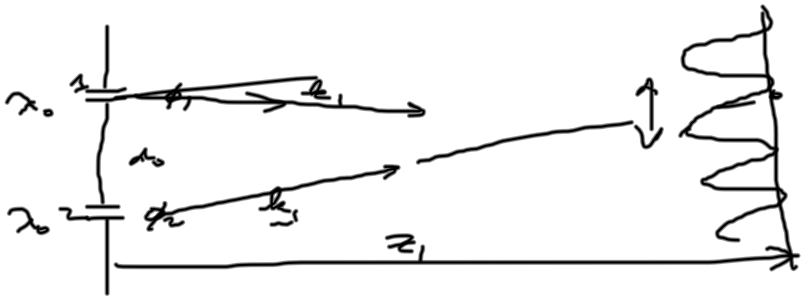
3 FEBRUARY 206 -Ø

INTERFEROMETRY

D.W YOUNG'S TWO-APERTURE → SINUSOIDAL, SPATIAL COHERENCE

D.A MICHELSON → TEMPORAL COHERENCE — TEMPORAL FRINGE



$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda z_1}\right) \cdot \langle \cos \Delta\phi(t) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda z_1}\right) \cdot \langle \cos \Delta\phi \rangle}$$

$$\langle \cos \Delta\phi \rangle = 1 - \frac{\langle (\Delta\phi)^2 \rangle}{2!} + \cdots$$

$$A_0 \cos(\underset{\sim}{k}_1 \cdot \underset{\sim}{r} - \omega_0 t + \phi_1) + A_0 \cos(\underset{\sim}{k}_2 \cdot \underset{\sim}{r} - \omega_0 t + \phi_2)$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

AVERAGE PHASE   MODULATION

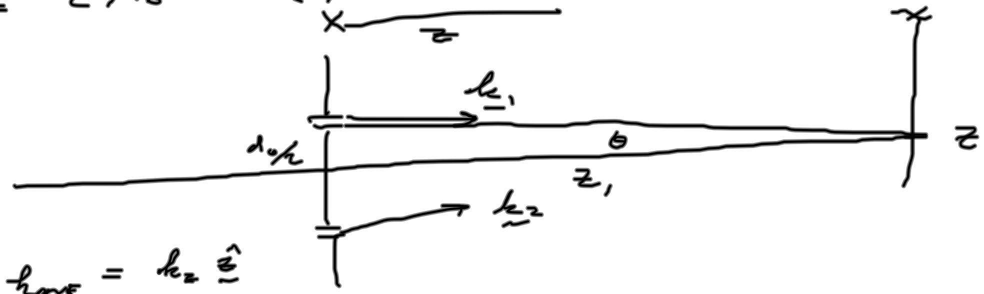
$$\cos(\underset{\sim}{k}_1 \cdot \underset{\sim}{r} - \omega_0 t) + \cos(\underset{\sim}{k}_2 \cdot \underset{\sim}{r} - \omega_0 t)$$

$$= 2A_0 \underbrace{\cos\Big(\underbrace{\frac{\underset{\sim}{k}_1+\underset{\sim}{k}_2}{2}}_{\underset{\sim}{k}_{AVG}} \cdot \underset{\sim}{r} - \underbrace{\frac{\omega_0+\omega_0}{2}}_{\omega_{AVG}} t\Big)}_{\text{AVERAGE WAVE}} \cdot \underbrace{\cos\Big(\underbrace{\frac{\underset{\sim}{k}_1-\underset{\sim}{k}_2}{2}}_{\underset{\sim}{k}_{MOD}} \cdot \underset{\sim}{r} - \underbrace{0 \cdot t}_{\omega_{MOD}=0}\Big)}_{\text{INTERFERENCE STANDING WAVE}}$$

$$\omega_1 - \omega_2 = \omega_0 \Rightarrow \frac{\omega_1 - \omega_2}{2} = 0$$

$$g[x,y,z,t] = 2A_0 \cos(\underbrace{\underset{\sim}{k}_{AVG} \cdot \underset{\sim}{r} - \omega_0 t}_{z}) \cdot \cos(\underbrace{\underset{\sim}{k}_{MOD} \cdot \underset{\sim}{r}}_{x})$$

$$\underset{\sim}{k}_1 = \begin{bmatrix} -k_x \\ 0 \\ +k_z \end{bmatrix}$$



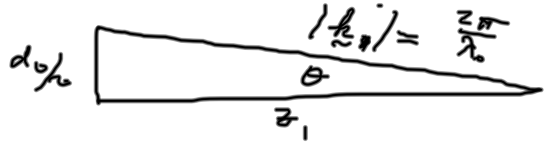
$$\underset{\sim}{k}_2 = \begin{bmatrix} +k_x \\ 0 \\ +k_z \end{bmatrix}$$

$$k_{AVG} = k_z \hat{\underset{\sim}{z}}$$

$$k_{MOD} = -k_x \hat{\underset{\sim}{x}}$$

$$g[x,y,z,t] = 2A_0 \cos(k_z z - \omega_0 t)\cos(k_x x)$$

k



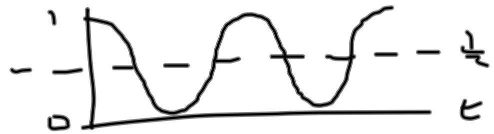
$$\sin\theta = \frac{d_0}{2}\cdot\frac{1}{|k_0|} = \frac{d_0}{2}\frac{\lambda_0}{2\pi}$$

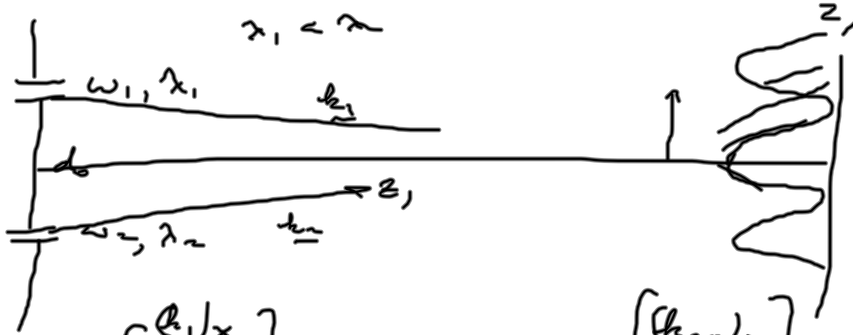
$$\left.\begin{aligned} k_x &= |k_0| \sin\theta \\ k_z &= |k_0| \cos\theta \end{aligned}\right\}$$

$$\langle |g|^2 \rangle = 4A_0^2 \underbrace{\langle \cos^2(\;) \rangle}_{\frac{1}{2}} \langle \cos^2(k_x x) \rangle$$

$$= 2A_0^2 \cdot \cos^2(k_x x)$$

$$= A_0^2 \cdot (1 + \cos(2k_x x))$$





$$|\underset{\sim}{k}_1| = \frac{2\pi}{\lambda_1} \qquad \begin{bmatrix} (k_1)_x \\ 0 \\ (k_1)_z \end{bmatrix}$$

$$|\underset{\sim}{k}_2| = \frac{2\pi}{\lambda_2} \qquad \begin{bmatrix} (k_2)_x \\ 0 \\ (k_2)_z \end{bmatrix}$$

$$k_{avg} = \begin{bmatrix} (k_{avg})_x \\ 0 \\ (k_{avg})_z \end{bmatrix} \qquad k_{mod} = \begin{bmatrix} (k_{mod})_x \\ 0 \\ (k_{mod})_z \end{bmatrix}$$

BOTH AVERAGE & MOD WAVES TRAVEL

$\Delta\tau$ — TIME AVERAGE, SHORT COMPARED TO MODS 1/2 CYCLE   TIME FOR $|\psi|^2$

2/3/10 (3)

$$\underbrace{\Delta t < \frac{1}{\Delta \nu}} = \frac{2\pi}{\Delta \omega}$$ COHERENCE TIME

$\omega_1 \Rightarrow \nu_1$

$\Delta \nu = |\nu_1 - \nu_2|$

$\omega_2 \Rightarrow \nu_2$

COHERENCE TIME

$\Delta \tau$ — TIME PERIOD OVER WHICH PHASE CHANGES BY $< \pi$

PHASES ARE ~~CORRELATED~~ CORRELATED

~~Ed~~ $\dfrac{2\Delta l}{c}$

$c \cdot \Delta \tau < 2 \cdot \Delta l$

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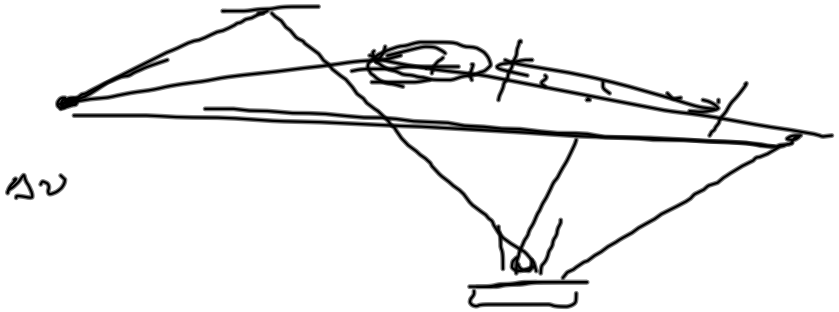
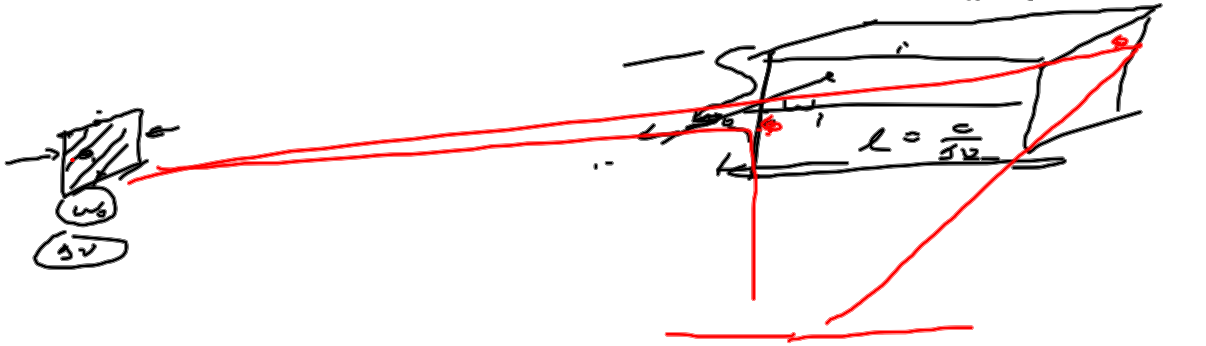
$$\frac{1}{\Delta\nu} = \Delta\tau$$

$c \cdot \Delta\tau = \Delta l$ = COHERENCE LENGTH OF SOURCE



$$\cos\left(\pi \frac{d_0 l_0}{\lambda z_0}\right)$$

COHERENCE VOLUME



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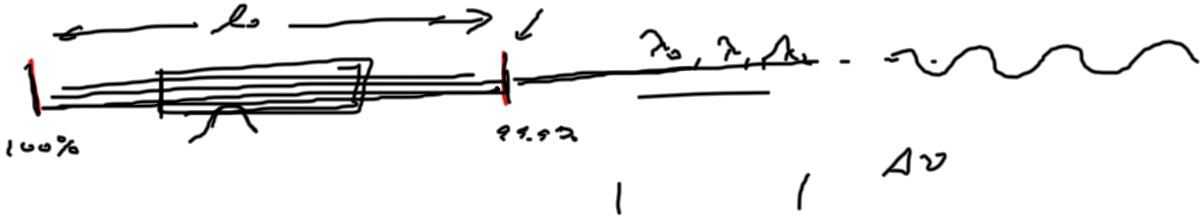
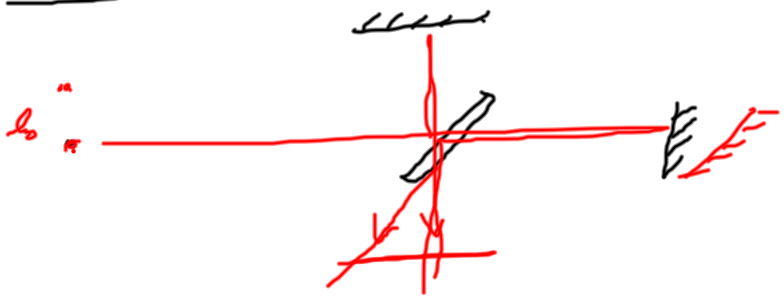


$$m\lambda_0 = 2l_0$$

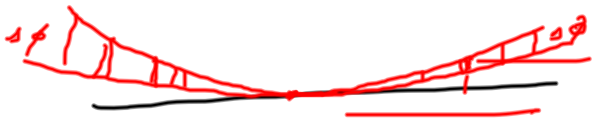
$$\lambda_1\left(m+\tfrac{1}{2}\right) = 2l_0$$

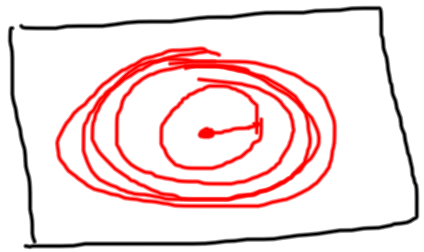


$$\frac{r^2}{\lambda_0 z_1} - \frac{r^2}{\lambda_0 z_2} = \frac{r^2}{\lambda}\left(\frac{1}{z_1} - \frac{1}{z_2}\right) = \frac{r^2}{\lambda}\,\frac{z_2 - z_1}{z_1 z_2} = \frac{r^2}{\lambda \frac{z_1 z_2}{z_2 - z_1}}$$

$\Delta\phi \propto r^2$

$\propto^2$





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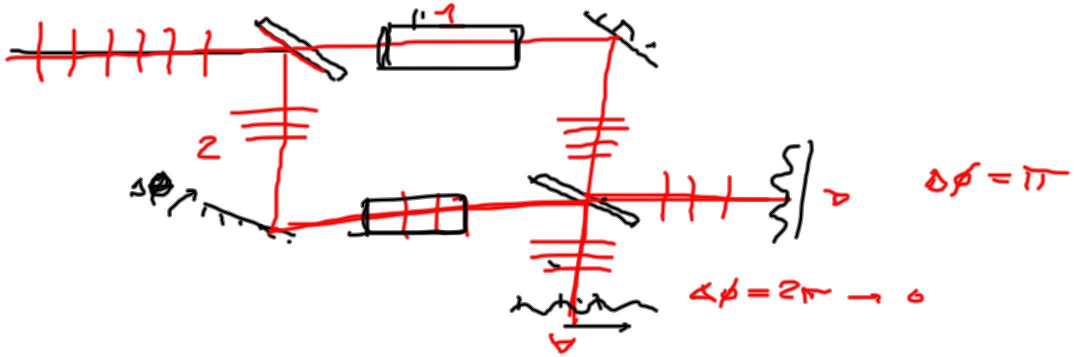
VARIANTS OF MICHELSON

MACH – ZEHNDER INTERFEROMETER – "UNFOLDED" MICHELSON

$n_{air} = 1.00027$ @ STP



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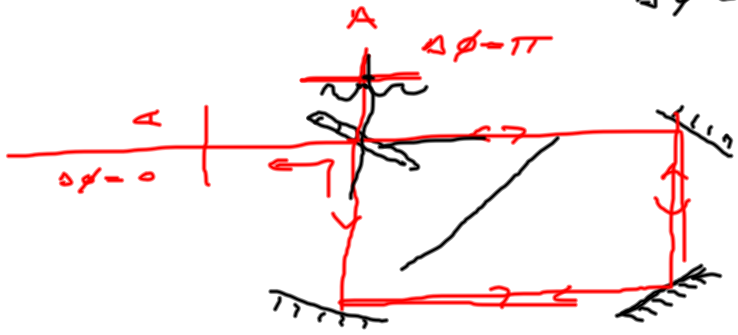
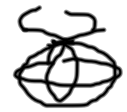
SAGNAC INTERFEROMETER

$\Delta\phi = \pi + \varepsilon$



DOPPLER SHIFT



LASER GYROSCOPE

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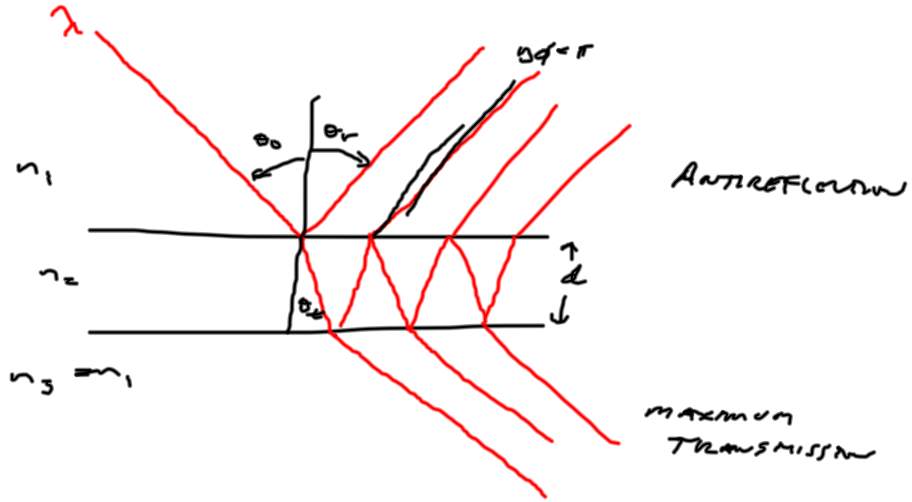
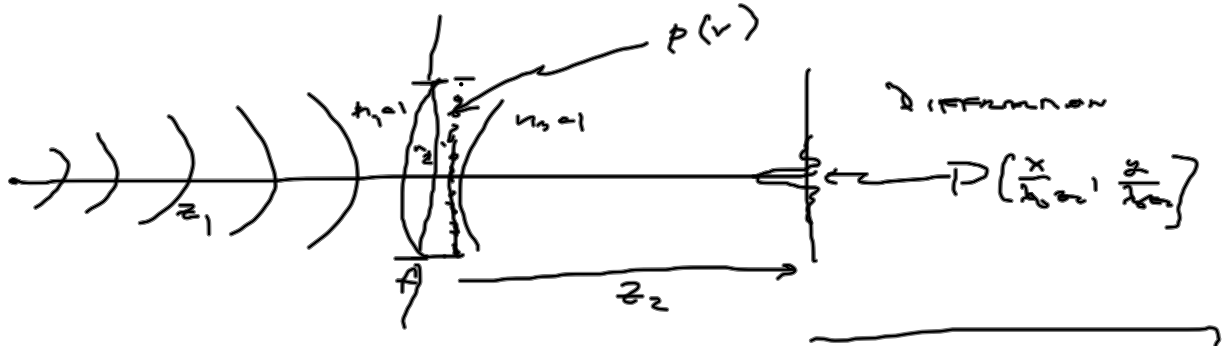


λ
δφ = π
θ₀
θr
n1
n2
θt
d
n3 = n1
ANTIREFLECTION
MAXIMUM TRANSMISSION

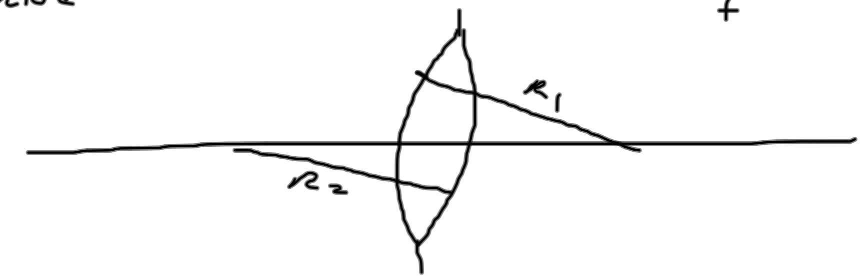
RAY OPTICS — GEOMETRICAL OPTICS



$$\boxed{\frac{1}{z_2} = \frac{1}{f} - \frac{1}{z_1}} \Rightarrow \boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}}$$

DERIVE LENSMAKER'S EQUATION

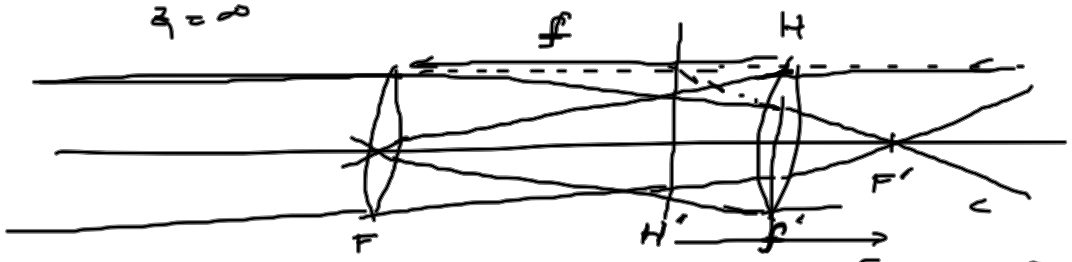
$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



PRINCIPAL PLANES

2/3/~~??~~ 10 — 13

$f[x,y]$



PRINCIPAL PLANE ⟹ LOCATION OF EQUIVALENT SINGLE THIN LENS