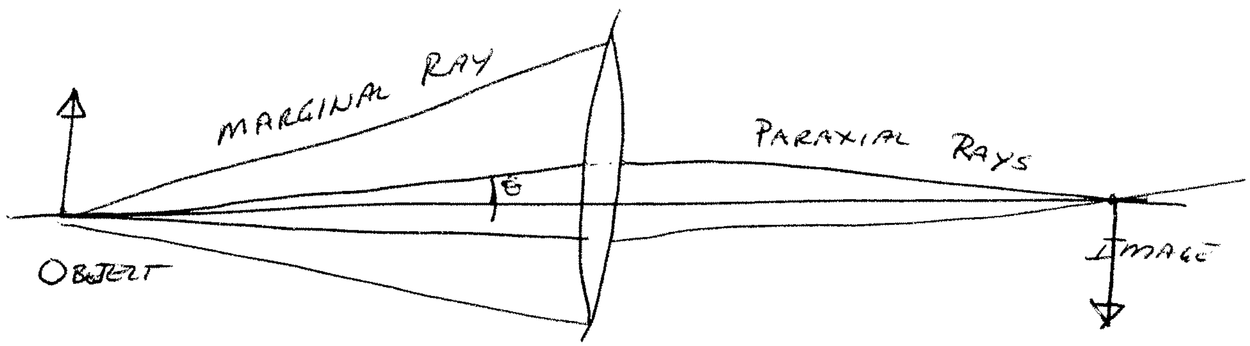
10 FEBRUARY 2010

RAY OPTICS MODEL OF IMAGING

FIND (LOCATE) IMAGES, PARAXIAL APPROXIMATION

MAGNIFICATIONS → QUALITY (ABERRATIONS)



LOGARITHMS

1970S → COMPUTERS, OSLO, CODE V

HERO OF ALEXANDRIA LIGHT RAY TRAVELS SHORTEST PATH

FERMAT - LIGHT RAY TRAVELS PATH ~~...~~ THAT TAKES SHORTEST

OPTICAL PATH LENGTH $\phi = \frac{2\pi}{\lambda'} z = \frac{2\pi}{\lambda/n} z = \cancel{2\pi}$

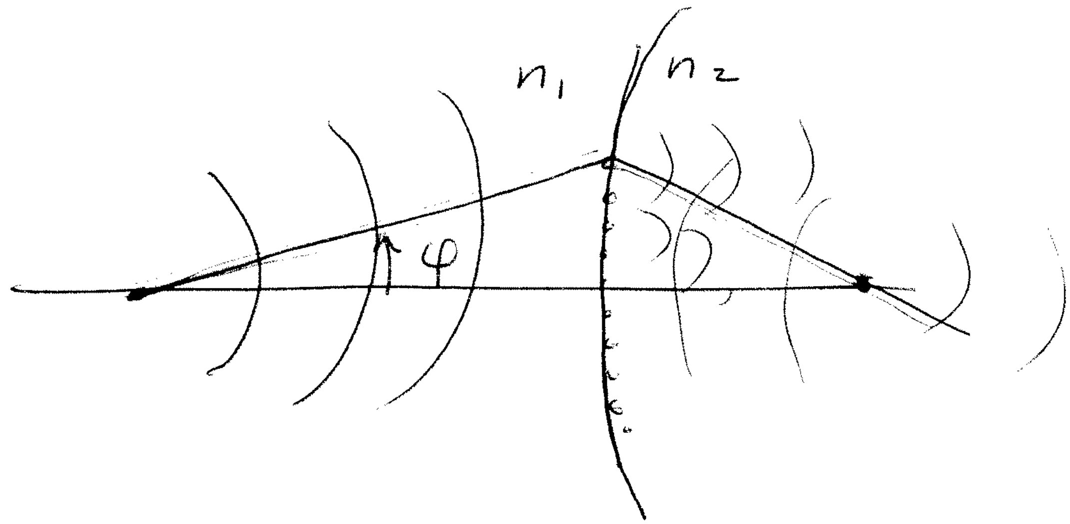
$$\phi = \frac{2\pi (nz)}{\lambda_0}$$

OPTICAL PATH LENGTH = nz

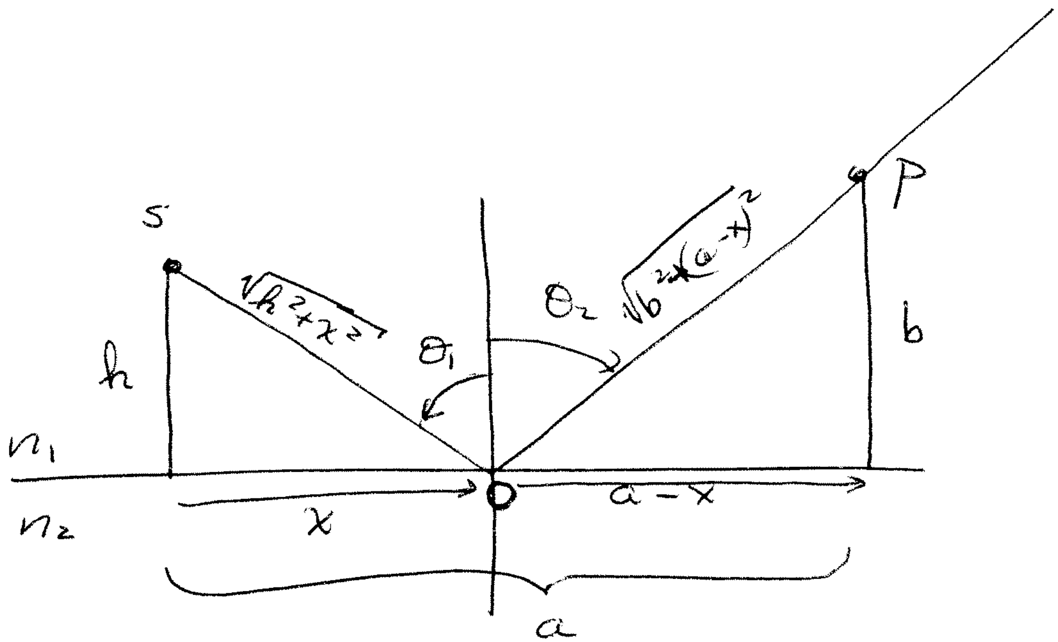
OPL = DISTANCE LIGHT TRAVELS IN VACUUM IN SAME TIME AS WOULD TRAVEL z IN INDEX n

OPL $> z$

$$\frac{d\,\mathrm{OPL}}{d\varphi} = 0$$



FERMAT'S PRINCIPLE AND REFLECTION
(SAME INDEX)

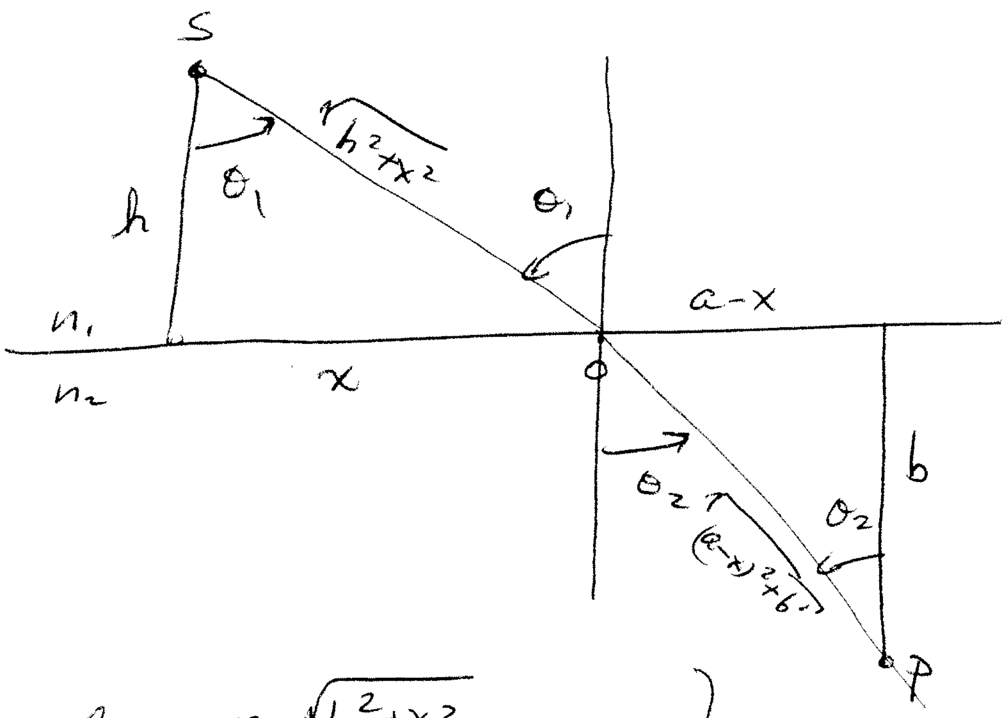


$\left.\begin{array}{l}\theta_1 > 0 \\ \theta_2 < 0\end{array}\right\}$ AS DRAWN

$$\left.\begin{array}{l} l_1 = n_1\sqrt{h^2+x^2} \\ l_2 = n_1\sqrt{b^2+(a-x)^2} \end{array}\right\} \quad OPL = l_1 + l_2$$

$$\frac{d\,OPL}{d\theta_1} = 0 = \frac{d\,OPL}{dx}\cdot\frac{dx}{d\theta}$$

$$\Rightarrow \sin\theta_1 = \sin(-\theta_2) \Rightarrow \theta_2 = -\theta_1$$



$$l_1 = n_1\sqrt{h^2+x^2}$$

$$l_2 = n_2\sqrt{b^2+(a-x)^2}$$

$$OPL = l_1 + l_2$$

$$= n_1\sqrt{h^2+x^2} + n_2\sqrt{b^2+(a-x)^2}$$

$$\frac{d}{dx}OPL = n_1\frac{1}{2\sqrt{h^2+x^2}}\cdot 2x + n_2\frac{1\cdot 2\cdot(a-x)\cdot(-1)}{2\sqrt{b^2+(a-x)^2}}$$

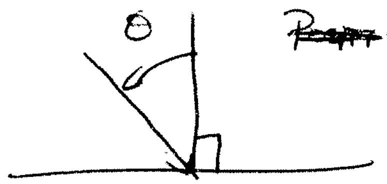
$$= n_1\frac{x}{\sqrt{h^2+x^2}} - n_2\frac{a-x}{\sqrt{b^2+(a-x)^2}}$$

$$= n_1\sin\theta_1 - n_2\sin\theta_2 = 0$$

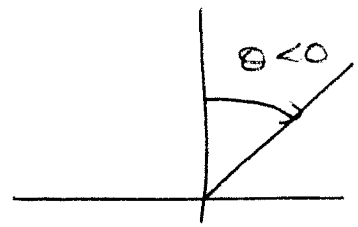
$$\frac{d}{dx}(OPL) = 0 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

SNELL'S LAW FOR REFRACTION

SIGN CONVENTIONS



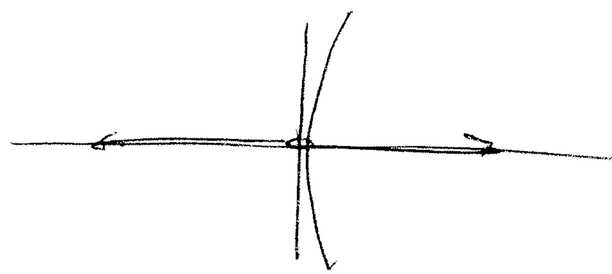
POSITIVE

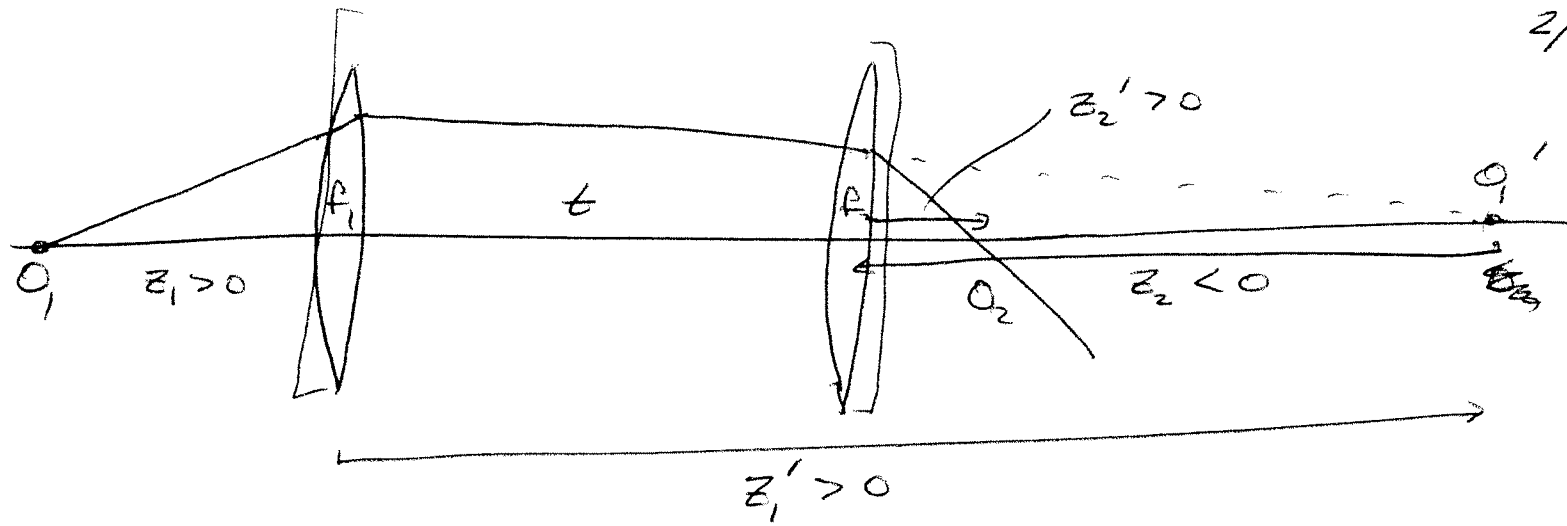


DISTANCE

(DIRECTED DISTANCE)

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}}$$





$$\boxed{\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f_{eff}}}$$

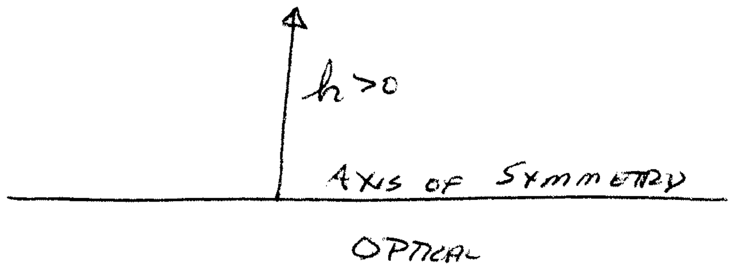
effective OBJECT DISTAN

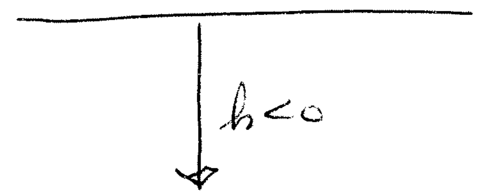
FROM OBJECT TO OBJECT-SPACE PRINCIPAL POIN

eff IMAGE DISTAN

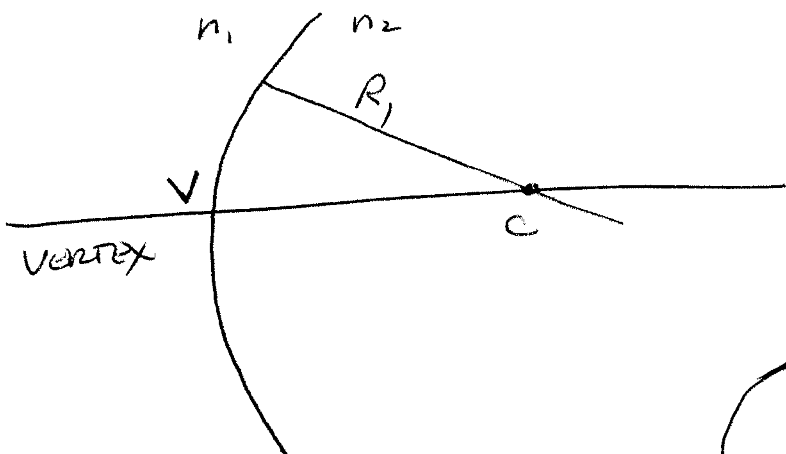
FROM H_2 TO O_2

HEIGHT

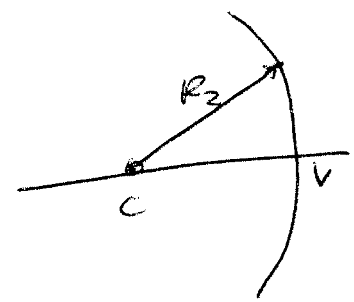


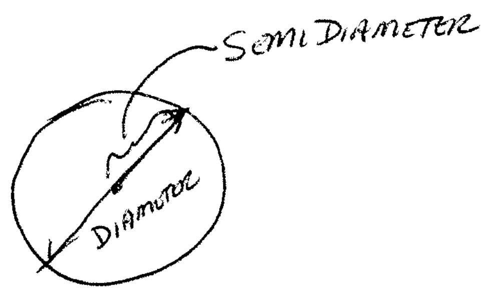


RADII OF CURVATURE



$\overline{VC} = R_1 > 0$



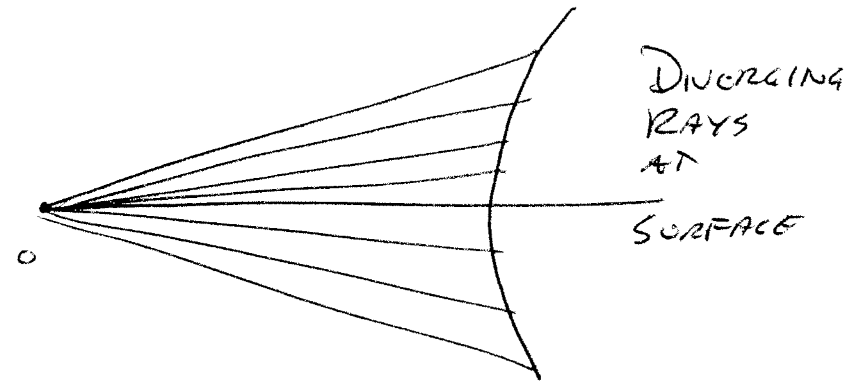


$R_2 < 0$

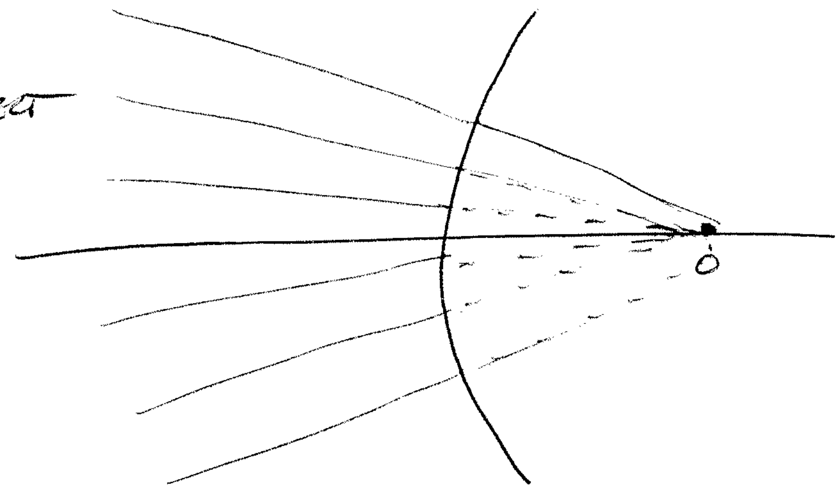
"QUALITY" OF OBJECT & IMAGE

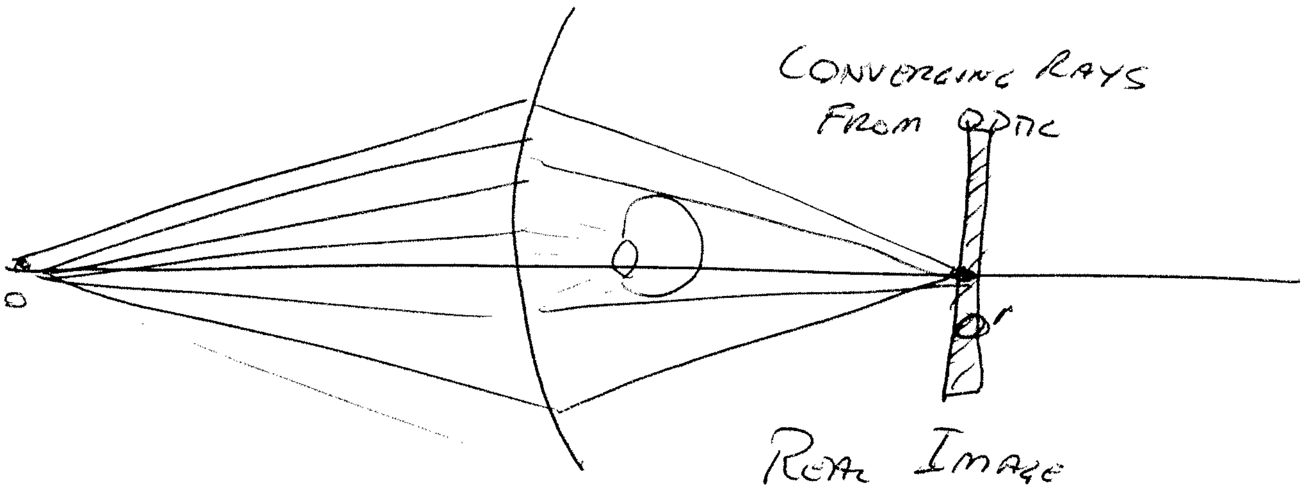
REAL, & VIRTUAL

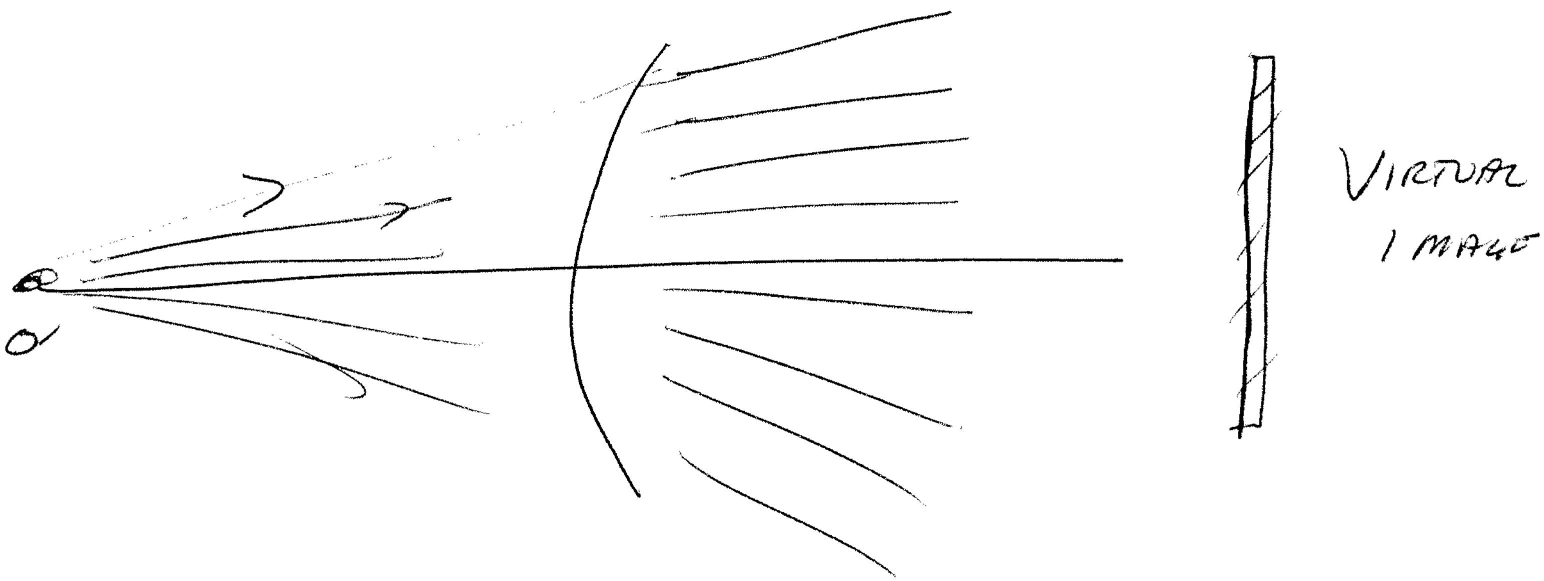
REAL OBJECT



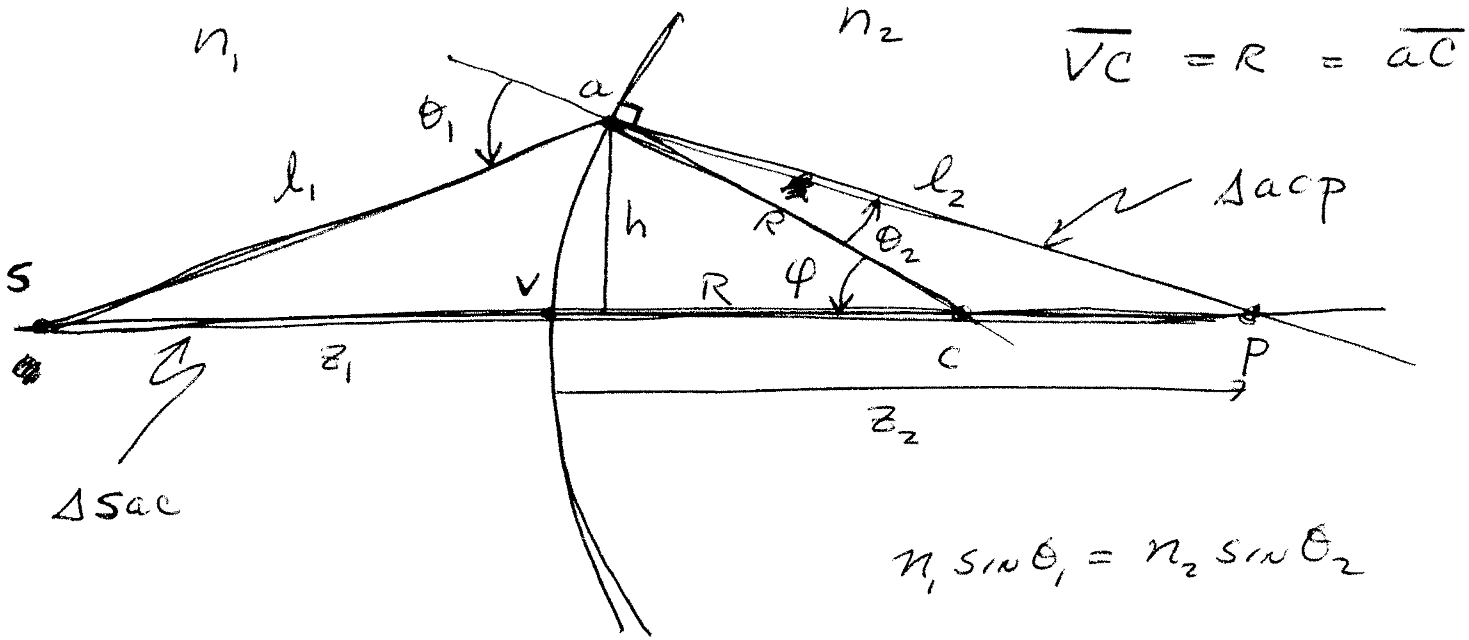
VIRTUAL OBJECT




CONVERGING RAYS
FROM OPTIC
O
O'
REAL IMAGE


O'
VIRTUAL
IMAGE

Refraction from Single Surface



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$OPL = n_1 l_1 + n_2 l_2$$

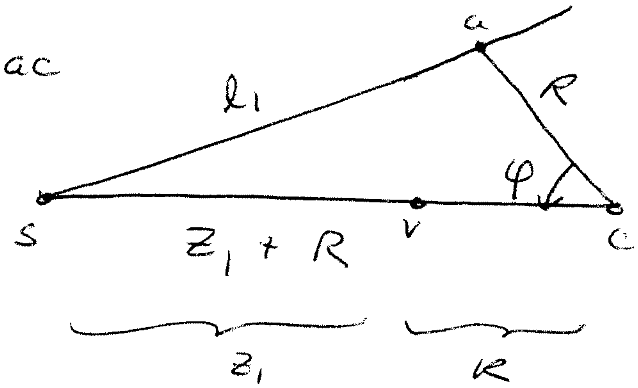
$$\frac{d}{d\varphi}(OPL) \Rightarrow (1)\ f[z_1, l_1, n_1, z_2, l_2, n_2]$$

(2) IF $\underline{\varphi \gtrsim 0}$, $z_1 \simeq l_1$, $z_2 \simeq l_2$ $\Rightarrow$ PARAXIAL

— Δsac

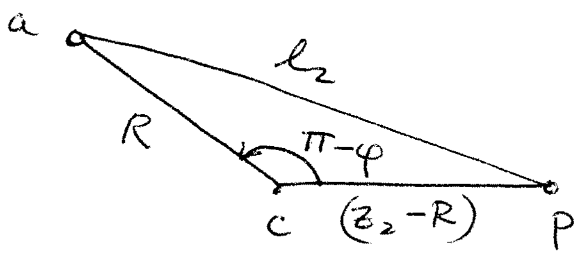
$\sin\varphi \simeq \tan\varphi \simeq \varphi \Rightarrow$ LINEAR EXPRESSION

$\Delta\, sac$



$$l_1^2 = R^2 + (z_1+R)^2 - 2R(z_1+R)\cos\varphi$$

$\Delta\, acp$



$$l_2^2 = R^2 + (z_2-R)^2 - 2R(z_2-R)\cos(\pi-\varphi)$$

$$\cos(\pi-\varphi) = \cos\pi\cos\varphi + \cancelto{0}{\sin\pi}\,\sin\varphi$$
$$= -\cos\varphi$$

$$l_2^2 = R^2 + (z_2-R)^2 + 2R(z_2-R)\cos\varphi \Rightarrow l_2 = \sqrt{R^2+(z_2-R)^2+2R(z_2-R)\cos\varphi}$$

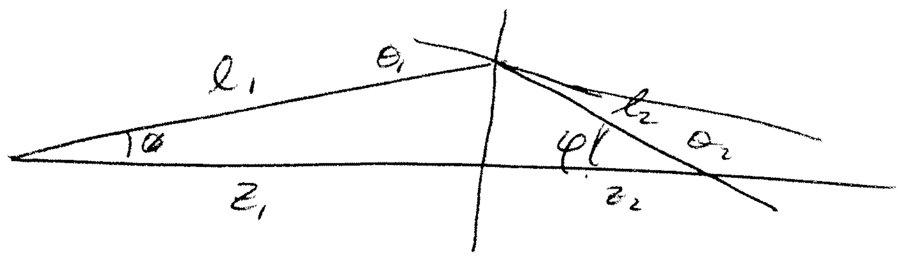
$$l_1^2 = R^2 + (z_1+R)^2 - 2R(z_1+R)\cos\varphi \qquad l_1 = \sqrt{(\qquad)}$$

$$OPL = n_1 l_1 + n_2 l_2$$

$$\frac{d\,OPL}{d\varphi} = n_1 \frac{1}{2\sqrt{\qquad}} + 2R(z_1+R)\sin\varphi \;\cancel{d} \; + n_2 \frac{1}{\sqrt{(\qquad)}} \cdot -2R(z_2-R)\sin\varphi$$

$$\cancel{\frac{n_1}{\ell_1}} \qquad \frac{n_1(R+z_1)}{\ell_1} + \frac{n_2(R-z_2)}{\ell_2} = 0$$

$$\frac{n_1}{\ell_1} + \frac{n_2}{\ell_2} = \frac{1}{R}\left(\frac{n_2 z_2}{\ell_2} - \frac{n_1 z_1}{\ell_1}\right)$$



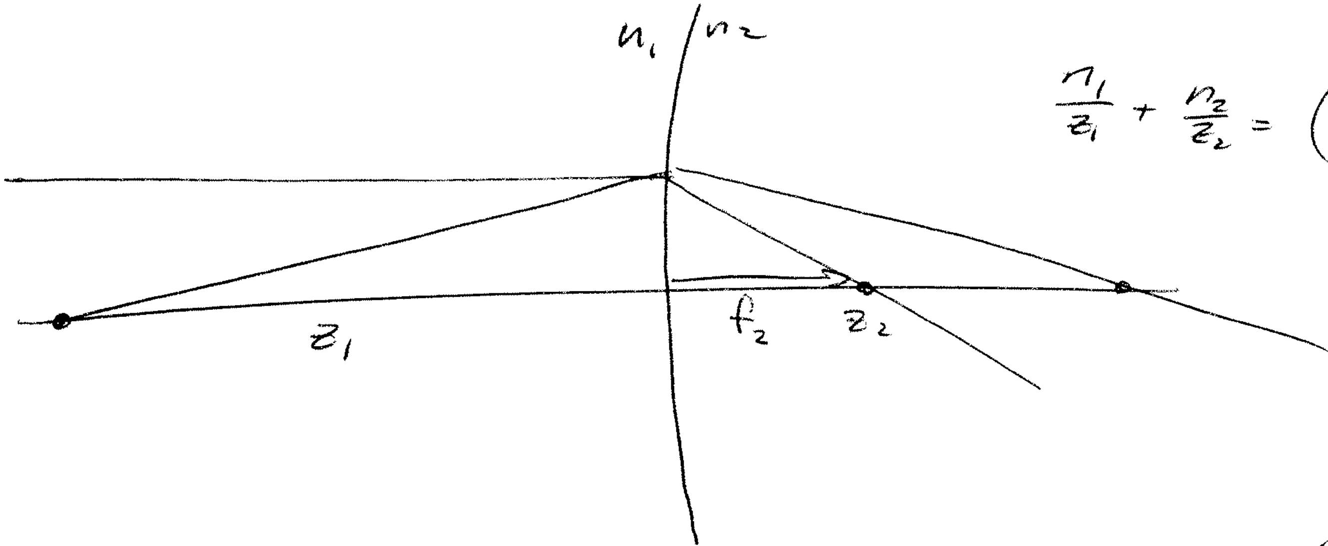
$$\cancel{\frac{\ell_2}{z_1}} \qquad \frac{\ell_2}{z_2} = \frac{1}{\cos\varphi}$$

IF $\varphi \gtrsim 0$, $\cos\varphi \simeq 1 - \frac{\varphi^2}{2!} + \cdots$

$\sin\varphi \sim \varphi$

$$\left.\begin{array}{l} \ell_2 \sim z_2 \\ \ell_1 \sim z_1 \end{array}\right\} \Longrightarrow \boxed{\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{1}{R}(n_2 - n_1)}$$

PARAXIAL APROX FOR REFRACTION FROM SINGLE SURF



$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \left(\frac{n_2 - n_1}{R}\right)$$

(1) $z_1 = \infty$ $\quad \frac{n_1}{\infty} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R_1} \Rightarrow z_2 = R_1 \cdot \left(\frac{n_2}{n_2 - n_1}\right)$

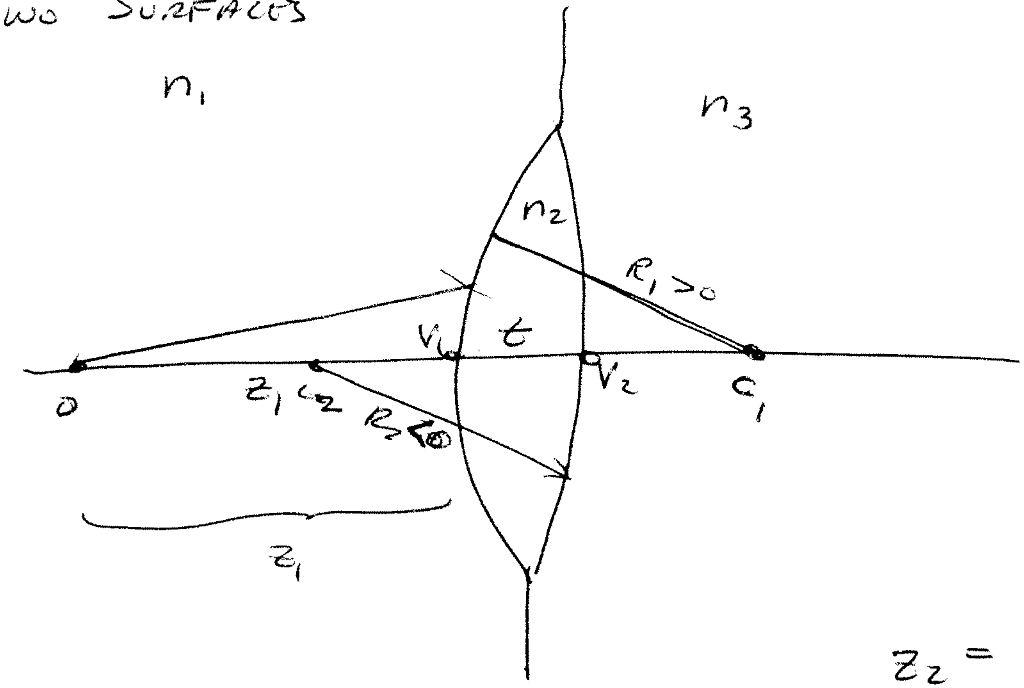
$$\uparrow$$

$$f_2 = \left(\frac{n_2}{n_2 - n_1}\right) R$$

(2) $z_2 = \infty$ $\quad \frac{n_1}{z_1} + 0 = \frac{n_2 - n_1}{R_1} \Rightarrow z_1 = f_1 = R_1 \left(\frac{n_1}{n_2 - n_1}\right)$

$$\boxed{\frac{f_1}{f_2} = \frac{n_1}{n_2}}$$

TWO SURFACES



(1) $$\frac{n_1}{z_1} + \frac{n_2}{z_1'} = \frac{n_2 - n_1}{R_1}$$

(2) $$\frac{n_2}{z_2} + \frac{n_3}{z_2'} = \frac{n_3 - n_2}{R_2}$$

$t = 0 \Rightarrow z_1' = -z_2$

$z_2 = t - z_1'$

$z_2 = -z_1'$ IF $t = 0$ (THIN LENS)

$z_1' = t - z_2$

THIN LENS

$$\frac{n_1}{z_1} + \frac{n_2}{-z_2} = \frac{n_2 - n_1}{R_1}$$

$$\frac{n_2}{z_2} + \frac{n_3}{z_2'} = \frac{n_3 - n_2}{R_2}$$

$$\underset{\uparrow\ \text{OBJECT}}{\frac{n_1}{z_1}} + \underset{\uparrow\ \text{IMAGE}}{\frac{n_3}{z_2'}} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\frac{n_1}{z_1} + \frac{n_3}{z_2'} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

IN AIR $n_1 = n_3 = 1$

$$\frac{1}{z_1} + \frac{1}{z_2'} = -\frac{1}{R_1} + \frac{1}{R_2} + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= -\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

THIN LENS, 3 MEDIA

$$\frac{n_1}{z_1} + \frac{n_3}{z_2'} = -\frac{n_1}{R_1} + \frac{n_3}{R_2} + n_2\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

IN AIR

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2'} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

LENSMAKER'S EQUATION (FOR THIN LENS)

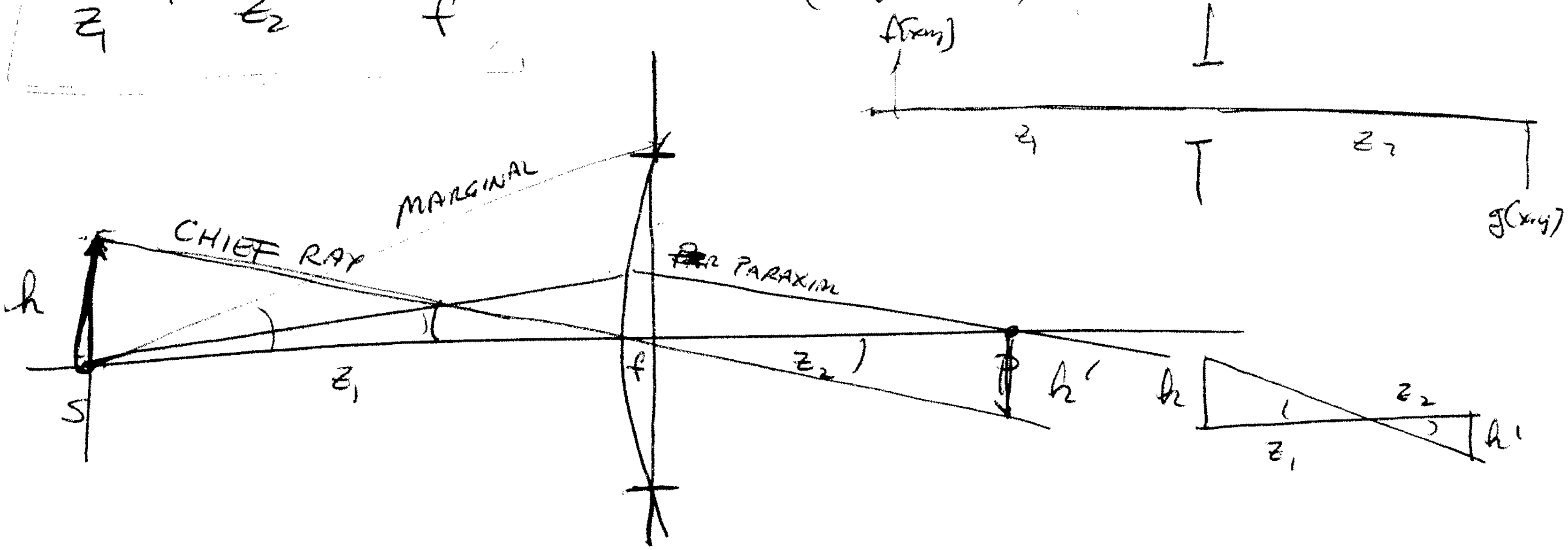
$z_2' \equiv z_2$

$$\frac{1}{z_1} + \frac{1}{z_2} \equiv \frac{1}{f}$$

FROM DIFFRACTION

$$\boxed{\frac{1}{\underline{\underline{f}}} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2}} = \frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



CHIEF RAY – EDGE OF OBJECT → CENTER OF LENS → EDGE OF IMAGE (PUPIL)

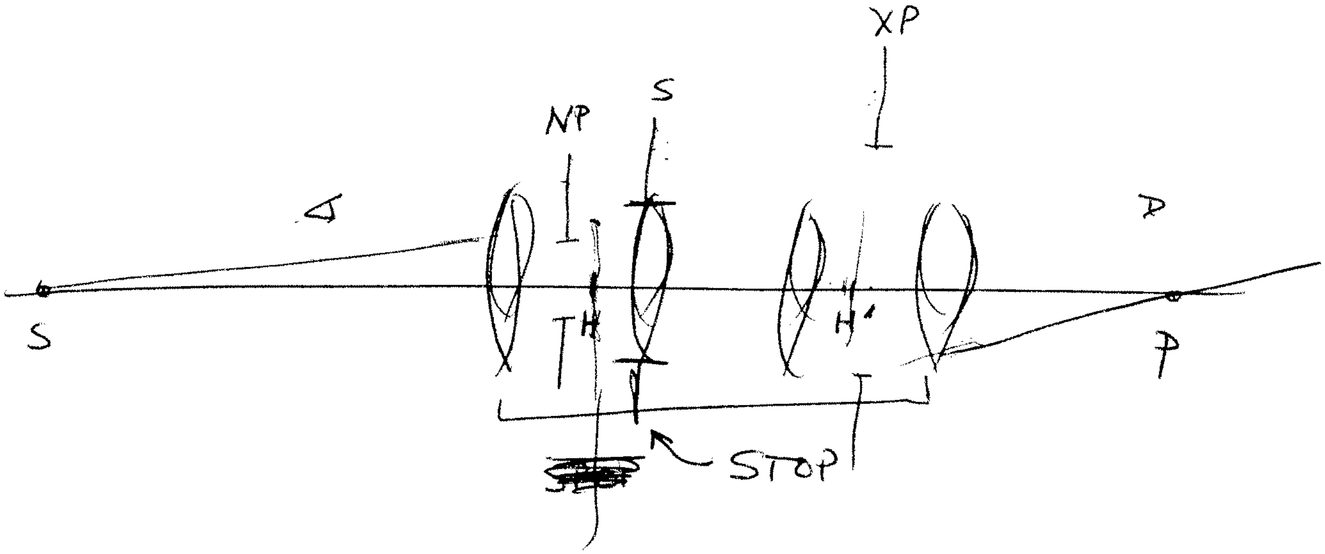
PARAXIAL RAY – CENTER OF OBJECT → INTERSECT LENS → CENTER OF IMAGE

MARGINAL RAY – SCALED REPLICA OF PARAXIAL

EDGE OF APERTURE STOP ⟺ ENTRANCE PUPIL

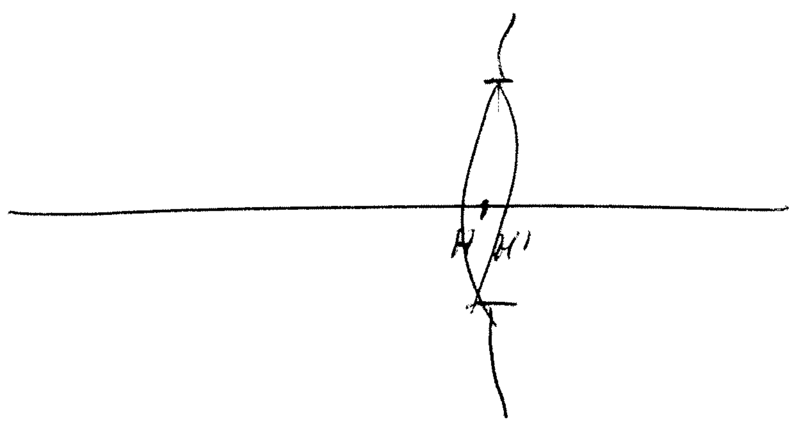
⟺ EXIT PUPIL

$$\boxed{\frac{h'}{h} = -\frac{z_2}{z_1}} = M_T \qquad M_\theta$$

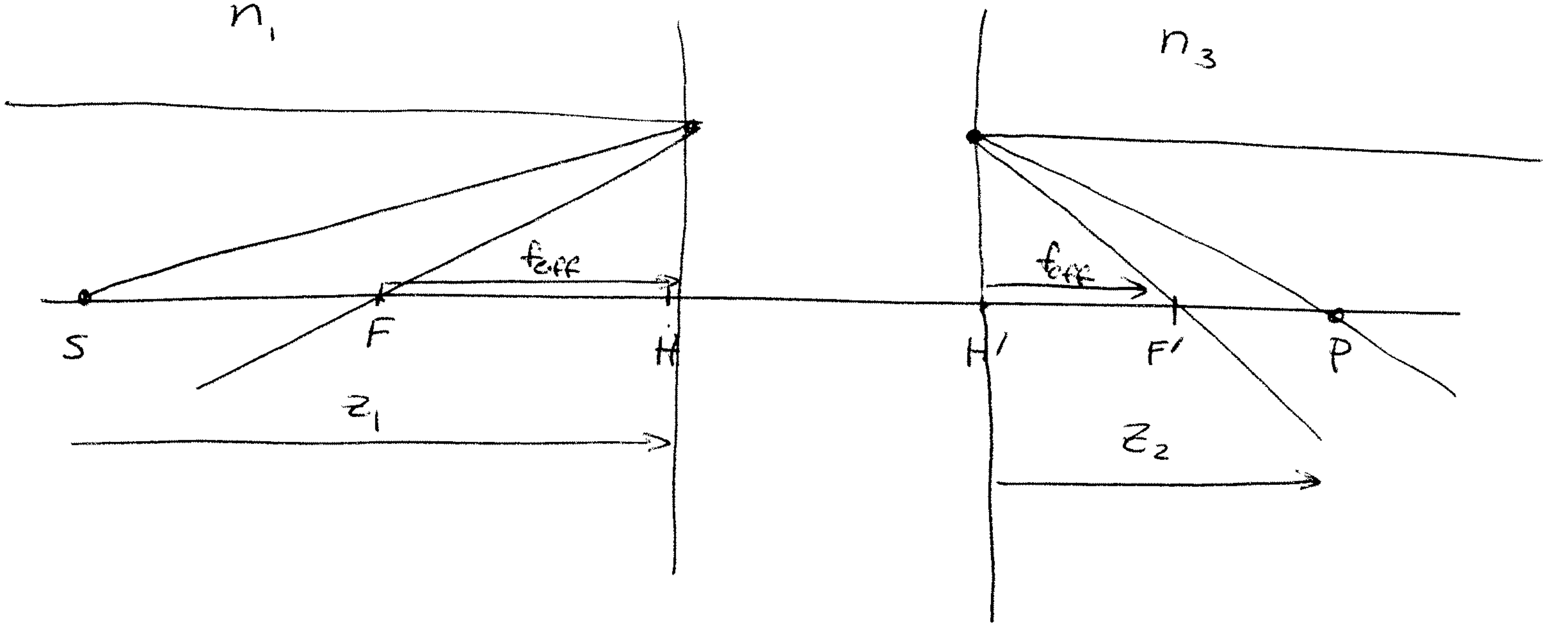


OBJECT SPACE PRINCIPAL PLANE

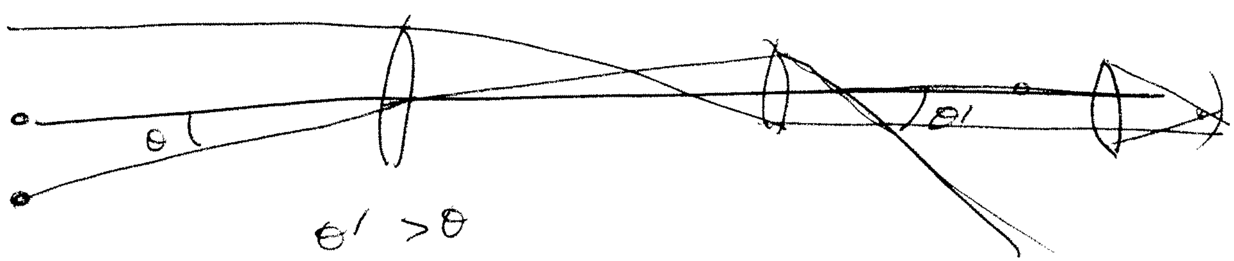
H' IS IMAGE OF H WITH $m_T = +1$



2/10 – (A)



$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f_{eff}}$$



$\theta' > \theta$