1 FEBRUARY 2010

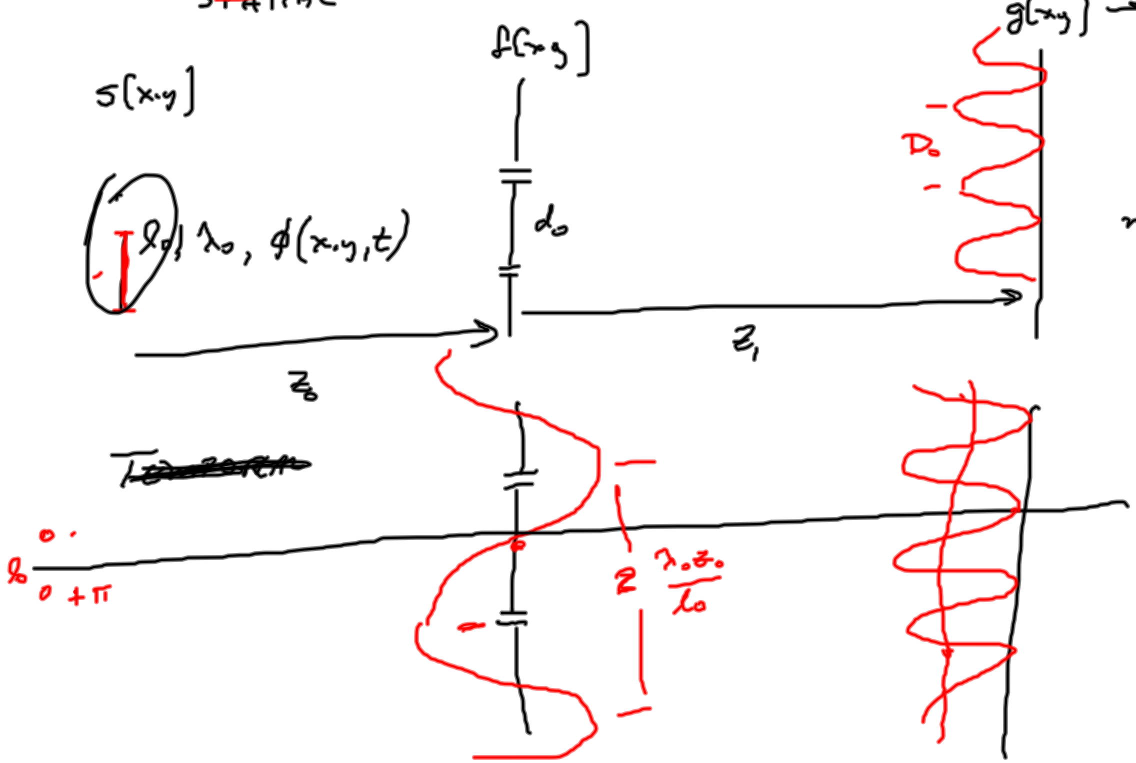
7

INTERFEROMETRY $\Rightarrow$ DIFFRACTION

COHERENCE – SPATIAL (TRANSVERSE)

TEMPORAL (LONGITUDINAL)

~~SPATIAL~~

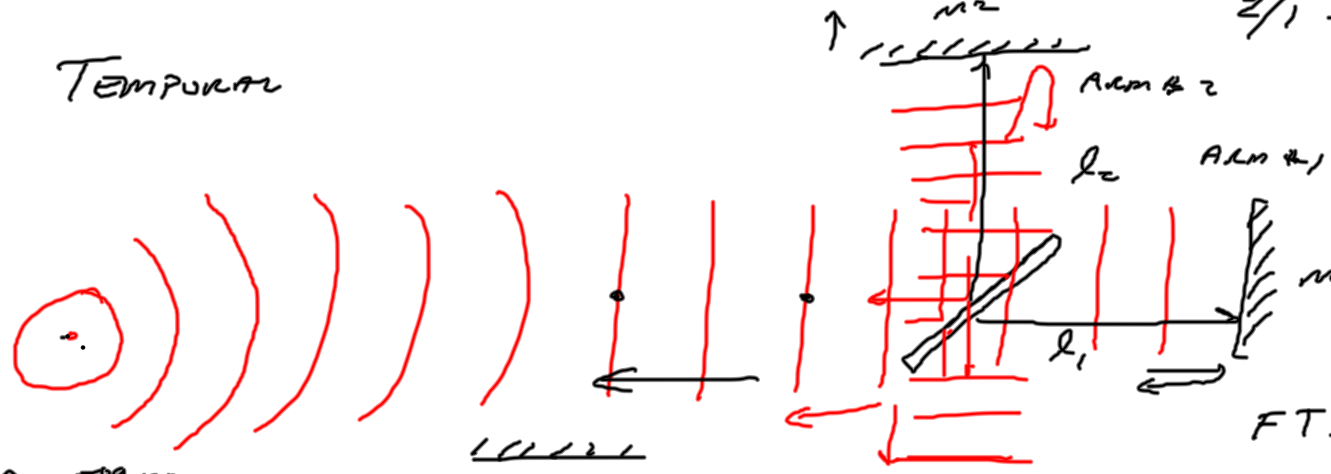


$$D_0 = \frac{\lambda_0 z_1}{d_0}$$

$$m = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

2/1 – 2

TEMPORAL

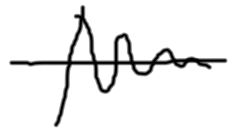


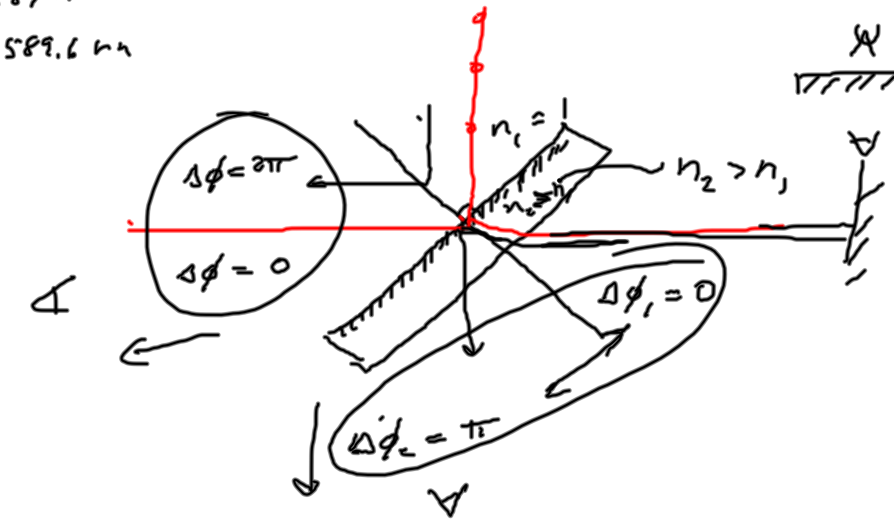
FTS

$\lambda_1 = 589$ nm

$\lambda_2 = 589.6$ nm

OUTPUT

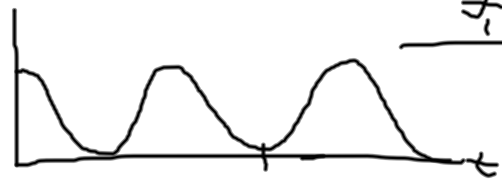




$$r = \frac{n_1 - n_2}{n_1 + n_2} < 0$$

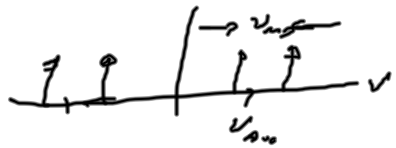
IFTS

$f(x, y, \lambda)$



$$\nu_{mod} = \frac{\nu_1 - \nu_2}{2}$$

$$\nu_{avg} = \frac{\nu_1 + \nu_2}{2}$$



2/1 - ④

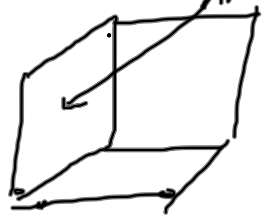
SPATIAL (TRANSVERSE) ⟹ DIVISION-OF-~~E~~ WAVEFRONT INTERFEROMETER
DoW

COHERENCE "WIDTH" → "AREA"

TEMPORAL (LONGITUDINAL) ⟹ DIVISION-OF-AMPLITUDE I
DoA

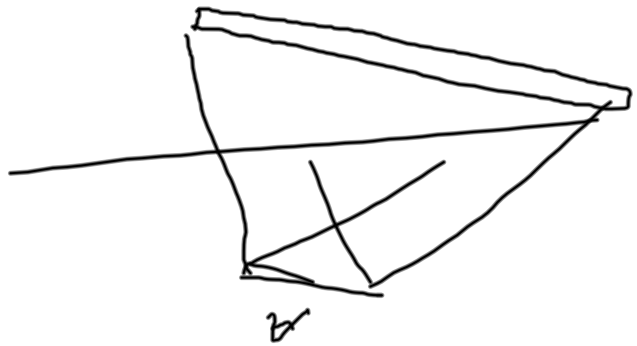
COHERENCE TIME ⟹ COHERENCE LENGTH → DISTANCE (TIME DELAY) AND STILL OBSERVE INTERFERENCE

AREA ⟹ SOURCE SIZE



LENGTH → $\Delta\nu$, $\Delta\lambda$

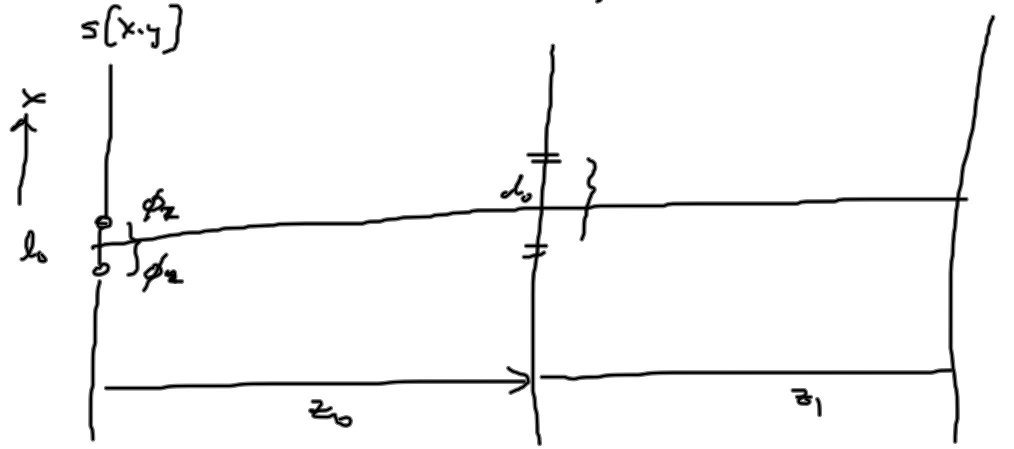
HOLOGRAPHY



2/1 (5)

SPATIAL COHERENCE

$g(x,y) \longrightarrow I[x,y] = \langle |g[x,y]|^2 \rangle$



$$s[x,y] = \delta\left[x + \frac{l_0}{2}\right] e^{i\phi_1(t)} + \delta\left[x - \frac{l_0}{2}\right] e^{i\phi_2(t)}$$

$$S[\xi,\eta] = e^{+2\pi i \frac{l_0}{2}\xi} e^{i\phi_1} + e^{-2\pi i \frac{l_0}{2}\xi} e^{i\phi_2} \qquad \xi \to \frac{x}{\lambda_0 z_0}$$

ϕ_{AVG}

$$= e^{+2\pi i \frac{l_0}{2}\xi} e^{i\phi_1/2} e^{i\frac{\phi_1}{2}} e^{i\frac{\phi_2}{2}} e^{-i\frac{\phi_2}{2}} + e^{-2\pi i \frac{l_0}{2}\xi} e^{i\frac{\phi_1}{2}} e^{-i\frac{\phi_1}{2}} e^{i\frac{\phi_2}{2}} e^{i\frac{\phi_2}{2}}$$

$$= e^{i\frac{\phi_1+\phi_2}{2}} \left[e^{+i\pi l_0 \xi} e^{+i\frac{\phi_1-\phi_2}{2}} + e^{-i\pi l_0 \xi} e^{-i\frac{\phi_1-\phi_2}{2}} \right]$$

$\frac{\Delta\phi}{2} = \phi_{mod}$

2/1 — 6

$$S[\xi,\eta] = e^{i\phi_{AVG}}\left[e^{+i(\pi l_0 \xi + \phi_{MOD})} + e^{-i(\pi l_0 \xi + \phi_{MOD})}\right]$$

$$= e^{i\phi_{AVG}} \cdot 2\cos\left(\pi l_0 \xi + \phi_{MOD}\right)$$

AMPLITUDE @ APERTURES $f[x,y]$

$$S\left[\frac{x}{\lambda_0 z_0}\right] = e^{+i\phi_{AVG}} \cdot 2\cos\left[\pi l_0 \frac{x}{\lambda_0 z_0} + \phi_{MOD}\right]$$

$$= \boxed{e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \frac{x}{\underbrace{\left(\frac{2\lambda_0 z_0}{l_0}\right)}} + \phi_{MOD}\right]}$$

PERIOD OF AMPLITUDE @ $f(x,y)$

$$= e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \frac{\left(x + \frac{x\lambda_0 z_0}{l_0}\cdot\frac{\Delta\phi}{x}\right)}{\left(\frac{2\lambda_0 z_0}{l_0}\right)}\right] \longrightarrow \cos\left(\frac{2\pi(x-x_0)}{X_0}\right)$$

COSINE AMPLITUDE IS TRANSLATED BY $x_0 = \left[-\frac{\lambda_0 z_0}{l_0}\cdot\Delta\phi\right]$

2/1 – 7

MULTIPLY BY APERTURE FUNCTION $f(x) = \delta\left(x+\frac{d_0}{2}\right) + \delta\left(x-\frac{d_0}{2}\right)$

$$e^{i\phi_{AVG}} \cdot 2\cos\left[2\pi \frac{x}{\left(\frac{2\lambda_0 z_0}{l_0}\right)} + \phi_{MOD}\right] \cdot \left(\delta\left(x+\frac{d_0}{2}\right) + \delta\left(x-\frac{d_0}{2}\right)\right)$$

$$= 2e^{i\phi_{AVG}}\left[\delta\left(x+\frac{d_0}{2}\right)\cos\left[2\pi \frac{-\frac{d_0}{2}}{\frac{2\lambda_0 z_0}{l_0}} + \phi_{MOD}\right] + \delta\left(x-\frac{d_0}{2}\right)\cos\left[2\pi \frac{+\frac{d_0}{2}}{\frac{2\lambda_0 z_0}{l_0}} + \phi_{MOD}\right]\right]$$

$$= 2e^{i\phi_{AVG}}\left(\delta\left(x+\frac{d_0}{2}\right)\cos\left[-\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right] + \delta\left(x-\frac{d_0}{2}\right)\cos\left[+\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right]\right)$$

$$= 2e^{i\phi_{AVG}}\left(\underline{\delta\left(x+\frac{d_0}{2}\right)}\,\underline{\cos\left(\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} - \phi_{MOD}\right)} + \underline{\delta\left(x-\frac{d_0}{2}\right)}\,\underline{\cos\left(\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right)}\right)$$

PROPAGATE TO OBSERVATION PLANE

$$g[x,y] = 2\, e^{i\phi_{AVG}} \left[\cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} - \phi_{MOD}\right] e^{+2\pi i \frac{x}{\lambda_0 z_1}\frac{d_0}{2}} + \cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} + \phi_{MOD}\right] e^{-2\pi i \frac{x d_0}{2\lambda_0 z_1}} \right]$$

$$\text{IF } \phi_{MOD} = \frac{\Delta\phi}{2} = \frac{\phi_1 - \phi_2}{2} = 0$$

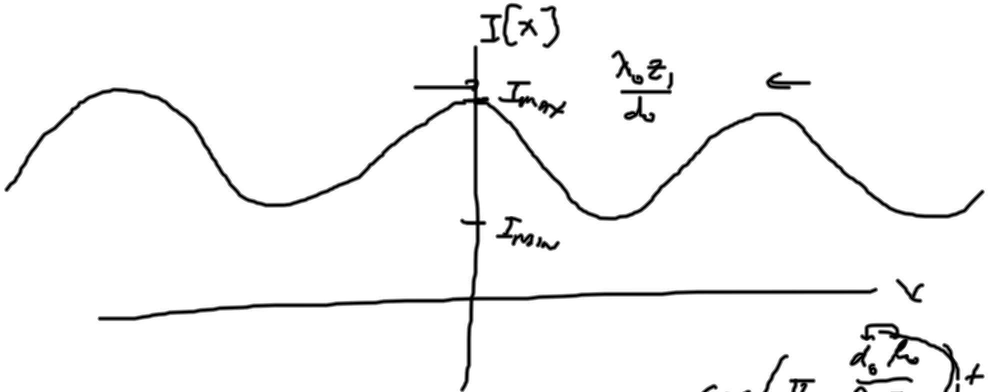
$$g[x,y;\Delta\phi=0] = 2\, \cancel{e^{i\phi_{AVG}}}\,\phi_0 \left\{ \cos\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right] \cdot \underbrace{2\cos\left[2\pi \frac{x}{\left(\frac{2\lambda_0 z_1}{d_0}\right)}\right]}\right.$$

$$\left|g[x;\Delta\phi=0]\right|^2 = 16\, \underbrace{\cos^2\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right]} \cdot \frac{1}{2}\Bigg(\underbrace{1 + \cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)}_{\text{BIASED COSINE}}\Bigg)$$

$$\cos^2\left[\frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0}\right] = 0 \quad \text{IF} \quad \frac{\pi}{2}\frac{l_0 d_0}{\lambda_0 z_0} = \pm\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{l_0 d_0}{\lambda_0 z_0} = 1 \qquad \text{IF} \quad \frac{l_0 d_0}{\lambda_0 z_0} < 1$$

MODULATION OF IRRADIANCE PATTERN



$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \equiv m \equiv V = \frac{\cos\left(\pi \cdot \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos(\Delta\phi(t)) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos(\Delta\phi(t)) \rangle}$$

EXAMPLES $\quad \langle \Delta\phi \rangle = 0 \quad V = \cancel{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)} \; \frac{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)} = 1$

if $\frac{d_0 l_0}{\lambda_0 z} = 1$, $I = 0$

2/1 - 10

RANDOM PHASE DIFFERENCE

$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) + \langle \cos(\Delta\phi(t)) \rangle}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot \langle \cos(\Delta\phi(t)) \rangle}$$

$$-1 \le \cos(\Delta\phi) \le +1$$

$$\langle \cos(\Delta\phi) \rangle = 0$$

IF $\Delta\phi$ IS RANDOM, $\Rightarrow -1 \le \cos(\Delta\phi) \le 1$

$$V = \frac{\cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) + 0}{1 + \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right) \cdot 0} = \cos\left(\pi \frac{d_0 l_0}{\lambda_0 z_0}\right)$$

$l_0 = 0 \Rightarrow$ ONE SOURCE

$d_0 = 0 \Rightarrow$ NO INTERFERENCE

$$-1 \le V \le 1$$

$$\frac{d_0 l_0}{\lambda z} = 1$$

$$\frac{d_0 l_0}{\lambda_0 z_0} < 1$$

$$l_0 < \frac{\lambda_0 z_0}{d_0}$$

Coherence WIDTH