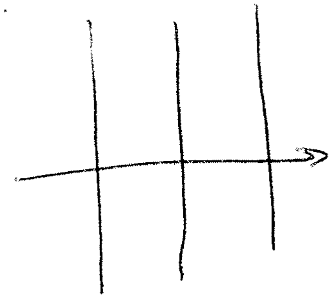
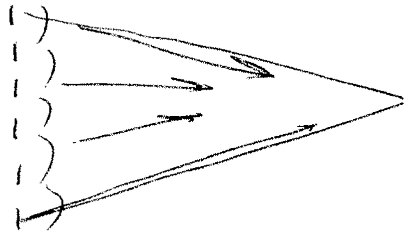
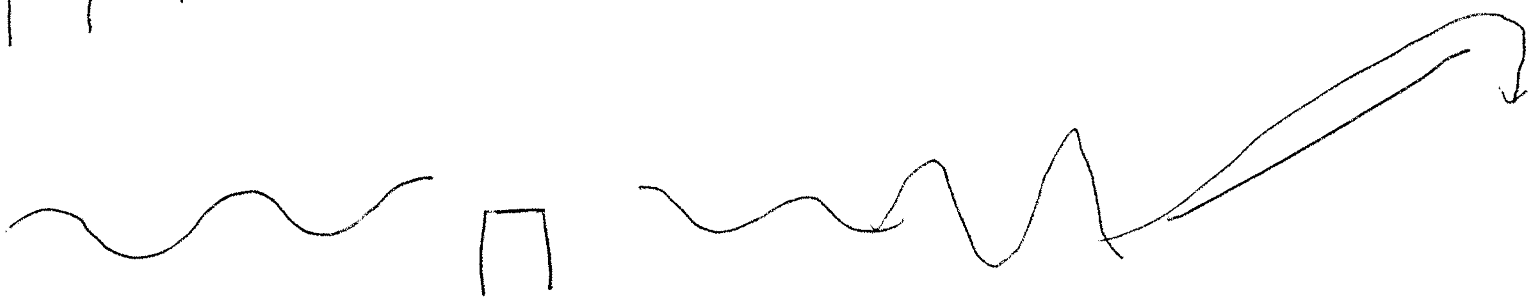
2 December 2009



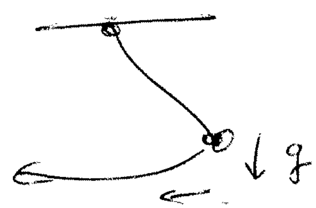




MAXWELL'S EQUATION - ELECTROMAGNETIC WAVES = EM WAVES

ELECTRIC
MAGNETIC FIELDS

WAVE = TRAVELING OSCILLATION



(1) INERTIA

(2) RESTORING   MEDIUM

AETHER

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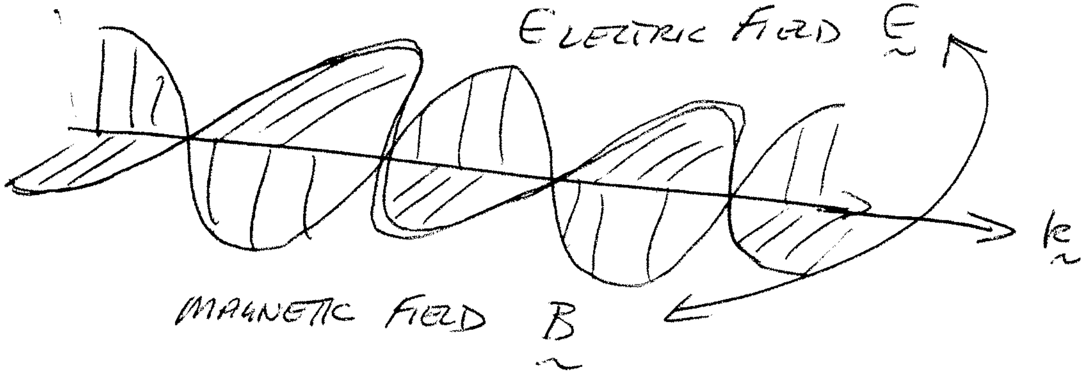
LIGHT IS EM WAVE



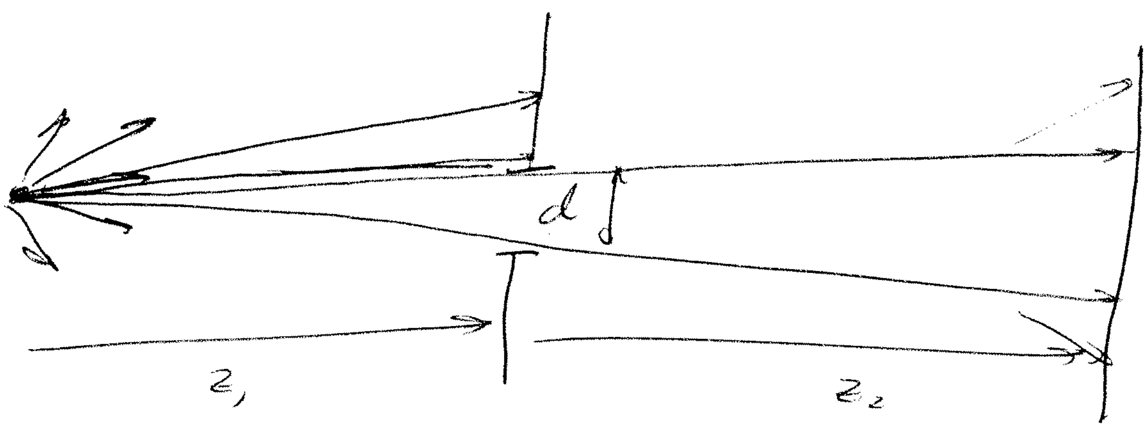
$$F = q\left(\underset{\sim}{E} + \underline{\frac{v}{c} \times \underset{\sim}{B}}\right)$$

$\Rightarrow$ ELECTRIC EFFECTS DOMINATE

MICHELSON - MORLEY $\longrightarrow$ NO MEDIUM (AETHER)

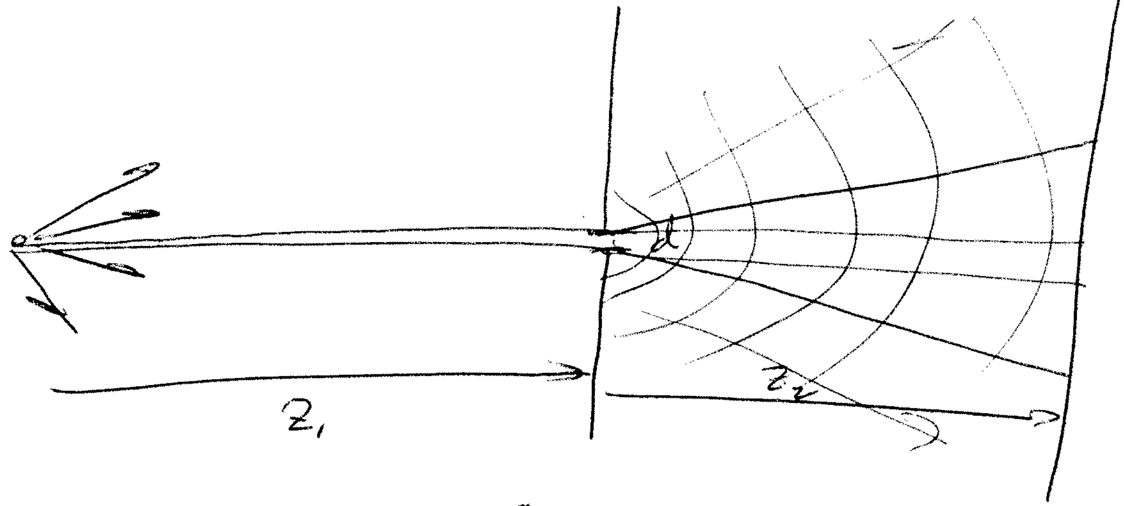
RAY

WAVE



$\boxed{\underset{\sim}{D} \propto d}$

$D \propto d \, \frac{z_2}{z_1 + z_2}$



D

$D \not\propto d$

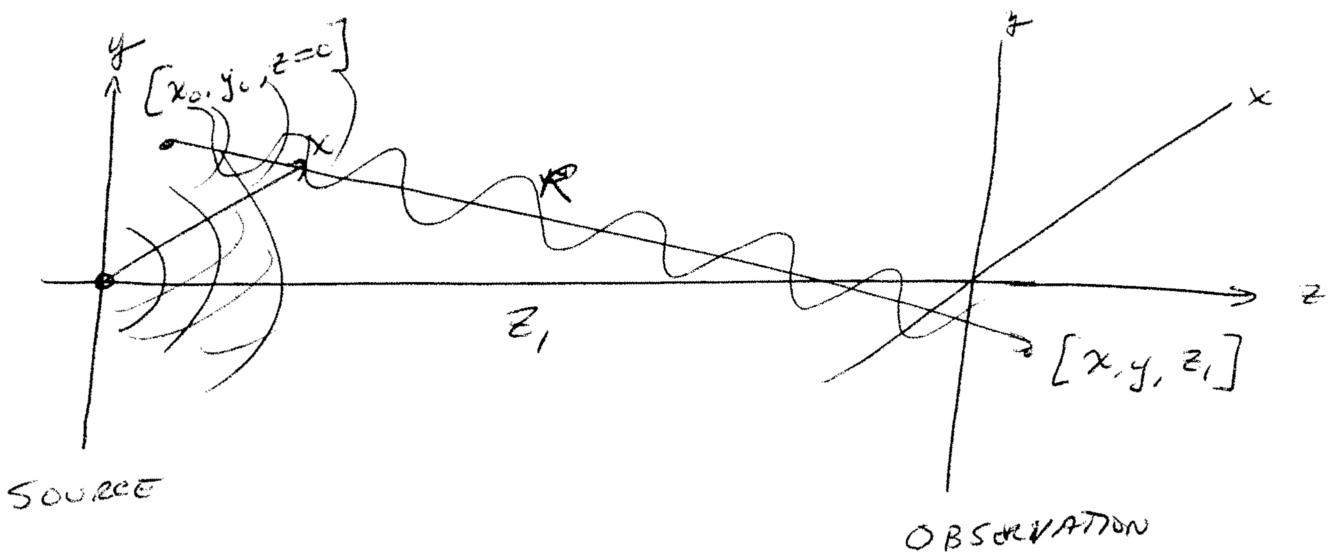
SMALL APERTURE

$d \sim \lambda_0$

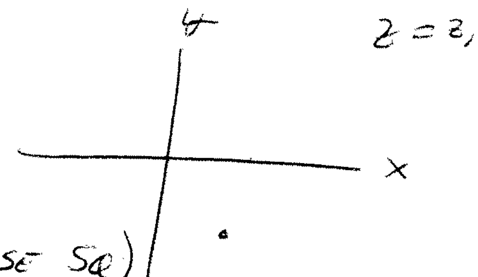
MEASUREMENT $D \propto \frac{1}{d}$

SCALING THEOREM

DIFFRACTION OF LIGHT



$\delta[x - x_0, y - y_0, z]$



AMPLITUDE AT $[x, y, z_1] \propto \frac{1}{R}$ (INVERSE SQ)

PHASE $\propto$ # OF ~~WAVELEN~~ WAVELENGTHS IN R $\quad 2\pi \frac{R}{\lambda_0}$

$$g[x, y, z_1] \propto \frac{1}{R} \exp\left[2\pi i\left(\frac{R}{\lambda_0} - \nu_0 t\right) + i\varphi_0\right]$$

STRICT CALCULATION → SPHERICAL WAVES → RAYLEIGH - SOMMERFELD DIFFRACTION
LSV

1ST - ORDER APPROXIMATION → PARABOLOIDAL WAVES → FRESNEL DIFFRACTION
LSI → CONVOLUTION

→ FRAUNHOFER DIFFRACTION
LSV

2ND - ORDER APPROXIMATION → PLANE WAVES

INPUT PLANE

$$R^2 = (x - x_0)^2 + (y - y_0)^2 + (\cancel{0 - z_1})^2 \qquad z_1 - 0$$

OUTPUT PLANE

$$R = \sqrt{z_1^2 + (x - x_0)^2 + (y - y_0)^2}$$

$$= \sqrt{z_1^2}\sqrt{1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2}} = z_1\left(1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2}\right)^{1/2}$$

AMPLITUDE PHASE 12/2 6

$$\frac{1}{R} = R^{-1} \qquad e^{i\left(2\pi \frac{R}{\lambda_0}\right)}$$

$\lambda_0 \sim 0.5\ \mu m$

$\sim 0.5 \cdot 10^{-6} m$

SMALL ERROR IN R IN PHASE $\Rightarrow$ BIG ERROR IN PHASE

SMALL ERROR IN AMPLITUDE $\Rightarrow$ NOT MUCH EFFECT

$$\frac{R}{R + \Delta r} = \frac{1}{1 + \frac{\Delta r}{R}} = \left(1 + \frac{\Delta r}{R}\right)^{-1} \doteq 1 - \frac{\Delta R}{R} + \frac{1}{2}\left(\frac{\Delta r}{R}\right)^2 - \cdots$$

SMALL

DIFFERENT APPROXIMATIONS FOR R

(1) COARSE APPROX. IN AMPLITUDE

(2) "FINE" APPROX. IN PHASE

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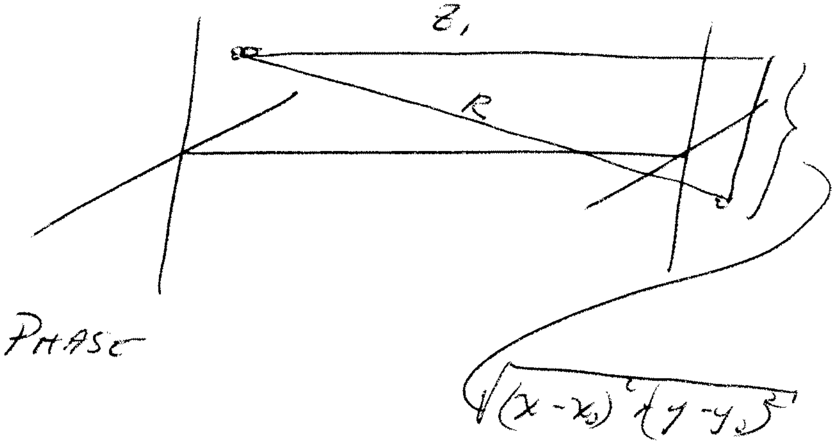
$$R = z_1 \left(1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right)^{1/2} \; ; \quad (1+\alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!}\alpha^2 + \frac{n(n-1)(n-2)}{3!}\alpha^3 + \cdots$$

IF $|\alpha| < 1$

$$= z_1 \left(1 + \frac{1}{2}\left(\frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right) + \left(-\frac{1}{8}\right)\left(\frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right)^2 + \cdots \right)$$

$$R = z_1 + \frac{(x-x_0)^2 + (y-y_0)^2}{2 z_1} + \cdots$$

"DOWNSTREAM PROPAGATION"   OFF-AXIS "CORRECTION"



USE TWO TERMS IN SUM FOR PHASE

1^{ST} TERM FOR AMPLITUDE

OBSERVED AMPLITUDE AT $[x, y, z_1]$ DUE TO SOURCE AT $[x_0, y_0, z=0]$

$$g[x, y; z = z_1] \propto \underbrace{\frac{1}{z_1}}_{\text{COARSE APPROXIMATION}} \exp\left[2\pi i \underbrace{\left(\frac{z_1 + \frac{(x-x_0)^2+(y-y_0)^2}{2z_1}}{\lambda_0} - \nu_0 t\right)}_{\text{FINE APPROXIMATION}} \cancel{+i\varphi_0} \right] \rightarrow 0$$

FRESNEL APPROXIMATION FOR DIFFRACTION

$$\underbrace{\frac{1}{z_1}}_{\text{INVERSE SQUARE LAW}} \underbrace{e^{+2\pi i \frac{z_1}{\lambda_0}}}_{\text{CONSTANT PHASE DOWN AXIS}} \underbrace{e^{+i \cancel{2}\pi \frac{(x-x_0)^2+(y-y_0)^2}{\cancel{2}\lambda_0 z_1}}}_{\text{QUADRATIC PHASE OFF-AXIS PROPAGATION}} \underbrace{e^{-2\pi i \nu_0 t}}_{\text{TEMPORAL PART}}$$

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$$\underbrace{}_{\text{OBSERVATION}}\quad \underbrace{}_{\text{SOURCE}}$$

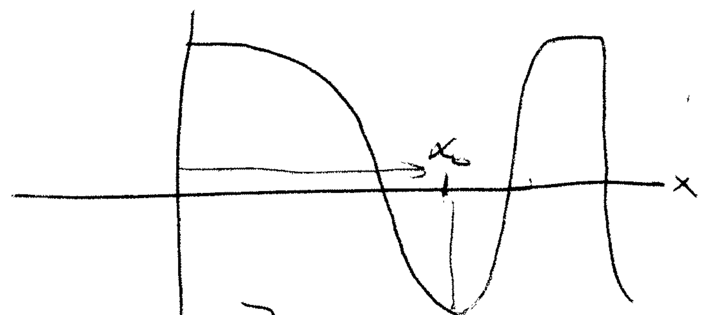
$$g[x, y; z=z_1, x_0, y_0] = \left(\frac{1}{i\lambda_0}\right)\frac{1}{z_1}\, e^{+2\pi i \frac{z_1}{\lambda_0}}\, e^{+i\pi \frac{(x-x_0)^2+(y-y_0)^2}{\left(\sqrt{\lambda_0 z_1}\right)^2}} \cdot e^{-2\pi i \nu_0 t}$$

CONSTANT

$$e^{+i\pi\left(\frac{x}{x_0}\right)^2}$$

RATE

CHIRP RATE IS $\sqrt{\lambda_0 z_1}$



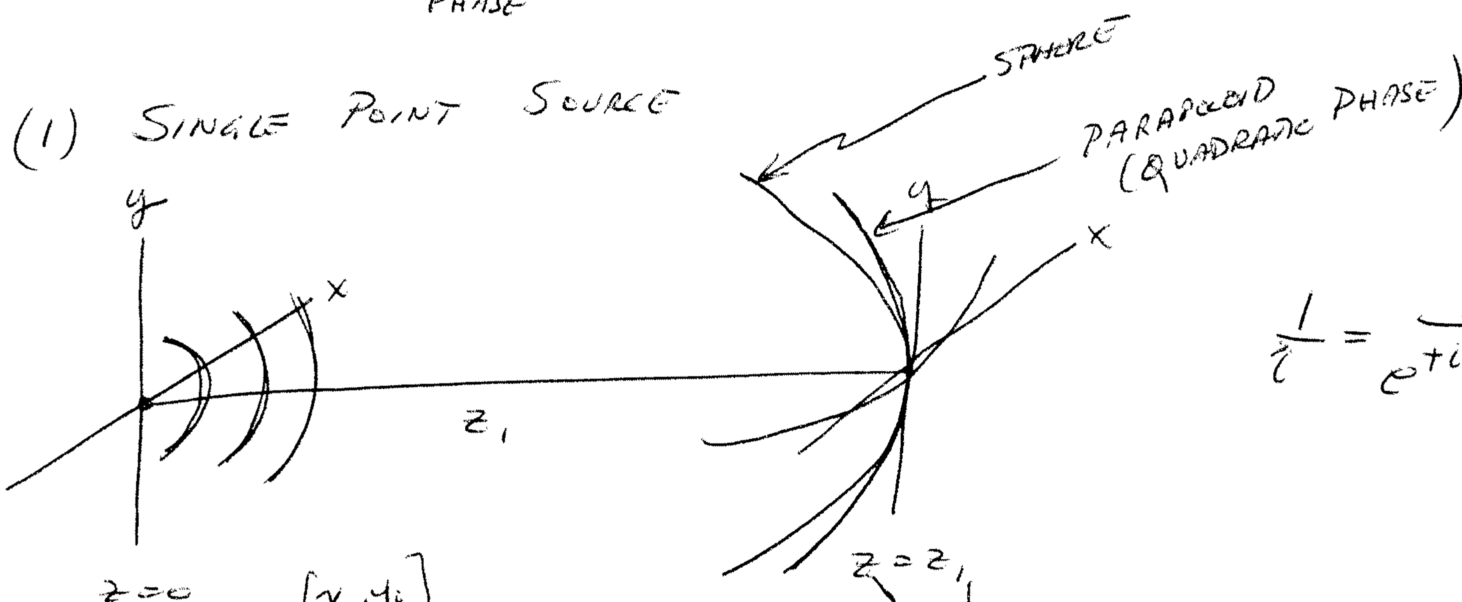
$$\delta[x-x_0, y-y_0]\,\delta[z] \longrightarrow g[x, y; x_0, y_0; z_1]$$

$$\delta[x, y]\,\delta[z] \longrightarrow g[x, y; 0, 0, z_1] = \left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i \frac{z_1}{\lambda_0}}\right) e^{+i\pi \frac{x^2+y^2}{\left(\sqrt{\lambda_0 z_1}\right)^2}} \cdot \text{TIME}$$

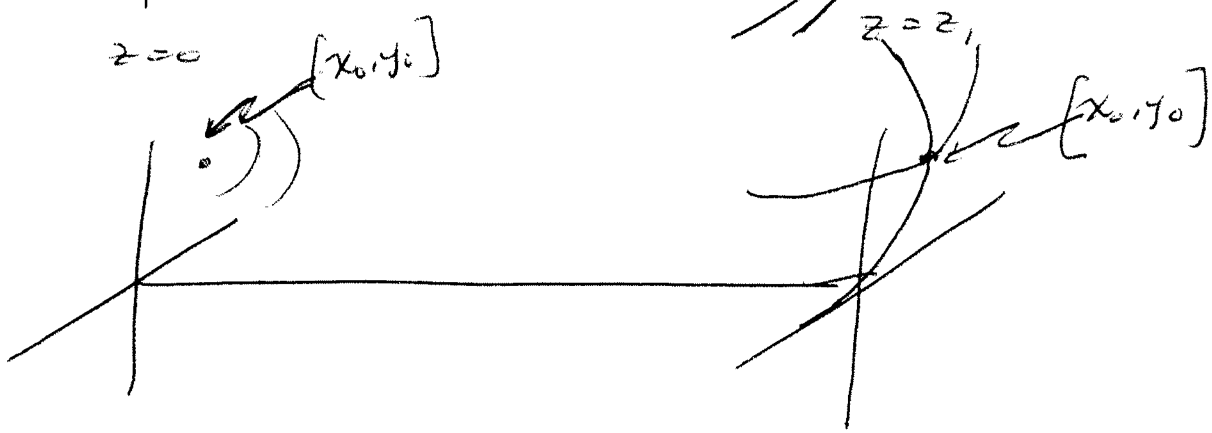
LSI SYSTEM

$$h[x, y; z_1] = \underbrace{\frac{1}{i\lambda_0 z_1}}_{\text{CONSTANT PHASE}} \underbrace{e^{+2\pi i \frac{z_1}{\lambda_0}} \; e^{+i\pi \frac{x^2+y^2}{(\lambda_0 z_1)}}}_{\text{SPATIAL PART}} \; e^{-2\pi i \nu_0 t}$$

(1) SINGLE POINT SOURCE



$$\frac{1}{i} = \frac{1}{e^{+i\frac{\pi}{2}}} = e^{-i\frac{\pi}{2}}$$

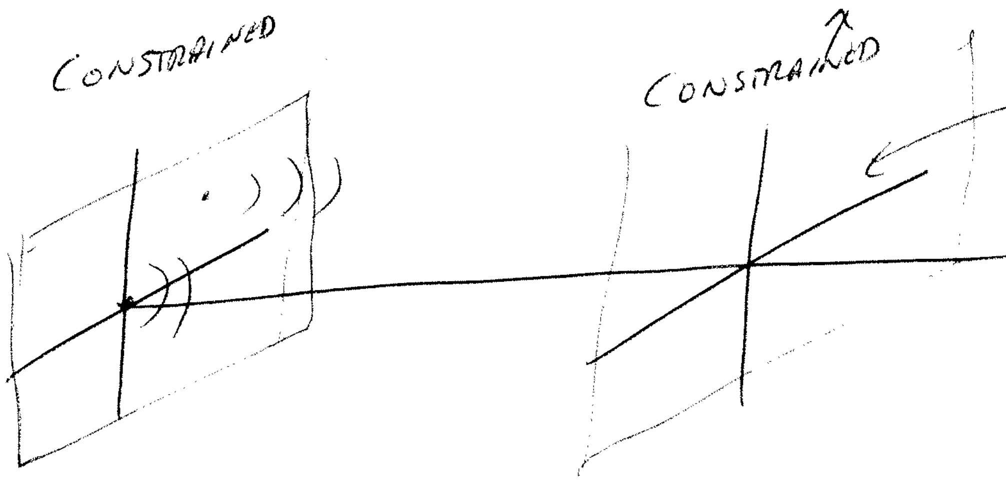


OBSERVE "POWER", NOT THE AMPLITUDE

TIME AVERAGE OF SQUARED MAGNITUDE

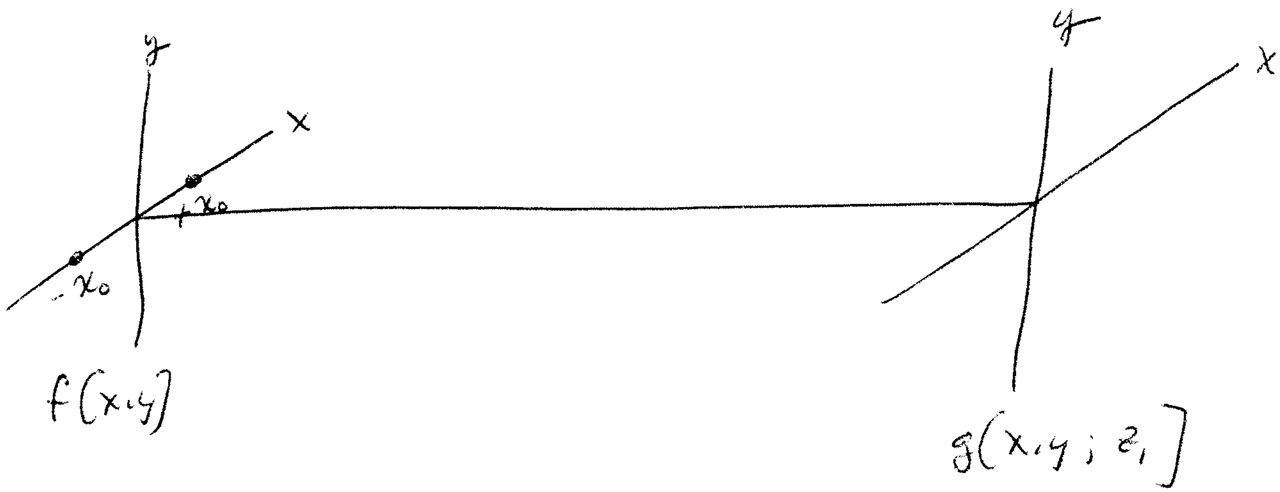
$$\left\langle \left| h[x,y;z_1,t] \right|^2 \right\rangle = \frac{1}{|i|^2}\left(\frac{1}{\lambda_0 z_1}\right)^2 \cancelto{1}{\left| e^{+i2\pi \frac{z_1}{\lambda}} \right|^2} \cancelto{1}{\left| e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \right|^2}$$

$$\propto \frac{1}{z_1^2}$$ INVERSE SQUARE LAW



CONSTANT $\propto \left(\frac{1}{z_1}\right)^2$

$$f[x,y] = \delta[x+x_0, y] + \delta(x-x_0, y)$$



$$g(x,y;z_1) = f(x,y) * h[x,y;z_1]$$

$$= \Big[\delta[x+x_0] + \delta(x-x_0)\Big]\delta(y] * \underbrace{\left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0}\right)}\right)}_{K_0} e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$= K_0\left(\Big(\delta[x+x_0] + \delta(x-x_0)\Big) * e^{+i\pi\frac{x^2}{\lambda_0 z_1}}\right)\left(\delta(y) * e^{+i\pi\frac{y^2}{\lambda_0 z_1}}\right)$$

$$g[x,y;z_1] = K_0 \left(e^{+i\pi \frac{(x+x_0)^2}{\lambda_0 z_1}} + e^{+i\pi \frac{(x-x_0)^2}{\lambda_0 z_1}} \right) \left(e^{+i\pi \frac{y^2}{\lambda_0 z_1}} \right)$$

$$= \left(K_0 \, e^{+i\pi \frac{y^2}{\lambda_0 z_1}} \right) \left(e^{+i\pi \frac{x^2+x_0^2+2xx_0}{\lambda_0 z_1}} + e^{+i\pi \frac{x^2+x_0^2-2xx_0}{\lambda_0 z_1}} \right)$$

$$= \left(K_0 \, e^{+i\pi \frac{y^2}{\lambda_0 z_1}} \right) \left(e^{+i\pi \frac{x^2+x_0^2}{\lambda_0 z_1}} \right) \left(\underbrace{e^{+i\pi \frac{2xx_0}{\lambda_0 z_1}} + e^{-i\pi \frac{2xx_0}{\lambda_0 z_1}}} \right)$$

$$= \underbrace{\left(K_0 \, \underbrace{e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}}_{\text{PARABOLOIDAL WAVE}} \; \underbrace{e^{+i\pi \frac{x_0^2}{\lambda_0 z_1}}}_{\text{CONSTANT PHASE}} \right)} \cdot 2\cos\left(2\pi \, x \underbrace{\frac{x_0}{\lambda_0 z_1}}_{\xi_0} \right)$$

$$= \left(\qquad \right) \boxed{2 \cos 2\pi \left(\frac{x}{\underbrace{\left(\frac{\lambda_0 z_1}{x_0}\right)}_{\text{PERIOD}}} \right)}$$

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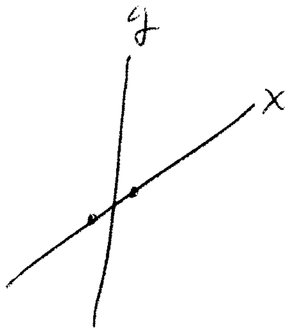
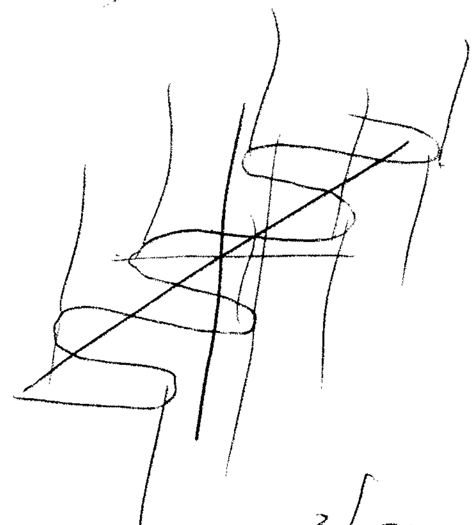
$$g[x,y;z_1,x_0,\lambda_0] = \left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i \frac{z_1}{\lambda_0}}\right) e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_1}} \, e^{+i\pi\frac{x_0^2}{\lambda_0 z_1}} \cdot 2\cos\left[2\pi \frac{x}{\underbrace{\left(\frac{\lambda_0 z_1}{x_0}\right)}_{\text{PERIOD}}}\right]$$





$$|g[x,y;z_1,\lambda_0,x_0]|^2 = \underbrace{\frac{1}{\lambda_0^2 z_1^2}}_{\text{IGNORE}} \cdot 4\cos^2\left[2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right]$$

$$\propto \frac{1}{z_1^2} \cdot 4\cos^2\left[2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right] \quad ; \quad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$= \frac{2}{z_1^2}\left(1 + \cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{2x_0}\right)}\right)\right)$$

PERIOD $\frac{\lambda_0 z_1}{2x_0}$

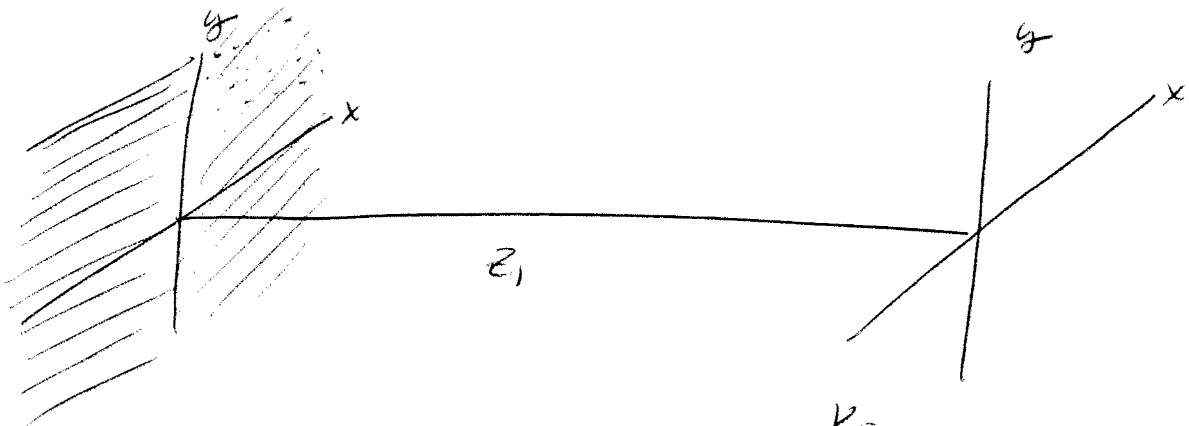
YOUNG'S TWO-~~APERTURE~~ SOURCE EXPERIMENT IN FRESNEL REGION

PERIOD $D_0 = \dfrac{\lambda_0 z_1}{2x_0}$ IRRADIANCE

$\underbrace{(2x_0)}_{\text{SEPARATION OF SOURCES}} D_0 = \lambda_0 z_1$

$2x_0 \equiv d_0 \implies \boxed{\underbrace{d_0}_{\text{TRANSVERSE}} D_0 = \underbrace{\lambda_0 z_1}_{\text{"LONGITUDINAL"}}}$

KNIFE EDGE



K_0

$$f[x,y] = \mathrm{STEP}[x] \cdot 1[y]$$

$$h[x,y;z_1,\lambda_0] = \boxed{\frac{1}{i\lambda_0 z_1}\, e^{+2\pi i \frac{z_1}{\lambda_0}}}\; e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$g[x,y;z_1,\lambda_0] = f[x,y] * h[x,y;z_1,\lambda_0]$$

$$= (K_0)\left(\mathrm{STEP}[x] * e^{+i\pi\frac{x^2}{\lambda_0 z_1}}\right)\left(\underbrace{1(y) * e^{+i\pi\frac{y^2}{\lambda_0 z_1}}}_{\cancel{\delta(x)}}\right)$$

$$1(y) * e^{+i\pi \frac{y^2}{(\sqrt{\lambda_0 z_1})^2}} \xrightarrow{\mathcal{F}_1} \delta(\eta) \cdot \left|\sqrt{\lambda_0 z_1}\right| e^{+i\frac{\pi}{4}} e^{-i\pi \lambda_0 z_1 \eta^2}$$

$$= \sqrt{\lambda_0 z_1}\, e^{+i\frac{\pi}{4}} \quad \delta(\eta)\, e^{-i\pi \lambda_0 z_1 \eta^2}$$

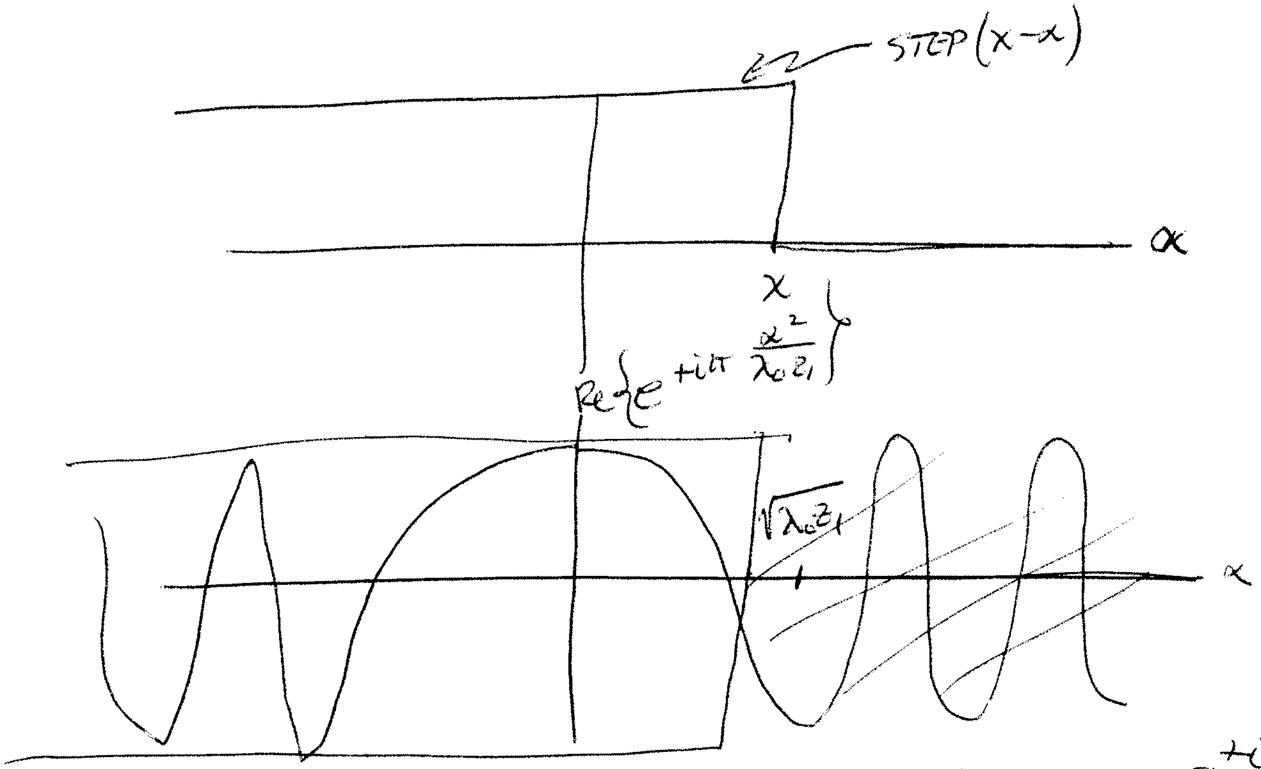
$$= \left(\sqrt{\lambda_0 z_1}\, e^{+i\frac{\pi}{4}}\right) \delta(\eta - 0)\, \cancel{e^{-i\pi \lambda_0 z_1 \cdot 0^2}}^{\,1}$$

$$= \sqrt{\lambda_0 z_1}\, e^{+i\frac{\pi}{4}}\, \delta(\eta)$$

$$\sqrt{\lambda_0 z_1}\, e^{+i\frac{\pi}{4}}\, 1(y) \xleftarrow{\mathcal{F}_1^{-1}}$$

$$\mathrm{STEP}(x) * e^{+i\pi \frac{x^2}{\lambda_0 z_1}} = \int_{-\infty}^{+\infty} \mathrm{STEP}(\alpha)\, e^{+i\pi \frac{(x-\alpha)^2}{\lambda_0 z_1}}\, d\alpha$$

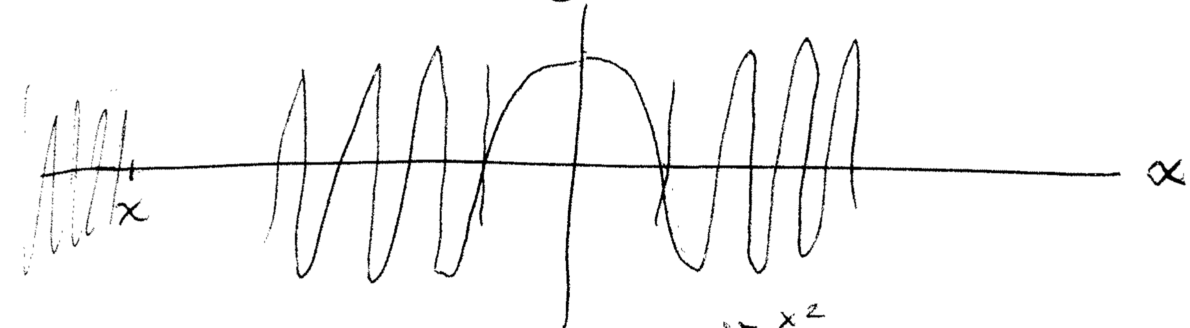
$$= \int_{-\infty}^{+\infty} \mathrm{STEP}(x-\alpha)\, e^{+i\pi \frac{\alpha^2}{\lambda_0 z_1}}\, d\alpha$$



AREA FROM $\alpha = -\infty$ to $\alpha = x$ OF $e^{+i\pi \frac{x^2}{\lambda_0 z_1}}$

$$STEP(x) \ast e^{+i\pi \frac{x^2}{\lambda_0 z_1}} = \int_{-\infty}^{x} e^{+i\pi \frac{\alpha^2}{\lambda_0 z_1}}\, d\alpha$$

$e^{+i\pi \frac{\alpha^2}{\lambda_0 z_1}}$



$STEP(x) \circledast e^{+i\pi \frac{x^2}{\lambda_0 z_1}}$