) * P\left[\frac{x}{\lambda_0 z_2}\right]} \quad (\quad)\, f\left(\frac{x}{-\frac{z_2}{z_1}}\right) * P\left[\frac{x}{\lambda_0 z_2}\right]$$

$$\frac{d_1}{d_0} = \left(-\frac{z_2}{z_1}\right)$$

$$h[x] \propto P\left[\frac{x}{\lambda_0 z_2}\right]$$



$$M_T = -\frac{z_2}{z_1}$$



$$M_T = -\frac{f}{z_1}$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$h[x] \propto P\left[\frac{x}{\lambda_0 f}\right]$$