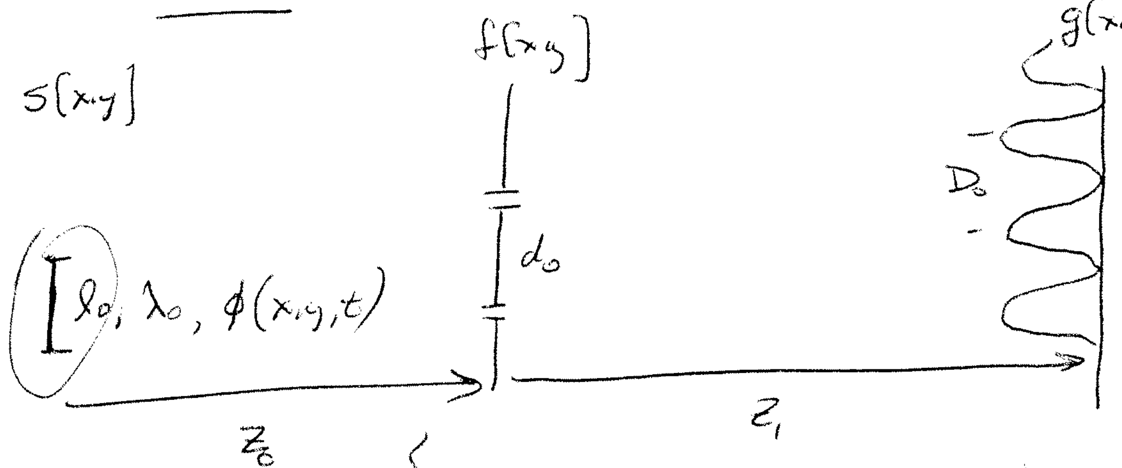
1 FEBRUARY 2010

INTERFEROMETRY $\Longrightarrow$ DIFFRACTION

COHERENCE – SPATIAL (TRANSVERSE)

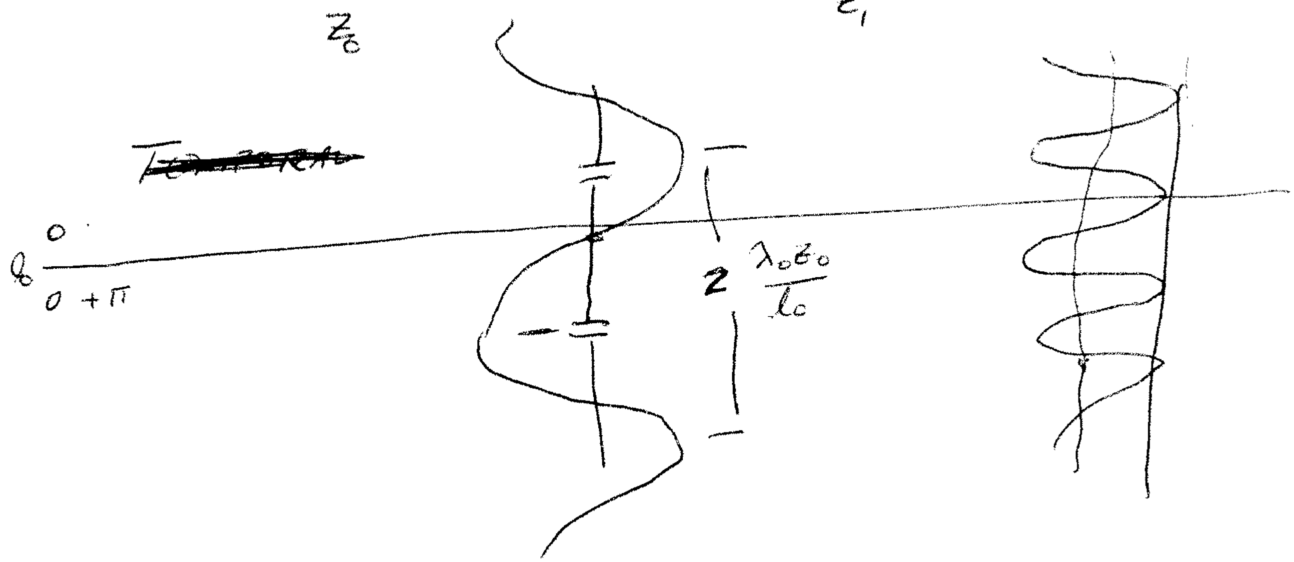
TEMPORAL (LONGITUDINAL)

SPATIAL



$$D_0 = \frac{\lambda_0 z_1}{d_0}$$

$$m = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$



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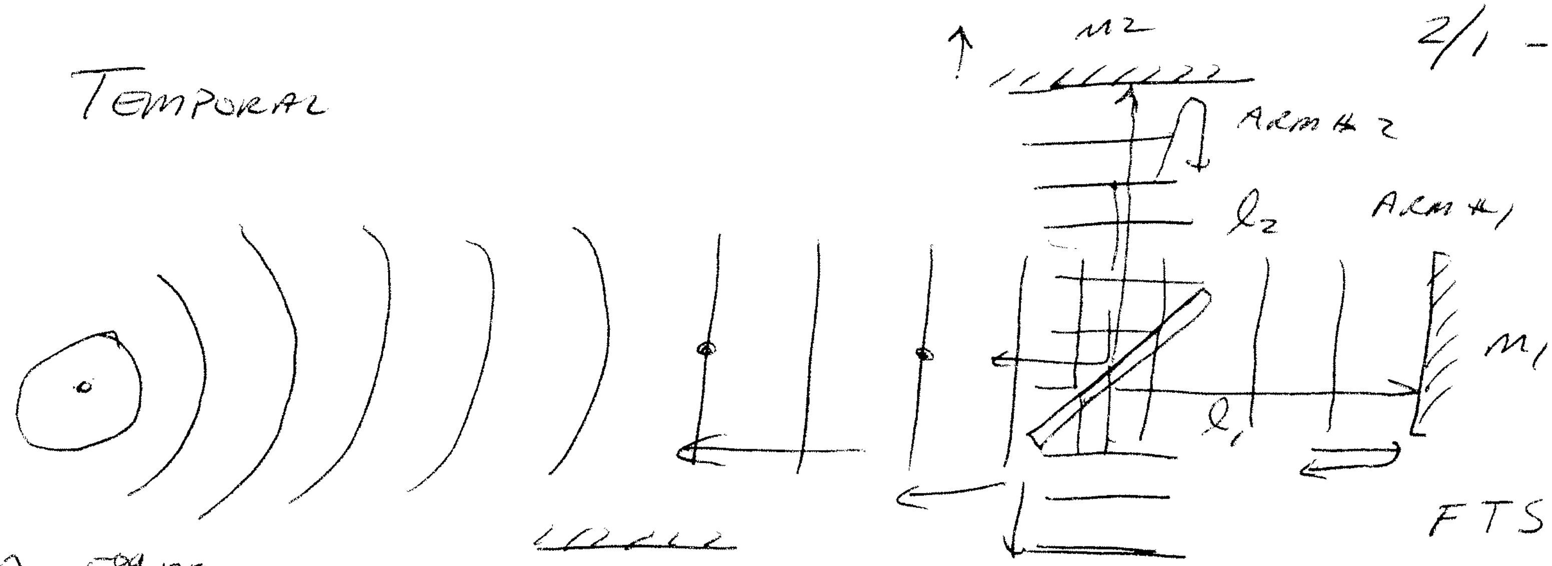
TEMPORAL

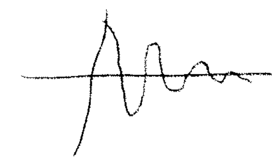


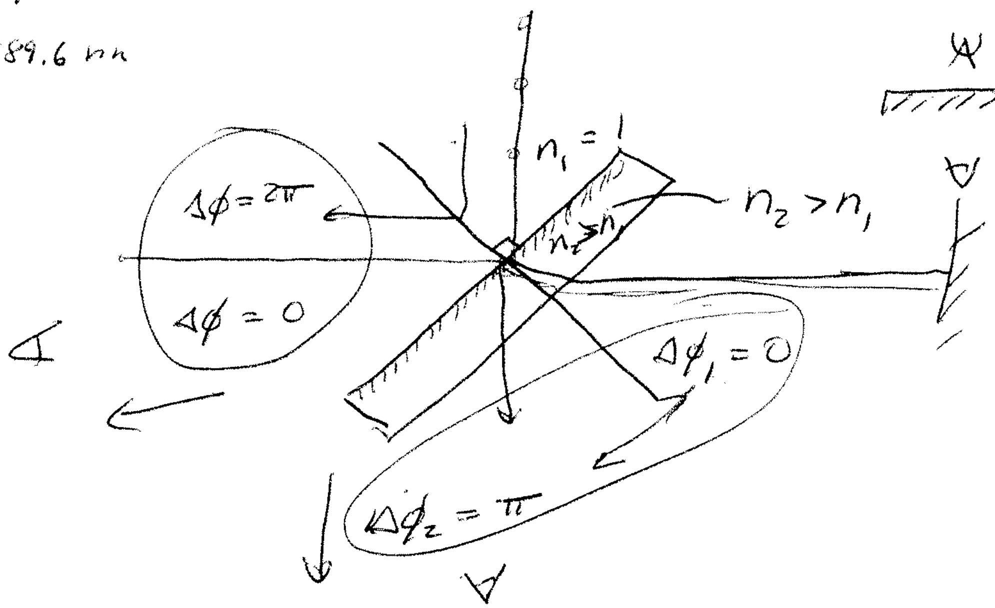
$\lambda_1 = 589$ nm

$\lambda_2 = 589.6$ nm

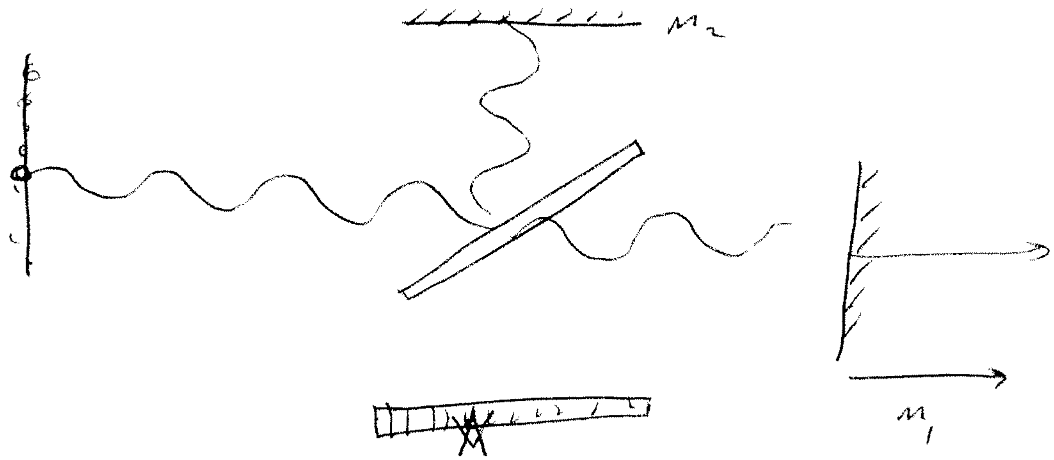
FTS

OUTPUT



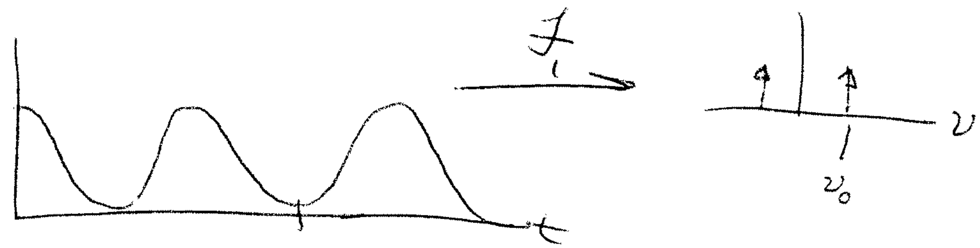


$$r = \frac{n_1 - n_2}{n_1 + n_2} < 0$$



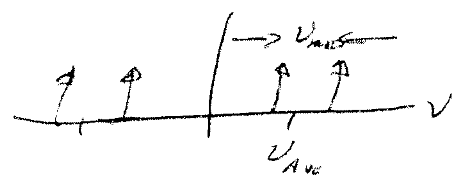
IFTS

$f(x, y, \lambda)$



$\nu_{AVG} = \frac{\nu_1 + \nu_2}{2}$

$\nu_{MOD} = \frac{\nu_1 - \nu_2}{2}$



SPATIAL (TRANSVERSE) $\Rightarrow$ DIVISION-OF-~~EN~~ WAVEFRONT INTERFEROMETER

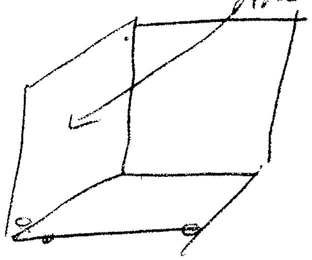
COHERENCE "WIDTH" → "AREA" DoW

TEMPORAL (LONGITUDINAL) $\Rightarrow$ DIVISION-OF-AMPLITUDE I

DoA

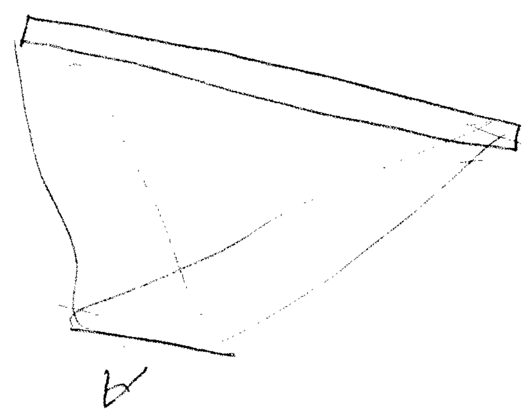
COHERENCE TIME $\Rightarrow$ COHERENCE LENGTH → DISTANCE (TIME DELAY) AND STILL OBSERVE INTERFERENCE

AREA $\Rightarrow$ SOURCE SIZE



LENGTH → $\Delta\nu$, $\Delta\lambda$

HOLOGRAPHY



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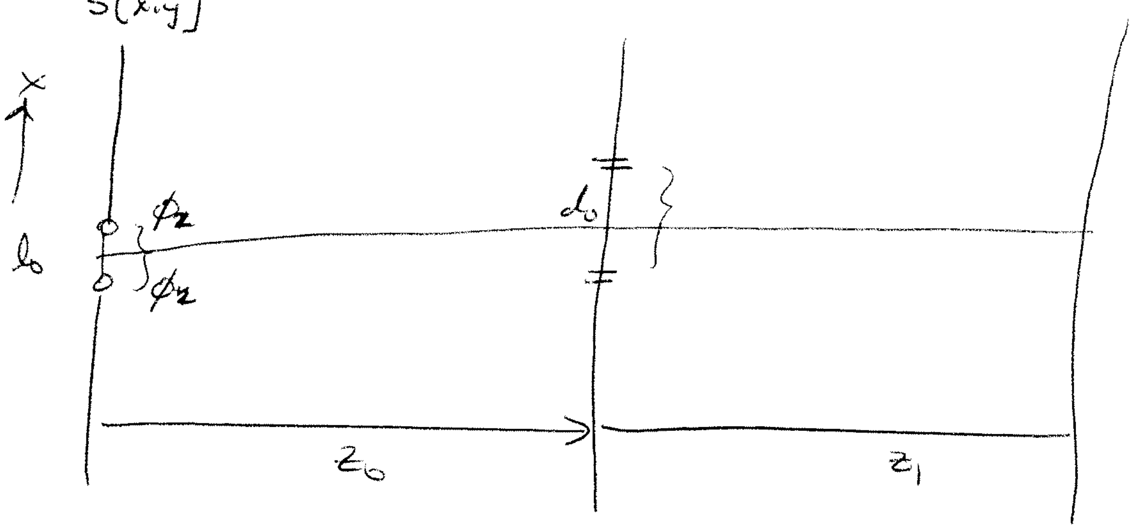
SPATIAL COHERENCE

$g(x,y) \longrightarrow I(x,y) = \langle |g[x,y]|^2 \rangle$



$$s[x,y] = \delta\left[x + \frac{l_0}{2}\right] e^{i\phi_1(t)} + \delta\left[x - \frac{l_0}{2}\right] e^{i\phi_2(t)}$$

$$S[\xi,\eta] = e^{+2\pi i \frac{l_0}{2}\xi} e^{i\phi_1} + e^{-2\pi i \frac{l_0}{2}\xi} e^{i\phi_2} \qquad \xi \rightarrow \frac{x}{\lambda_0 z_\phi}$$

$$= e^{+i 2\pi i \frac{l_0}{2}\xi} e^{i\phi_1/2} e^{i\frac{\phi_1}{2}} e^{i\frac{\phi_2}{2}} e^{-i\frac{\phi_2}{2}} + e^{-2\pi i \frac{l_0}{2}\xi} e^{i\frac{\phi_1}{2}} e^{-i\frac{\phi_1}{2}} e^{i\frac{\phi_2}{2}} e^{i\frac{\phi_2}{2}}$$

ϕ_{AVG}

$$= e^{i\frac{\phi_1+\phi_2}{2}} \left[e^{+i\pi l_0 \xi} e^{+i\frac{\phi_1-\phi_2}{2}} + e^{-i\pi l_0 \xi} e^{-i\frac{\phi_1-\phi_2}{2}} \right]$$

$$\frac{\Delta\phi}{2} = \phi_{mod}$$

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$$S[\xi,\eta] = e^{i\phi_{AVG}}\left[e^{+i(\pi l_0 \xi + \phi_{MOD})} + e^{-i(\pi l_0 \xi + \phi_{MOD})}\right]$$

$$= e^{i\phi_{AVG}} \cdot 2\cos\left(\pi l_0 \xi + \phi_{MOD}\right)$$

AMPLITUDE @ APERTURES $f(x,y)$

$$\cancel{S} \quad S\left[\frac{x}{\lambda_0 z_0}\right] = e^{+i\phi_{AVG}} \cdot 2\cos\left[\pi l_0 \frac{x}{\lambda_0 z_0} + \phi_{MOD}\right]$$

$$= \boxed{e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \underbrace{\frac{x}{\left(\frac{2\lambda_0 z_0}{l_0}\right)}} + \phi_{MOD}\right]}$$

PERIOD OF AMPLITUDE @ $f(x,y)$

$$= e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \frac{\left(x + \frac{2\lambda_0 z_0}{l_0} \cdot \frac{\Delta\phi}{2}\right)}{\left(\frac{2\lambda_0 z_0}{l_0}\right)}\right] \longrightarrow \cos\left(\frac{2\pi(x-x_0)}{X_0}\right)$$

COSINE AMPLITUDE IS TRANSLATED BY $x_0 = \left[-\frac{\lambda_0 z_0}{l_0} \cdot \Delta\phi\right]$

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MULTIPLY BY APERTURE FUNCTION $f(x) = \delta(x+\frac{d_0}{2}) + \delta(x-\frac{d_0}{2})$

$$e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \frac{x}{\left(\frac{2\lambda_0 z_0}{l_0}\right)} + \phi_{MOD}\right] \cdot \left(\delta(x+\frac{d_0}{2}) + \delta(x-\frac{d_0}{2})\right)$$

$$= 2e^{i\phi_{AVG}}\left[\delta\left[x+\frac{d_0}{2}\right]\cos\left[2\pi\frac{-\frac{d_0}{2}}{\frac{2\lambda_0 z_0}{l_0}} + \phi_{MOD}\right] + \delta\left(x-\frac{d_0}{2}\right)\cos\left[2\pi\frac{+\frac{d_0}{2}}{\frac{2\lambda_0 z_0}{l_0}} + \phi_{MOD}\right]\right]$$

$$= 2e^{i\phi_{AVG}}\left(\delta\left(x+\frac{d_0}{2}\right)\cos\left[-\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right] + \delta\left(x-\frac{d_0}{2}\right)\cos\left[+\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right]\right)$$

$$= 2e^{i\phi_{AVG}}\left(\underset{\sim}{\delta(x+\tfrac{d_0}{2})}\cos\underline{\left(\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} - \phi_{MOD}\right)} + \underset{\sim}{\delta(x-\tfrac{d_0}{2})}\cos\underline{\left(\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right)}\right)$$

PROPAGATE TO OBSERVATION PLANE

$$g[x,y] = 2e^{i\phi_{AVG}}\left[\cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} - \phi_{MOD}\right] e^{+2\pi i \frac{x}{\lambda_0 z_1}\frac{d_0}{2}} + \cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right] e^{-2\pi i \frac{x d_0}{2\lambda_0 z_1}}\right]$$

$$\text{IF } \phi_{MOD} = \frac{\Delta\phi}{2} = \frac{\phi_1 - \phi_2}{2} = 0$$

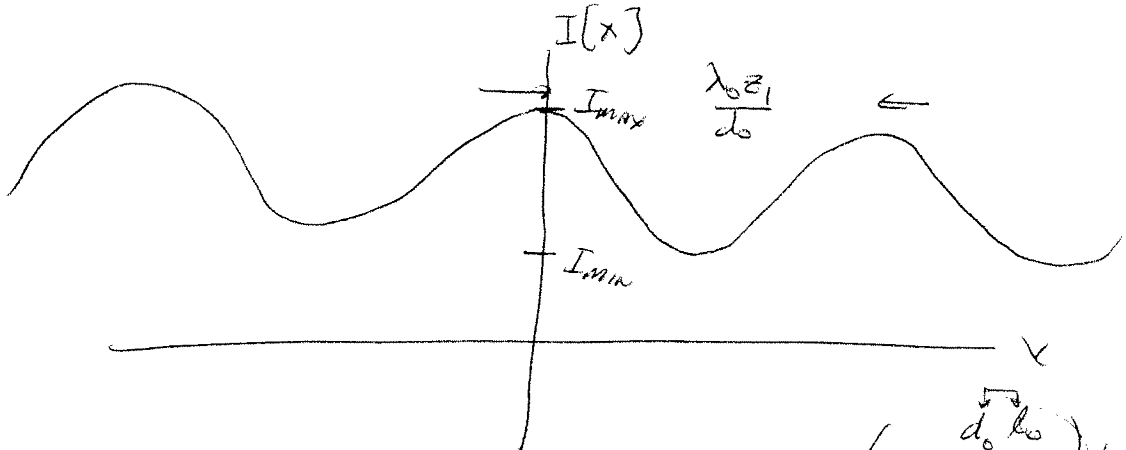
$$g[x,y;\Delta\phi=0] = 2e^{\cancel{i\phi_{AVG}}\ \phi_0}\left\{\cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right] \cdot \underbrace{2\cos\left[2\pi\frac{x}{\left(\frac{2\lambda_0 z_1}{d_0}\right)}\right]}\right\}$$

$$|g[x;\Delta\phi=0]|^2 = 16\underbrace{\cos^2\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right]} \cdot \cancel{\cos}\ \underbrace{\frac{1}{2}\left(1+\cos\left(2\pi\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)\right)}_{\text{BIASED COSINE}}$$

$$\cos^2\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right] = 0 \quad \text{IF} \quad \frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} = \pm\frac{\pi}{2}$$

$$\Rightarrow \frac{l_0 d_0}{\lambda_0 z_0} = 1 \quad \text{IF} \quad \frac{l_0 d_0}{\lambda_0 z_0} < 1$$

MODULATION OF IRRADIANCE PATTERN



$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \equiv m \equiv V = \frac{\cos\left(\pi \cdot \frac{d_0 l_0}{\lambda_0 z_0}\right) + \langle \cos(\Delta\phi(t)) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos(\Delta\phi(t)) \rangle}$$

EXAMPLES

$\langle \Delta\phi \rangle = 0$ $\quad V = \cancel{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)} \; \frac{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)} = 1$

IF $\frac{d_0 l_0}{\lambda_0 z_0} = 1$, $I = 0$

RANDOM PHASE DIFFERENCE

$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) + \langle \cos(\Delta\phi(t)) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos(\Delta\phi(t)) \rangle}$$

IF $\Delta\phi$ IS RANDOM, $\Rightarrow -1 \leq \cos(\Delta\phi) \leq 1$

$$-1 \leq \cos(\Delta\phi) \leq +1$$

$$\langle \cos(\Delta\phi) \rangle = 0$$

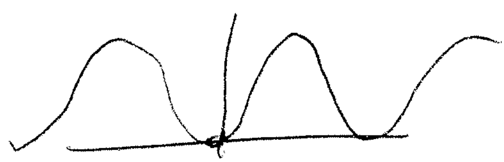
$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) + 0}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot 0} = \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)$$

$$-1 \leq V \leq 1$$

$l_0 = 0 \Rightarrow$ ONE SOURCE

$d_0 = 0 \Rightarrow$ NO INTERFERENCE

$$\frac{d_0 l_0}{\lambda z_0} = 1$$



$$\frac{d_0 l_0}{\lambda_0 z_0} < 1$$

$$l_0 < \frac{\lambda_0 z_0}{d_0}$$

Coherence WIDTH