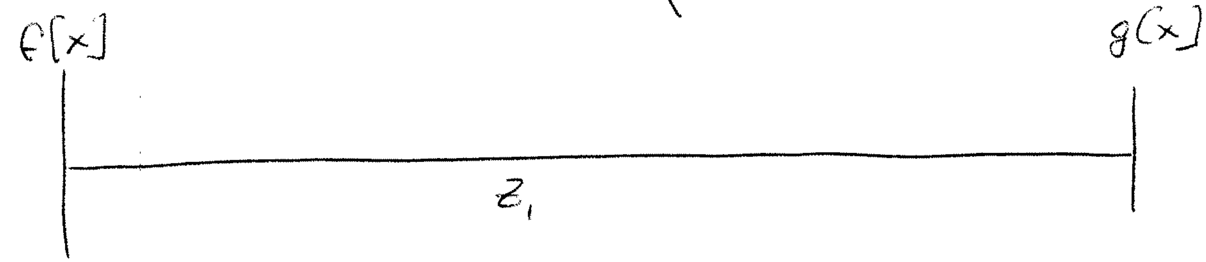
7 DECEMBER 2009

FRESNEL REGION = "NEAR FIELD" (FRAUNHOFER = FAR FIELD)

$$g[x] = \frac{1}{(i\lambda)z_1} \underline{e^{2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)}} \left(f(x) * e^{+i\pi \frac{x^2}{\lambda_0 z_1}} \right)$$



MEASURE "IRRADIANCE" — $\langle |g(x,y)|^2 \rangle$

TIME AVERAGE

$$I[x,y] = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} |g(x,y;t)|^2 \, dt \; ; \quad T_0 = \text{AVERAGING TIME}$$

$$\longrightarrow |g(x,y)|^2$$

$$f[x,y] = \delta(x-x_0,\ y-y_0] \longrightarrow g(x,y] = \underbrace{\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_0}{\lambda_0} - \nu_0 t\right)}}_{K_0} \; e^{+i\pi \frac{(x-x_0)^2+(y-y_0)^2}{\lambda_0 z_1}}$$

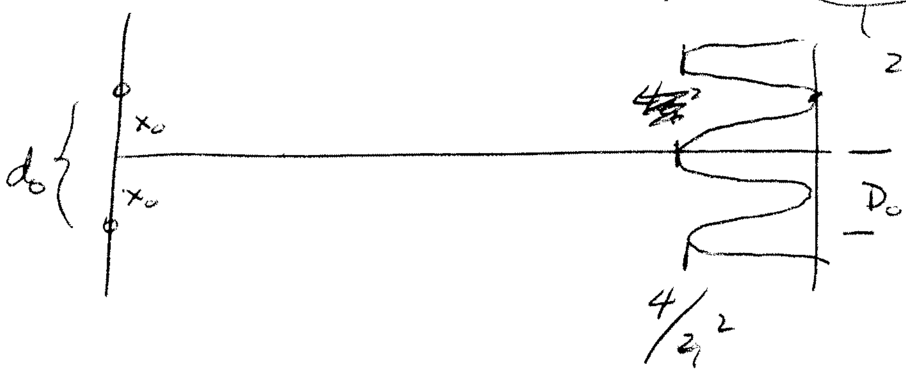
$$\left|g[x,y]\right|^2 = |K_0|^2 \cdot 1[x,y] = \frac{1}{\lambda_0^2 z_1^2} \cdot 1(x,y]$$

INVERSE SQUARE LAW

$$\delta[x+x_0] + \delta[x-x_0] \longrightarrow \frac{1}{z_1^2} \cdot 4 \cdot \frac{1}{2}\left(1+\cos\left(2\pi \frac{x}{\frac{\lambda_0 z_1}{2x_0}}\right)\right)$$

2

D_0 d_0



$$D_0 = \frac{\lambda_0 z_1}{d_0}$$

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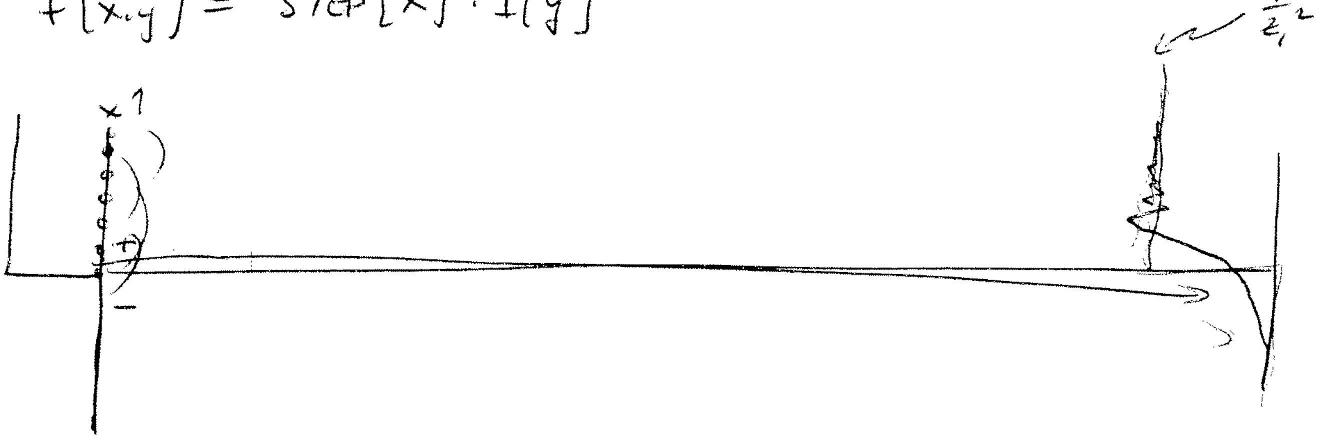
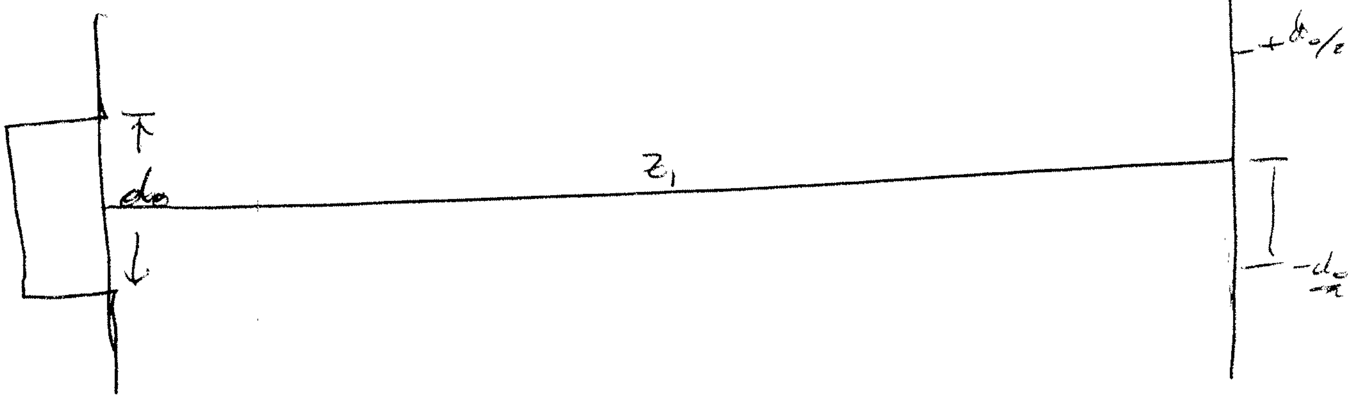
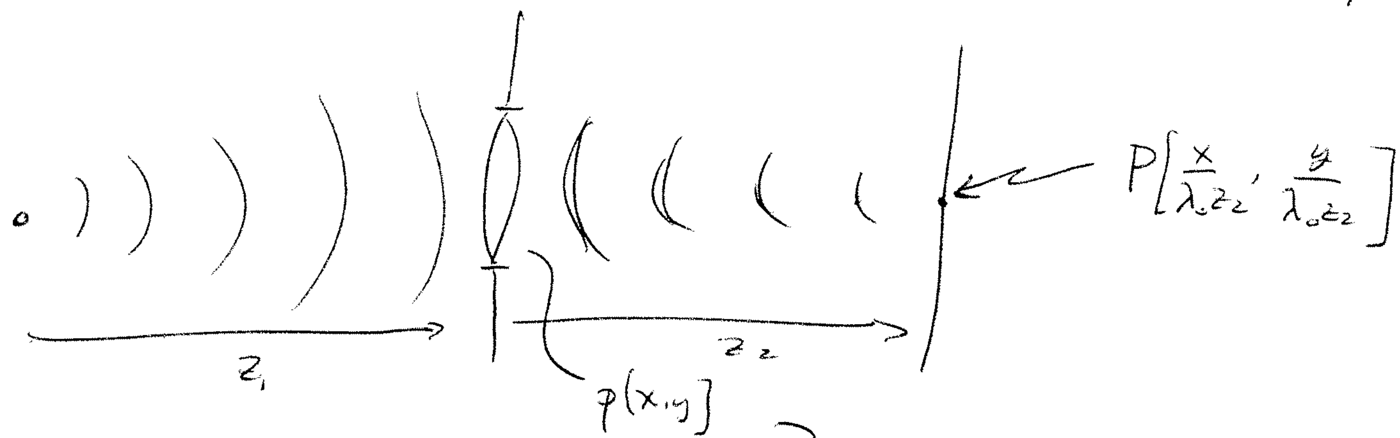
$f[x,y] = \mathrm{STEP}[x] \cdot 1[y]$



$\mathrm{RECT}\left[\frac{x}{d_0}\right] = \mathrm{STEP}\left(x + \frac{d_0}{2}\right) - \mathrm{STEP}\left(x - \frac{d_0}{2}\right) \rightarrow \left|\left(\mathrm{STEP}(x) * h(x)\right) * \left(\delta\left(x + \frac{d_0}{2}\right) - \delta\left(x - \frac{d_0}{2}\right)\right)\right|^2$





PROPERTIES OF FRESNEL DIFFRACTION PATTERNS

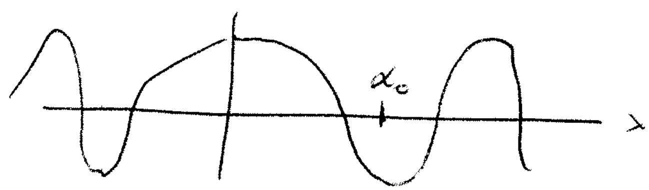
(1) LARGER OBJECT $\Rightarrow$ LARGER "IMAGE"

(2) PATTERN VARIES WITH z_1

$$e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

α_0^2

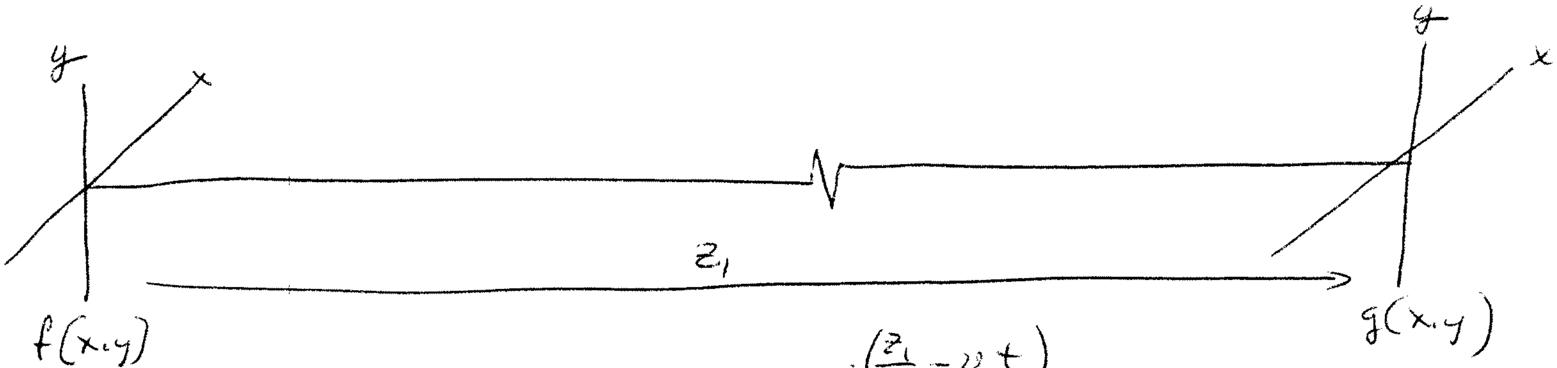
$$\alpha_0 = \sqrt{\lambda_0 z_1} \quad \text{- CHIRP RATE}$$



INCREASING $z_1 \Rightarrow$ COARSER "OSCILLATIONS" (RINGING)

(3) ALLPASS FILTER W/ QUADRATIC PHASE

"FAR FIELD" - FRAUNHOFER DIFFRACTION



IN NEAR FIELD $\quad \frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)}$

$$g(x,y) = K_0 \, f(x,y) * e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$= K_0 \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, e^{+i\pi\frac{(x-\alpha)^2+(y-\beta)^2}{\lambda_0 z_1}}\, d\alpha\, d\beta$$

$$g(x,y) = K_0 \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, \underbrace{e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_1}}}_{\text{QUADRATIC}}\, \underbrace{e^{+i\pi\frac{\alpha^2+\beta^2}{\lambda_0 z_1}}}_{\text{QUADRATIC}}\, \underbrace{e^{-\frac{2\pi i}{\lambda_0 z_1}(x\alpha+y\beta)}}_{\text{"BILINEAR"}}\, d\alpha\, d\beta$$

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$$g(x,y) = K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, \cancel{e^{+i\pi \frac{\alpha^2+\beta^2}{\lambda_0 z_1}}}\, e^{-2\pi i\left(\alpha \frac{x}{\lambda_0 z_1} + \beta \frac{y}{\lambda_0 z_1}\right)} d\alpha\, d\beta$$

NEW ASSUMPTION, CONSTRAIN SUPPORT OF $f(\alpha,\beta)$, i.e. $|\alpha| \cong 0$, $|\beta|$

IF $\lambda_0 z_1 >> \alpha^2+\beta^2 \Rightarrow \frac{\alpha^2+\beta^2}{\lambda_0 z_1} \simeq 0 \Rightarrow e^{+i\pi \frac{\alpha^2+\beta^2}{\lambda_0 z_1}} \simeq 1$

OBSERVATION PLANE IS FARTHER FROM OBJECT

$$g(x,y) \cong K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, e^{-2\pi i\left(\alpha \cdot \left(\frac{x}{\lambda_0 z_1}\right) + \beta\left(\frac{y}{\lambda_0 z_1}\right)\right)} d\alpha\, d\beta$$

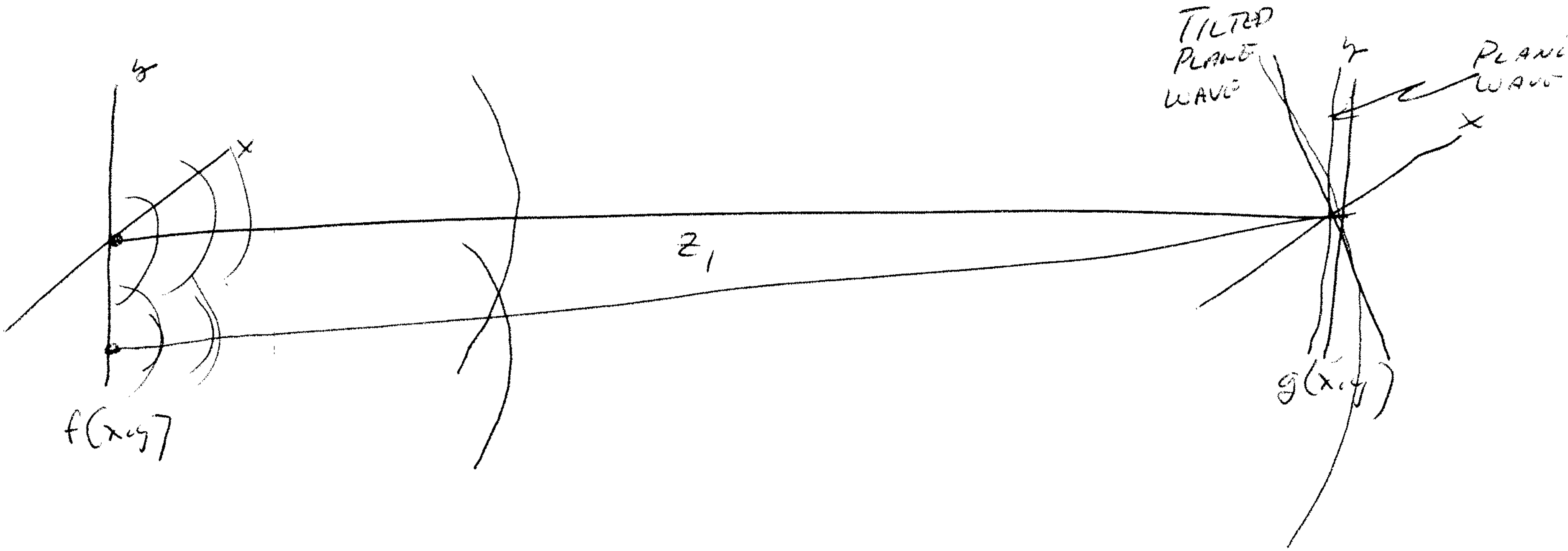
$\frac{x}{\lambda_0 z_1}$, $\frac{y}{\lambda_0 z_1}$ HAVE DIMENSIONS OF LENGTH^{-1}, e.g. $\frac{\text{CYCLES}}{\text{mm}}$

$$\boxed{g(x,y) \cong K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \cdot \underbrace{\mathcal{F}_2\{f(x,y)\}}_{F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]}\Bigg|_{\xi \to \frac{x}{\lambda_0 z_1},\ \eta \to \frac{y}{\lambda_0 z_1}}}$$

FRAUNHOFER DIFFRACTION OF $f(x,y)$

$$g[x,y;z_1,\lambda_0,\nu_0] \cong K_0\, e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_1}}\, F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]$$

$\angle SV \Longrightarrow$ NO IMPULSE RESPONSE

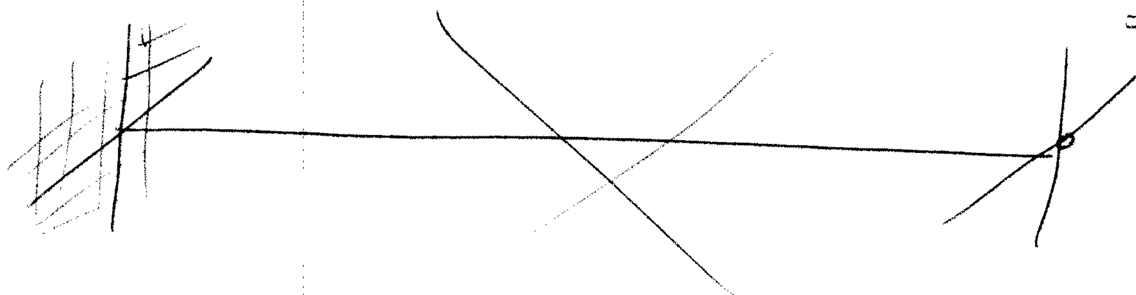


① $f[x,y] = 1[x,y] \longrightarrow F[\xi,\eta] = \delta(\xi,\eta]$

(NOT SMALL, COMPACT SUPPORT) $F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] = \delta\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]$

$\Longrightarrow$ VIOLATES ASSUMPTION

$= (\lambda_0 z_1)^2 \, \delta[x, y]$



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② $f(x,y] = \mathrm{RECT}\left(\frac{x}{b_0}, \frac{y}{d_0}\right] \longrightarrow F(\xi,\eta] = |b_0 d_0| \, \mathrm{SINC}(b_0\xi, d_0\eta)$

$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = |b_0 d_0| \, \mathrm{SINC}\left(b_0 \frac{x}{\lambda_0 z_1}, \frac{d_0 y}{\lambda_0 z_1}\right)$

$$g[x,y] = K_0 \, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \, |b_0 d_0| \, \mathrm{SINC}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{b_0}\right)}, \frac{y}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)$$

WIDTH PARAMETERS OF SINC ARE $\frac{\lambda_0 z_1}{b_0}, \frac{\lambda_0 z_1}{d_0}$

WIDTH $\uparrow$ IF $\lambda_0 \uparrow$, $z_1 \uparrow$, $b_0 \downarrow$

$$\text{IRRADIANCE} \propto |g(x,y)|^2 \propto \text{sinc}^2\left(\frac{x}{\left(\frac{\lambda_0 z_1}{b_0}\right)}, \frac{y}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)$$

3 PAIR OF POINT SOURCES

$$\delta(x,y) \longrightarrow 1[\xi,\eta] \longrightarrow 1\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] = 1(x,y]$$

$$\delta[x+x_0, y] + \delta(x-x_0, y] \longrightarrow \left(e^{-2\pi i x_0 \xi} + e^{+2\pi i x_0 \xi}\right) 1[\eta]$$

$$F(\xi,\eta] = 2\cos(2\pi x_0 \xi)\, 1[\eta]$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] = 2\cos\left(2\pi x_0 \frac{x}{\lambda_0 z_1}\right) 1\left(\frac{y}{\lambda_0 z_1}\right]$$

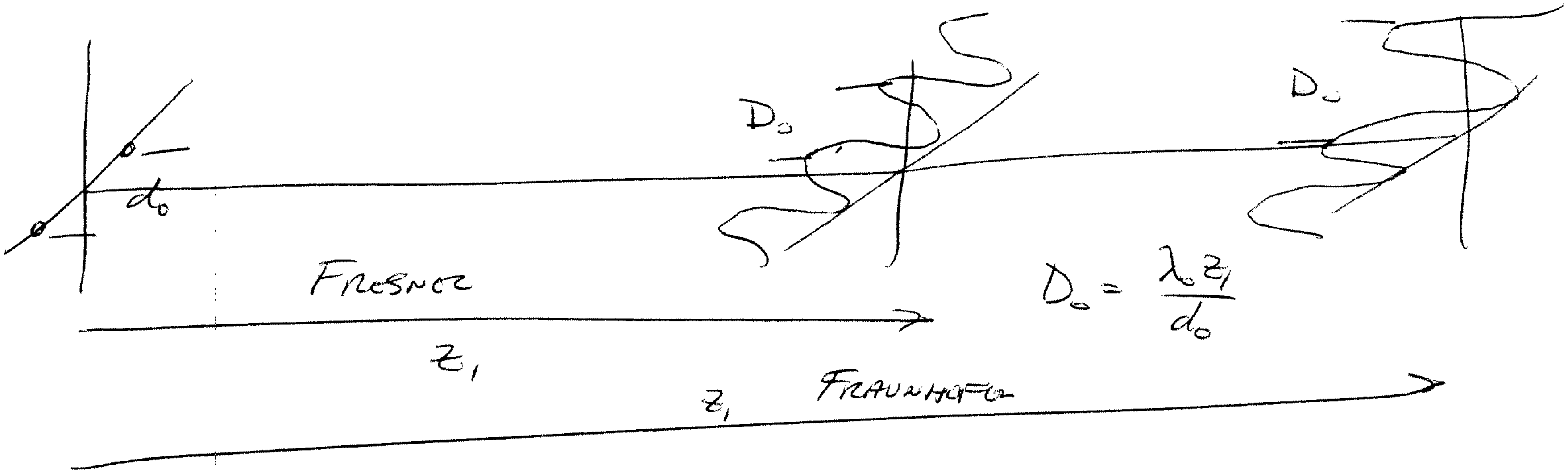
$$g(x,y] \propto 2\cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right) 1[y]$$

$$|g[x,y]|^2 \propto 4\cos^2\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right) 1(y]$$

$$= 4\left(\frac{1}{2} + \frac{1}{2}\cos\left(2\pi \frac{x}{\underbrace{\left(\frac{\lambda_0 z_1}{2x_0}\right)}_{D_0}}\right)\right) 1(y)$$

$$\frac{\lambda_0 z_1}{2x_0} \equiv D_0, \quad 2x_0 \equiv d_0 \Rightarrow \boxed{\lambda_0 z_1 = d_0 D_0}$$

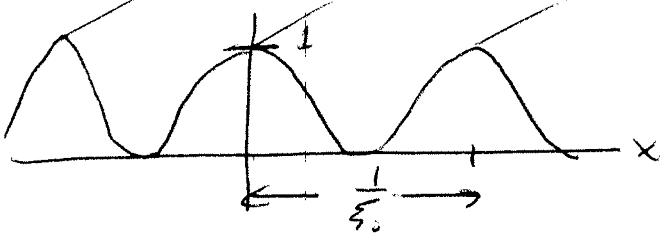
2 - APERTURE EXPERIMENT



$$D_0 = \frac{\lambda_0 z_1}{d_0}$$

PERIODIC "GRATING"

$$f[x,y] = \left(\cos(2\pi \xi_0 x) + 1\right) \cdot \frac{1}{2} \cdot 1[y]$$



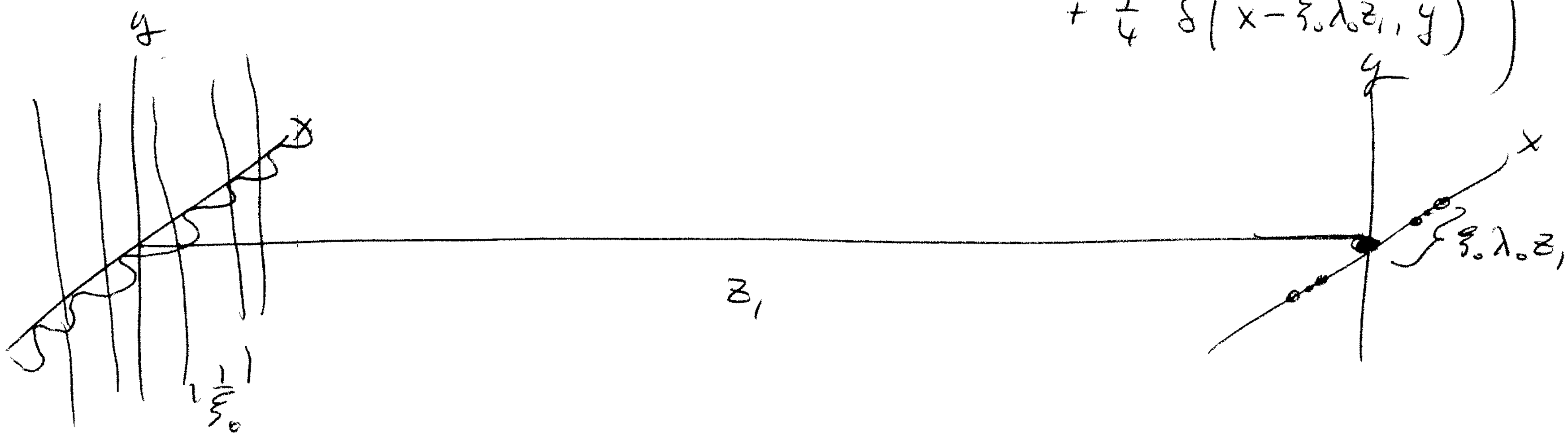
$$F[\xi,\eta] = \frac{1}{2}\left(\delta[\xi,\eta] + \frac{1}{2}\delta(\xi+\xi_0,\eta] + \frac{1}{2}\delta[\xi-\xi_0,\eta] \right)$$

$$= \frac{1}{2}\delta(\xi,\eta) + \frac{1}{4}\delta[\xi+\xi_0,\eta] + \frac{1}{4}\delta[\xi-\xi_0,\eta]$$

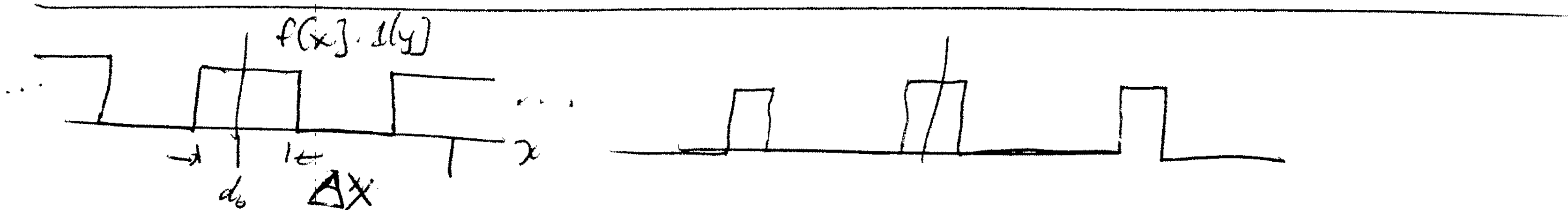
$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = \frac{1}{2}\delta\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] + \frac{1}{4}\delta\left[\frac{x}{\lambda_0 z_1}+\xi_0, \frac{y}{\lambda_0 z_1}\right] + \frac{1}{4}\delta\left(\frac{x}{\lambda_0 z_1}-\xi_0, \frac{y}{\lambda_0 z_1}\right)$$

$$= (\lambda_0 z_1)^2 \left(\frac{1}{2}\delta[x,y] + \frac{1}{4}\delta[x+\xi_0 \lambda_0 z_1, y] + \frac{1}{4}\delta[x-\xi_0\lambda_0 z_1, y] \right)$$

$$g(x,y) = K_0 \, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \cdot (\lambda_0 z_1)^2 \left(\frac{1}{2}\,\delta(x,y) + \frac{1}{4}\,\delta(x+\xi_0\lambda_0 z_1, y) + \frac{1}{4}\,\delta(x-\xi_0\lambda_0 z_1, y) \right)$$



DIFFRACTION GRATING



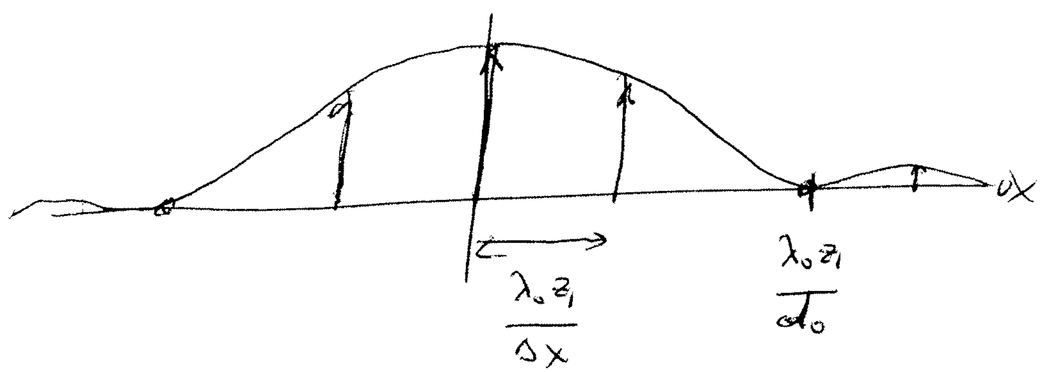
$$f(x,y) = \left(\mathrm{Rect}\left(\frac{x}{d_0}\right) * \frac{1}{\Delta x}\,\mathrm{comb}\left(\frac{x}{\Delta x}\right) \right) 1(y)$$

$$F(\xi, \eta) = \left(d_0 \,\mathrm{SINC}(d_0 \xi) \cdot \mathrm{COMB}(\Delta x \cdot \xi) \right) \cdot \delta(\eta)$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = \left(d_0 \,\mathrm{SINC}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{COMB}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)\right) \delta\left(\frac{y}{\lambda_0 z_1}\right)$$

$$= d_0 \lambda_0 z_1 \quad \mathrm{SINC}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{COMB}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)$$

$$|g(x,y)|^2 \propto \mathrm{SINC}^2\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{COMB}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)$$



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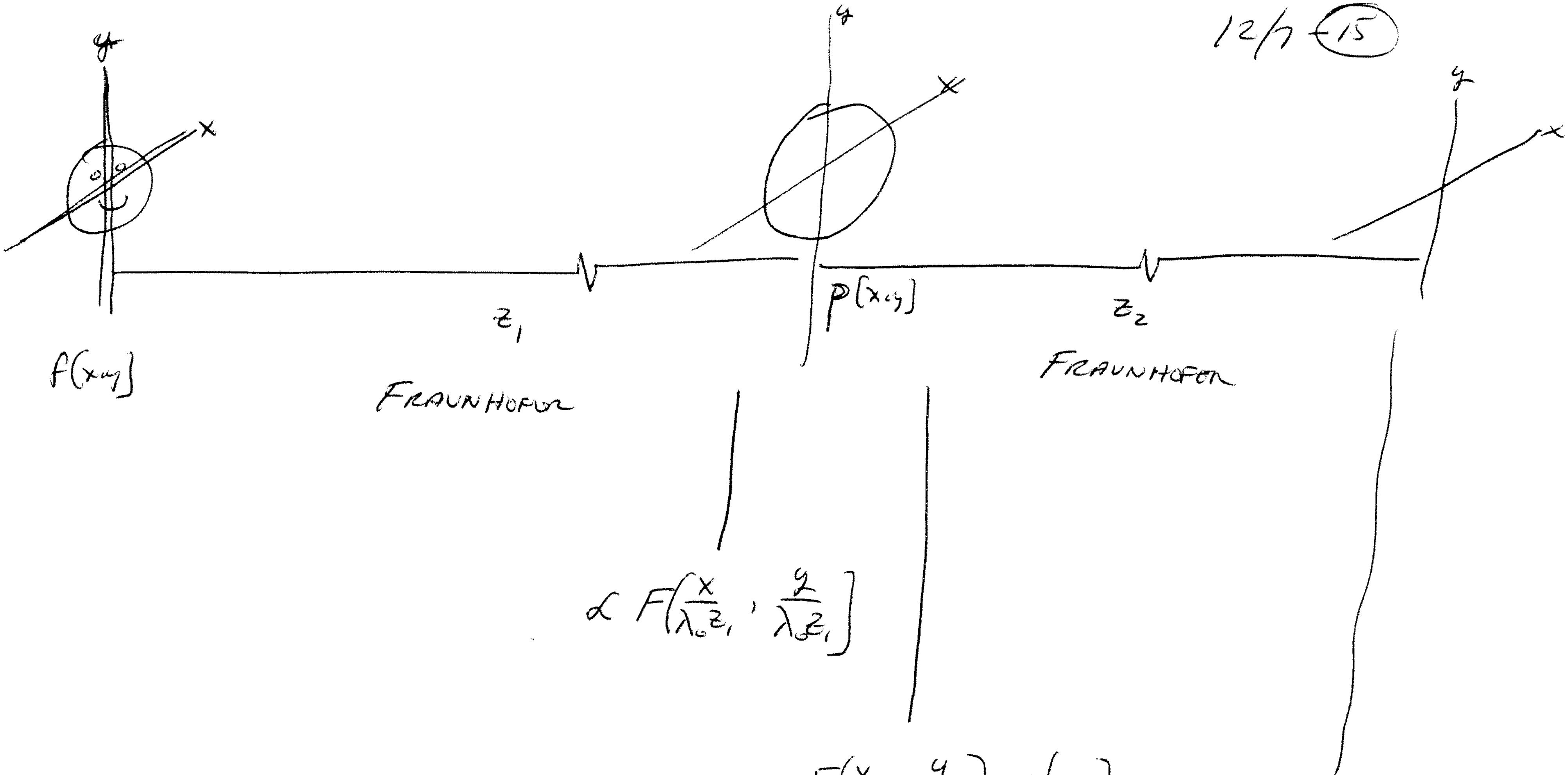

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) \cdot p(x,y)$$

$$\mathcal{F}_2\left\{ F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) \cdot p(x,y) \right\} = (\lambda_0 z_1)^2 f(-\lambda_0 z_1 \xi, -\lambda_0 z_1 \eta) * P(\xi, \eta)$$

$$\xi \to \frac{x}{\lambda_0 z_2}, \quad \eta \to \frac{y}{\lambda_0 z_2}$$

$$g[x,y] \propto (\lambda_0 z_1)^2 \, f\left[-\lambda_0 z_1 \cdot \frac{x}{\lambda_0 z_2}, -\lambda_0 z_1 \cdot \frac{y}{\lambda_0 z_2}\right] * P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

$$\propto f\left[\frac{x}{\left(-\frac{z_2}{z_1}\right)}, \frac{y}{\left(-\frac{z_2}{z_1}\right)}\right] * \underbrace{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]}_{h[x,y]}$$

SCALE FACTOR $= -\dfrac{z_2}{z_1}$

IF $z_2 = z_1 \Longrightarrow f[-x,-y]$