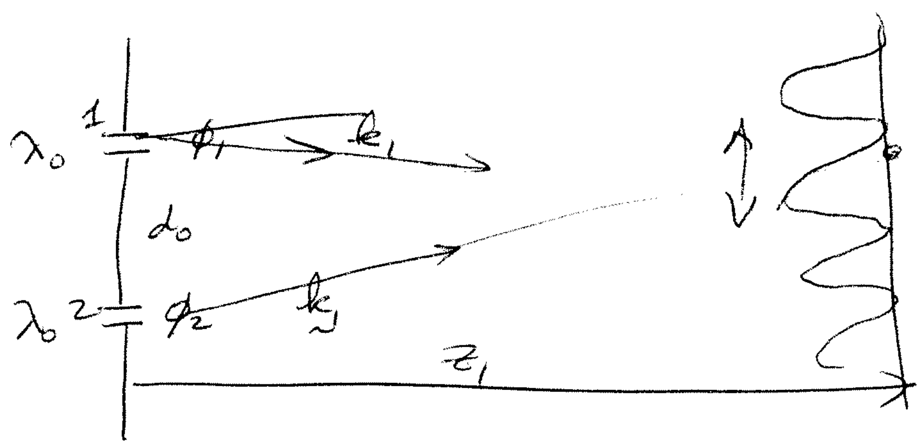
INFEROMETRY

DoW YOUNG'S TWO-APERTURE → SINUSOIDAL, SPATIAL COHERENCE

DoA MICHELSON → TEMPORAL COHERENCE - TEMPORAL FIELD



$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos\Delta\phi(t) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos\Delta\phi(t) \rangle}$$

$$\langle \cos\Delta\phi \rangle = 1 - \frac{\langle (\Delta\phi)^2 \rangle}{2!} + \cdots$$

$$A_0 \cos(\underset{\sim}{k}_1 \cdot \underset{\sim}{r} - \omega_0 t + \phi_1) + A_0 \cos(\underset{\sim}{k}_2 \cdot \underset{\sim}{r} - \omega_0 t + \phi_2)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

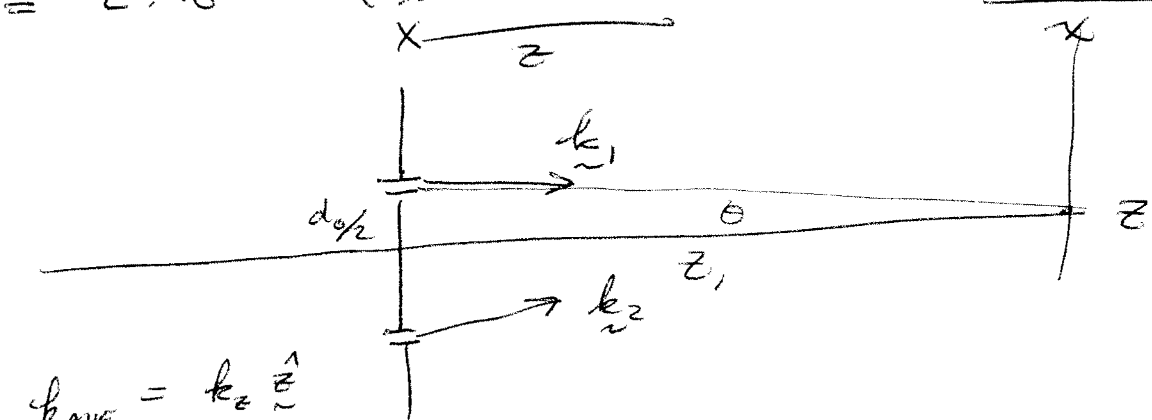
↑ AVERAGE PHASE ↑ MODULATION

$$\cos(\tilde{k}_1 \circ \tilde{r} - \omega_0 t) + \cos(\tilde{k}_2 \circ \tilde{r} - \omega_0 t)$$

$$= \underbrace{2 A_0 \cos\left(\underbrace{\frac{\tilde{k}_1 + \tilde{k}_2}{2}}_{\tilde{k}_{AVG}} \circ \tilde{r} - \underbrace{\frac{\omega_0 + \omega_0}{2}}_{\omega_{AVG}} t\right)}_{\text{AVERAGE WAVE}} \cdot \underbrace{\cos\left(\underbrace{\frac{\tilde{k}_1 - \tilde{k}_2}{2}}_{\tilde{k}_{mod}} \circ \tilde{r} - \underbrace{0 \cdot t}_{\omega_{MOD} = 0}\right)}_{\text{INTERFERENCE STANDING WAVE}}$$

$$\omega_1 = \omega_2 = \omega_0 \Rightarrow \frac{\omega_1 - \omega_2}{2} = 0$$

$$g[x, y, z, t] = 2 A_0 \cos\left(\tilde{k}_{AVG} \circ \tilde{r} - \omega_0 t\right) \cdot \cos\left(\tilde{k}_{MOD} \circ \tilde{r}\right)$$



$$\tilde{k}_1 = \begin{bmatrix} -k_x \\ 0 \\ +k_z \end{bmatrix}$$

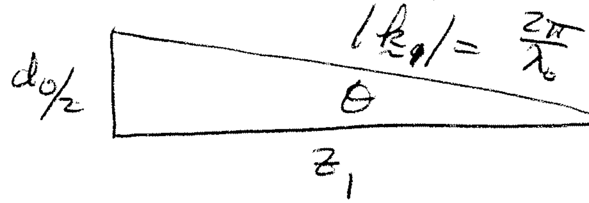
$$\tilde{k}_2 = \begin{bmatrix} +k_x \\ 0 \\ +k_z \end{bmatrix}$$

$$\tilde{k}_{AVE} = k_z \hat{\tilde{z}}$$

$$\tilde{k}_{MOD} = -k_x \hat{\tilde{x}}$$

$$g(x,y,z,t] = 2A_0 \cos(k_z z - \omega_0 t)\,\cos(k_x x)$$

k



$$\sin\theta = \frac{d_0}{2}\cdot\frac{1}{|k_0|} = \frac{d_0}{2}\frac{\lambda_0}{2\pi}$$

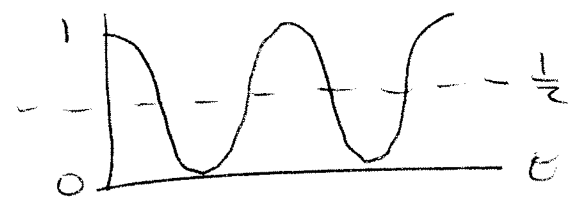
$$k_x = |k_0|\sin\theta$$

$$k_z = |k_0|\cos\theta$$

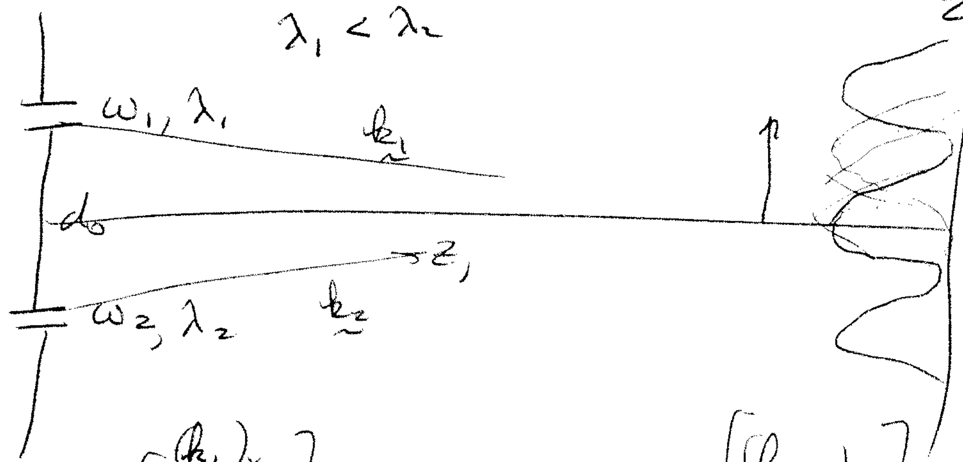
$$\langle|g|^2\rangle = 4A_0^2\,\underbrace{\langle\cos^2(\;)\rangle}_{\frac{1}{2}}\,\langle\cos^2(k_x x)\rangle$$

$$= 2A_0^2\cdot\cos^2(k_x x)$$

$$= A_0^2\cdot(1+\cos(2k_x x))$$



$\lambda_1 < \lambda_2$



$$|k_1| = \frac{2\pi}{\lambda_1} \qquad \begin{bmatrix} (k_1)_x \\ 0 \\ (k_1)_z \end{bmatrix}$$

$$|k_2| = \frac{2\pi}{\lambda_2} \qquad \begin{bmatrix} (k_2)_x \\ 0 \\ (k_2)_z \end{bmatrix}$$

$$> \quad k_{AVG} = \begin{bmatrix} (k_{AVG})_x \\ 0 \\ (k_{AVG})_z \end{bmatrix}$$

$$k_{MOD} = \begin{bmatrix} (k_{MOD})_x \\ 0 \\ (k_{MOD})_z \end{bmatrix}$$

BOTH AVERAGE & MOD WAVES TRAVEL

$\Delta\tau$ — TIME AVERAGE, SHORT COMPARED TIME FOR $|g|^2$ TO MOVE 1/2 CYCLE

2/3/10 (5)

$$\Delta \tau < \frac{1}{\Delta \nu} = \frac{2\pi}{\Delta \omega}$$ COHERENCE TIME

$\omega_1 \Rightarrow \nu_1$

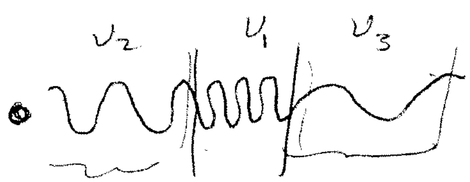
$\Delta \nu = |\nu_1 - \nu_2|$

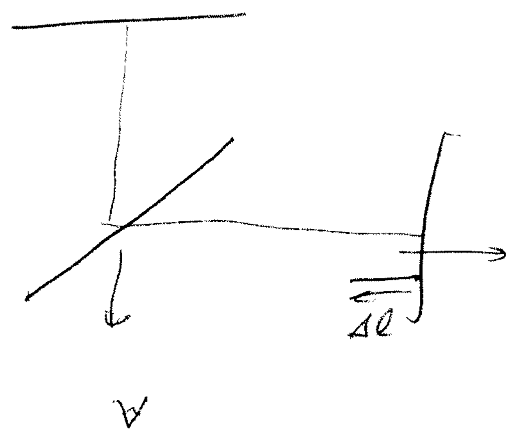
$\omega_2 \Rightarrow \nu_2$

COHERENCE TIME

$\Delta \tau$ — TIME PERIOD OVER WHICH PHASE CHANGES BY $< \pi$

PHASES ARE ~~CORRELATION~~ CORRELATED





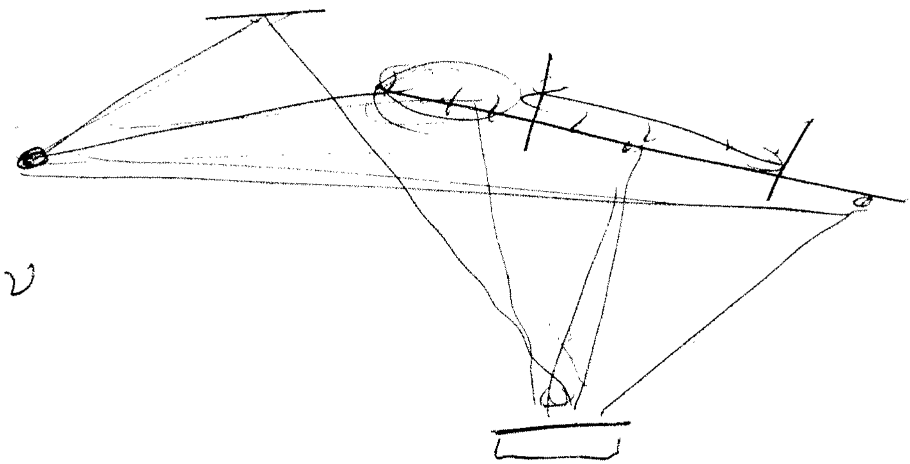
~~El~~ $2\Delta l$

c

$c \cdot \Delta \tau < 2 \cdot \Delta l$

$$\frac{1}{\Delta\nu} = \Delta\tau$$

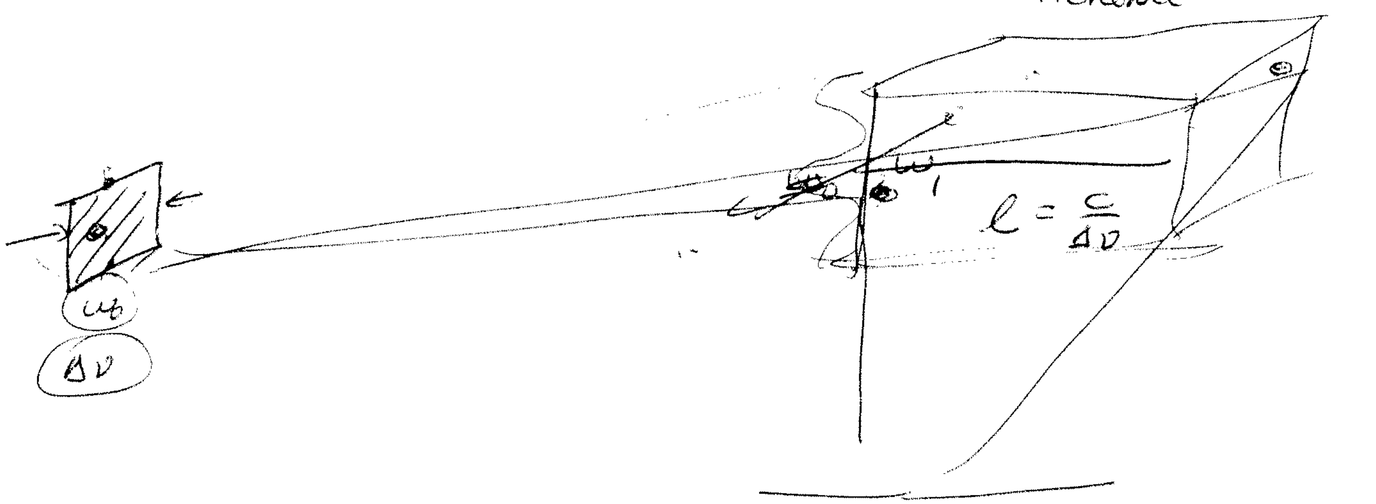
$$c \cdot \Delta\tau = \Delta l$$ = COHERENCE LENGTH OF SOURCE

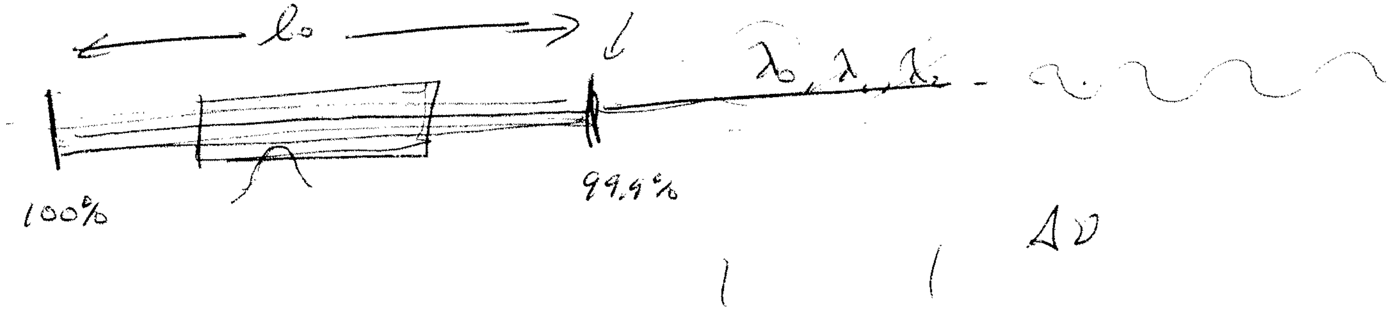


$\Delta\nu$

$$\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)$$

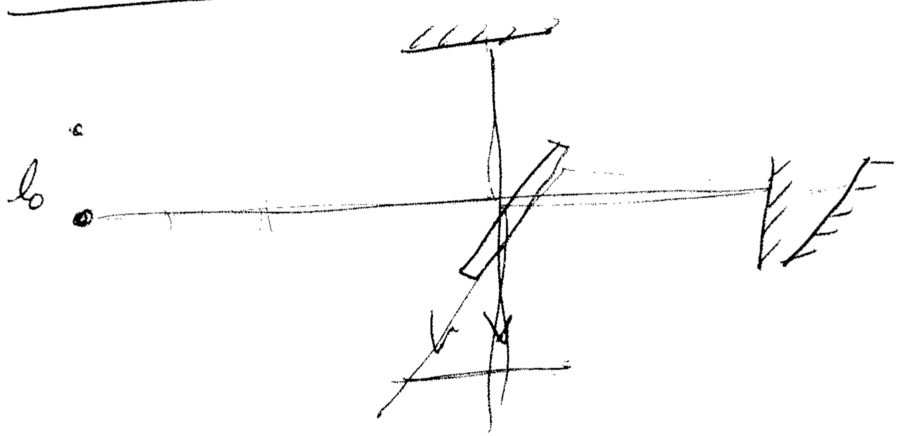
COHERENCE VOLUME

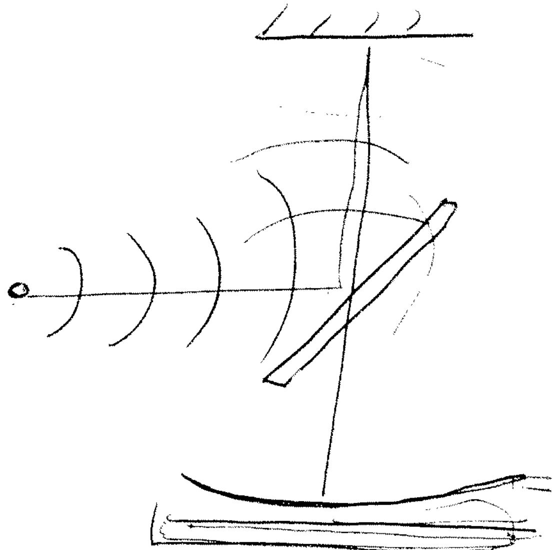


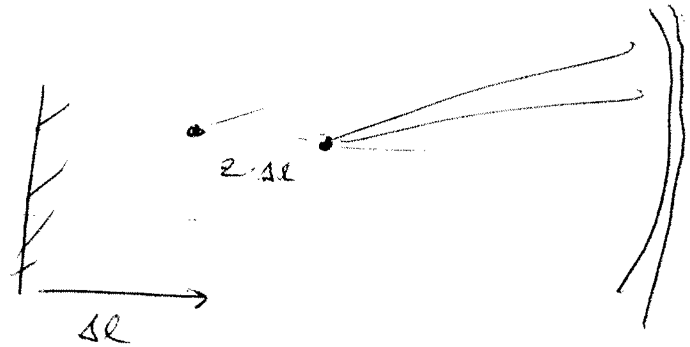


$$m\lambda_0 = 2l_0$$

$$\lambda_1\left(m+\tfrac{1}{2}\right) = 2l_0$$



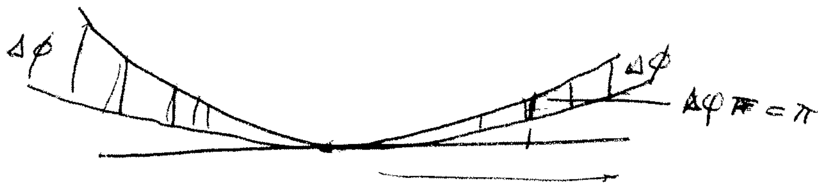


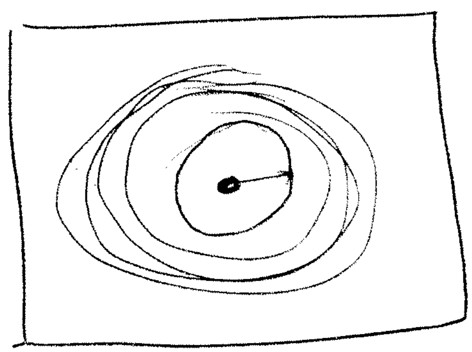


$$\frac{r^2}{\lambda_0 z_1} - \frac{r^2}{\lambda_0 z_2} = \frac{r^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{z_2}\right) = \frac{r^2}{\lambda_0}\,\frac{z_2 - z_1}{z_1 z_2} = \frac{r^2}{\lambda_0 \frac{z_1 z_2}{z_2 - z_1}}$$

α^2

$$\Delta\phi \propto r^2$$

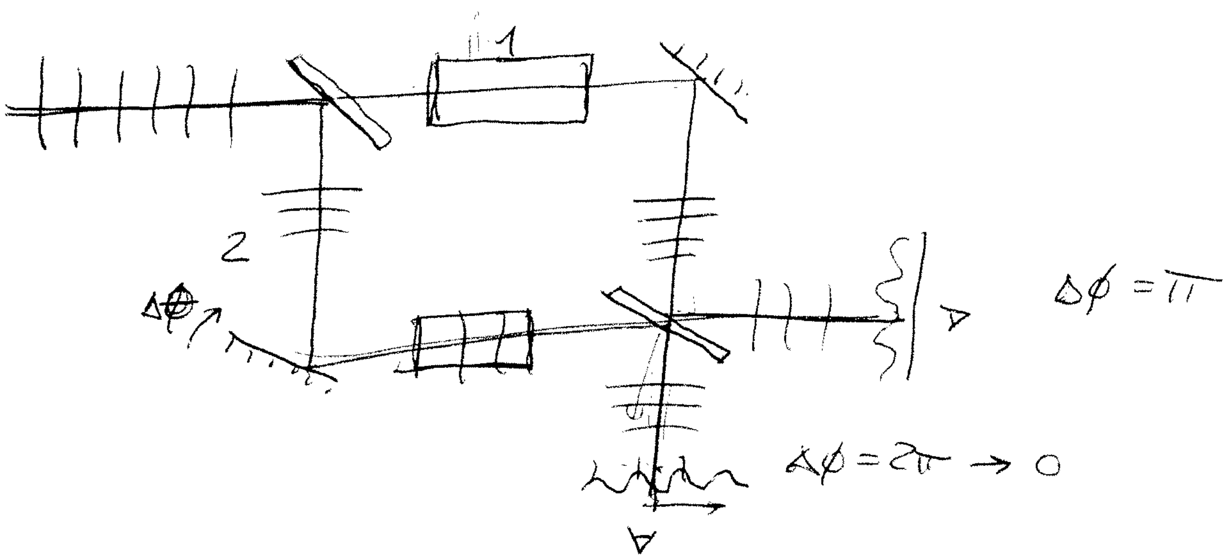




VARIANTS OF MICHELSON

MACH - ZEHNDER INTERFEROMETER - "UNFOLDED" MICHELSON

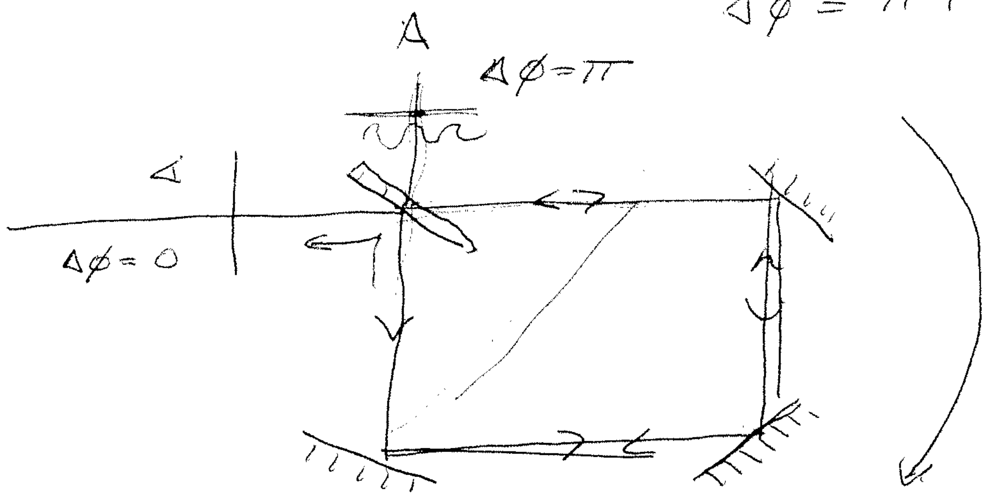
$n_{AIR} = 1.00027$ @ STP

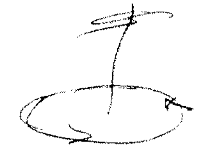


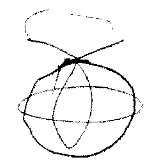
2/3/10 (10)

SAGNAC INTERFEROMETER

$\Delta\phi = \pi + \varepsilon$

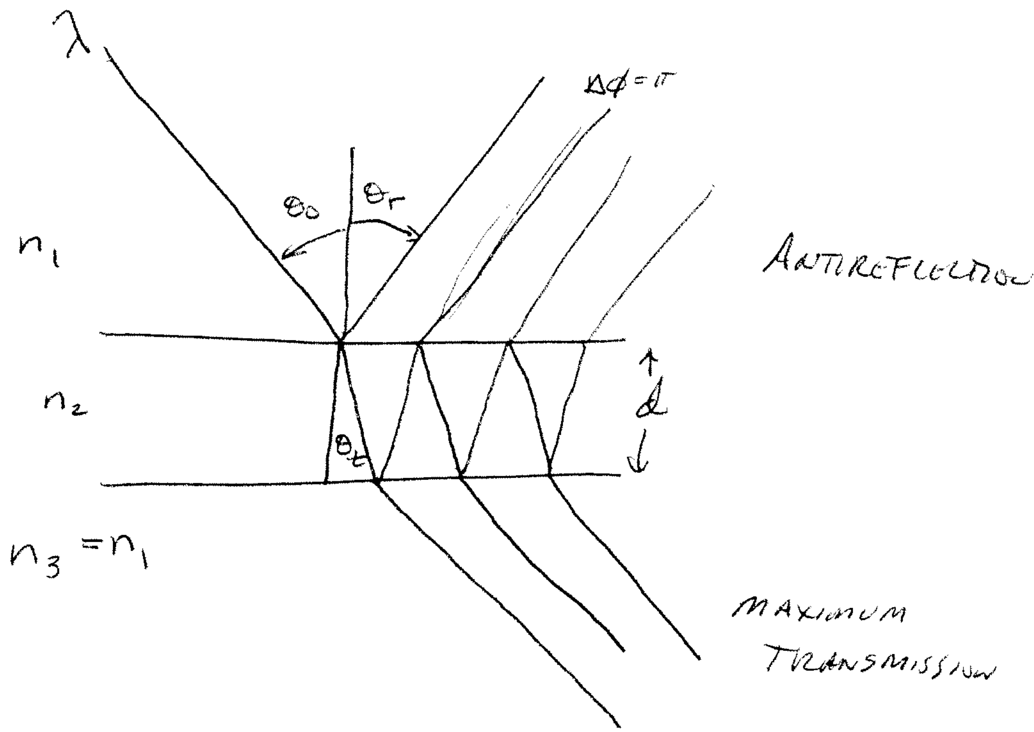






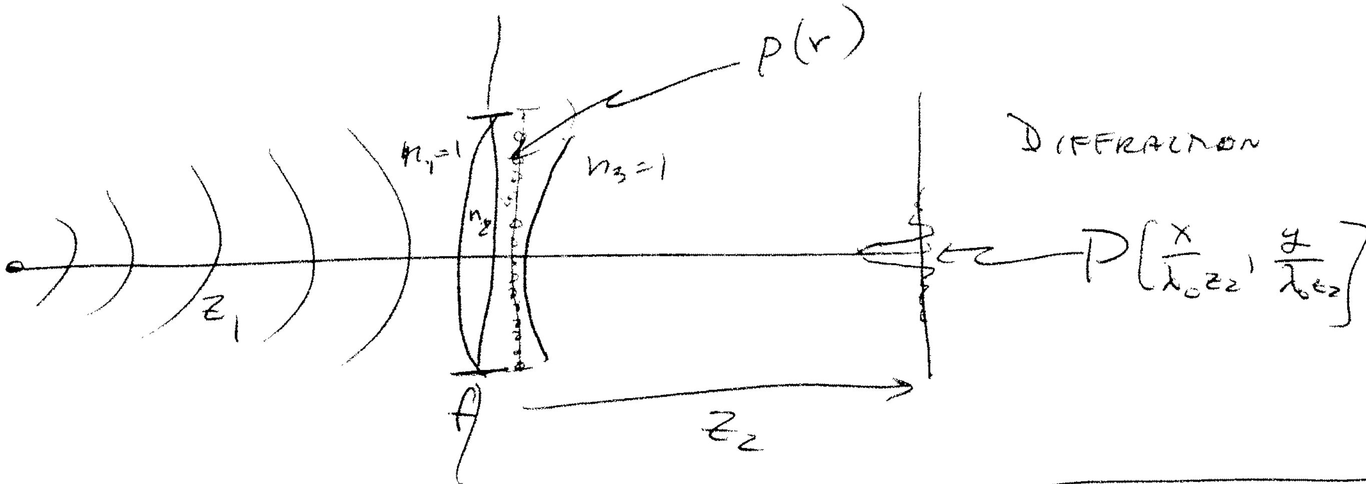
LASER GYROSCOPE

2/3/10 - (11)



λ
$\Delta\phi = \pi$
θ_0
θ_r
n_1
ANTIREFLECTION
n_2
θ_t
d
$n_3 = n_1$
MAXIMUM
TRANSMISSION

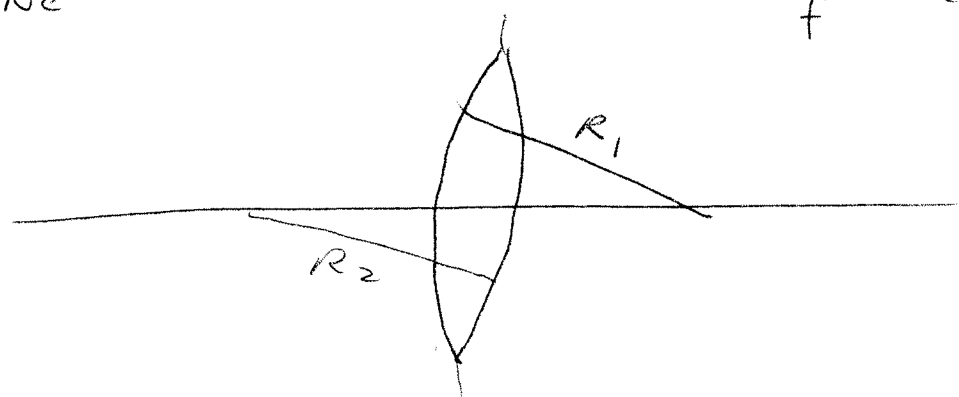
RAY OPTICS - GEOMETRICAL OPTICS



$$\frac{1}{z_2} = \frac{1}{f} - \frac{1}{z_1} \Rightarrow \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

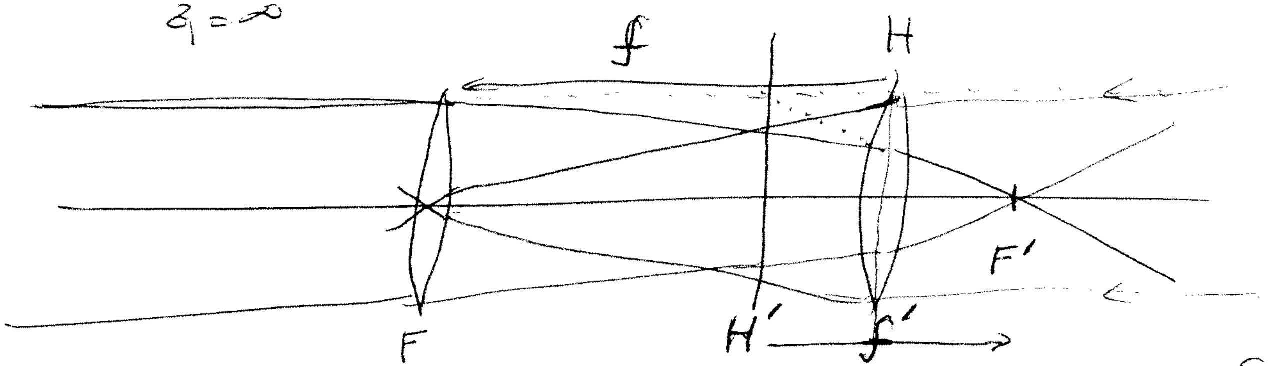
DERIVE LENSMAKER'S EQUATION

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



PRINCIPAL PLANES

$z_1 = \infty$



$f(x, y)$

PRINCIPAL PLANE ⇒ LOCATION OF EQUIVALENT SINGLE THIN LENS