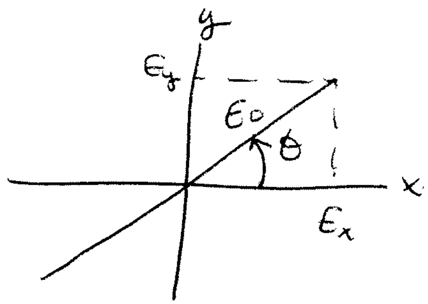
1/27/10 -① POLARIZED LIGHT

$$\underset{\sim}{E}[x,y,z,t] = \left(\hat{\underset{\sim}{x}} E_x + \hat{y} E_y\right) \cos\left(\frac{2\pi}{\lambda_0} z - 2\pi\nu_0 t + \varphi_0\right)$$

$e^{-i\delta}$



LINEAR POLARIZATION

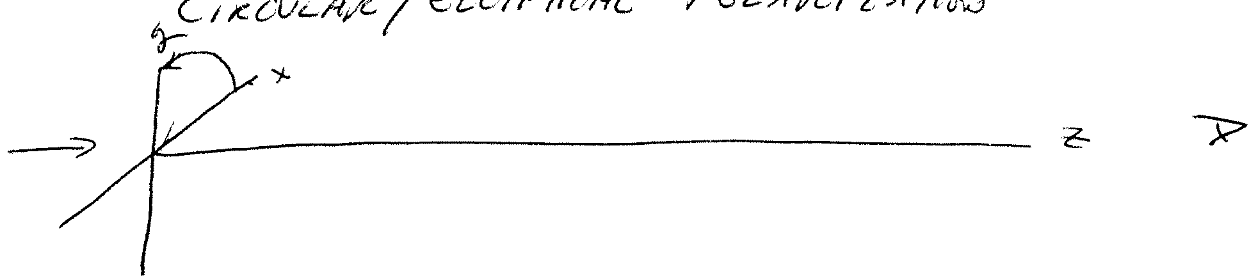
SELECTIVE EMISSION → RADIO

SELECTIVE ABSORPTION → POLAROID

REFLECTION → BREWSTER'S ANGLE TE

SCATTERING → BLUE SKY, NAVIGATION BY BIRDS

CIRCULAR/ELLIPTICAL POLARIZATION



CP ⇒ $\underline{E}$ ROTATES

CIRCULAR POLARIZATION

$$E[x,y,z,t] = \left(\underline{E_0}\,\hat{\underset{\sim}{x}} + \underline{E_0}\,\hat{\underset{\sim}{y}}\,e^{-i\delta}\right)\cos\left(2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)\right)$$

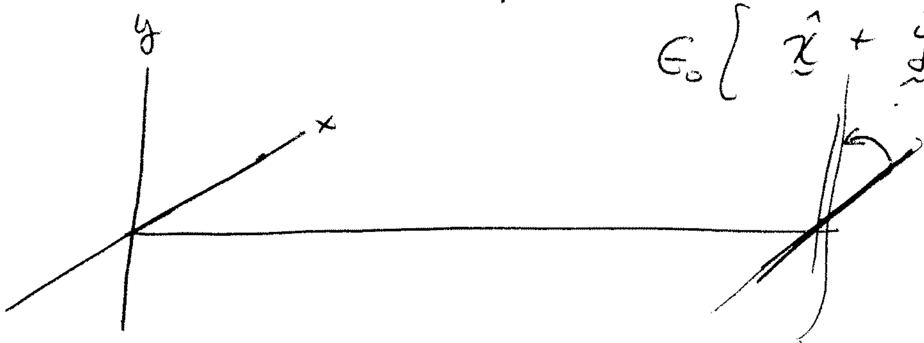
$\delta = \pm\frac{\pi}{2} \Longrightarrow E_y$ OSCILLATE IN QUADRATURE

E_x AND

$\Delta\phi = \frac{1}{4}$ CYCLE

$\delta = +\frac{\pi}{2}$

$$E_0\left[\hat{\underset{\sim}{x}} + \hat{\underset{\sim}{y}}\,e^{-i\frac{\pi}{2}}\right]\cos\left(2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)\right)$$



$\boxed{RHCP}$

CCW ROTATION IF $\delta = +\frac{\pi}{2}$

CW ROTATION IF $\delta = -\frac{\pi}{2}$

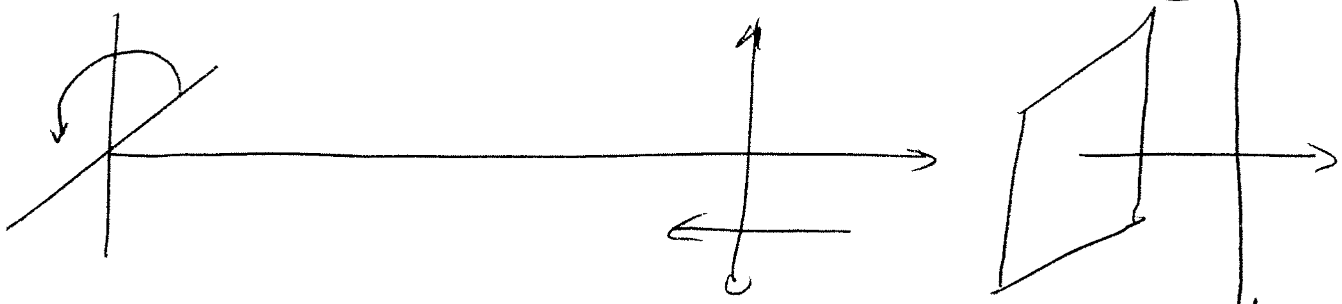
$\boxed{LHCP}$

~~OPTICS CONVENTION (SCREWY)~~

ANGULAR MOMENTUM CONVENTION

1/27/10 - 5

$$E_{IN} = E_0\left(\hat{x} + e^{+i\frac{\pi}{2}}\hat{y}\right)\cos(k_0 z - \omega_0 t)$$



$$E_0\left(\hat{x} + e^{+i\frac{\pi}{2}}\hat{y}\right)\cos(k_0 z + \omega_0 t)$$

ANGULAR

$$\cos(\theta) + i\sin(\theta)$$

RHCP

JONES VECTOR

$$\begin{bmatrix}1\\+i\end{bmatrix} \quad e^{-i\delta}$$

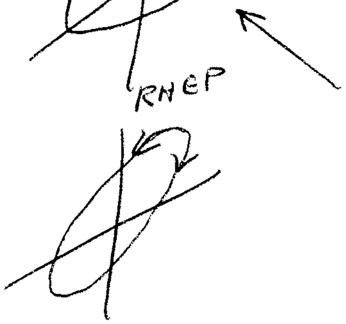
$$\cos(\theta - \underbrace{\tfrac{\pi}{2}}_{\delta})$$

$$-(\delta) = -\left(-\frac{\pi}{2}\right) = +\frac{\pi}{2}$$

ELLIPTICAL POLARIZATION

$\delta \leq 0 \Rightarrow$ RHEP $\ni -\delta > 0$

$\delta > 0 \Rightarrow$ LHEP



DESCRIPTION OF POLARIZATION STATES

- JONES VECTOR – 2-D VECTOR COMPLETELY POLARIZED LIGHT, LP, CP, EP
- COHERENCY MATRIX – 2×2 MATRIX – UNPOLARIZED + POLARIZED
- * MUELLER MATRIX – 4×4 MATRIX – UN "

1/27/10 - 5

<u>JONES VECTOR</u> $\quad \left[\hat{\underset{\sim}{x}} E_x + \hat{\underset{\sim}{y}} E_y e^{-i\delta} \right] \cos(k_0 z - \omega_0 t)$

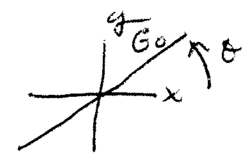
$$\underset{\sim}{\mathcal{E}} = \begin{bmatrix} E_x \\ E_y e^{-i\delta} \end{bmatrix} \qquad \mathcal{O}\{\underset{\sim}{\mathcal{E}}_{IN}\} = \underset{\sim}{\mathcal{E}}_{OUT}$$

$$M \underset{\sim}{\mathcal{E}}_{IN} = \underset{\sim}{\mathcal{E}}_{OUT}$$

M ↑ OPTIC = 2×2 MATRIX

LP ALONG $\hat{\underset{\sim}{x}} = \underset{\sim}{\mathcal{E}} = \begin{bmatrix} E_x \\ 0 \end{bmatrix} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\hat{\underset{\sim}{y}} \Rightarrow \underset{\sim}{\mathcal{E}} = E_0 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$\underset{\sim}{\mathcal{E}} = E_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

RHCP $\Rightarrow \underset{\sim}{\mathcal{E}} = E_0 \begin{bmatrix} 1 \\ +i \end{bmatrix}$, LHCP $= E_0 \begin{bmatrix} 1 \\ -i \end{bmatrix}$

IRRADIANCE $= I_x + I_y = \langle \mathcal{E}_x \mathcal{E}_x^* \rangle + \langle \mathcal{E}_y \mathcal{E}_y^* \rangle$

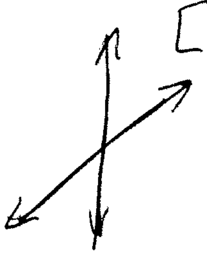
$$I = \underset{\sim}{E}^{*} \circ \underset{\sim}{E} \quad \left\langle \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y e^{-i\delta} \end{bmatrix}^{*} \circ \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y e^{-i\delta} \end{bmatrix} \right\rangle$$

$$= \langle \mathcal{E}_x^{*} \mathcal{E}_x \rangle + \cancel{\langle \mathcal{E}_y^{*} e^{+i\delta} \cdot \mathcal{E}_y e^{-i\delta} \rangle}$$

$$= \langle \mathcal{E}_x^{*} \mathcal{E}_x \rangle + \langle \mathcal{E}_y^{*} \mathcal{E}_y \rangle = |\mathcal{E}_x|^2 + |\mathcal{E}_y|^2$$

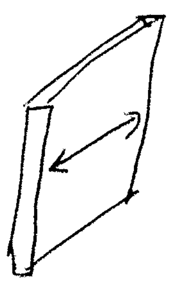
NOT A VALID/USEFUL NOTATION FOR UNPOLARIZED/PARTIAL POLARIZED LIGHT

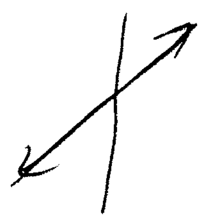
EXAMPLES OF OPTICS



$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$





$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\underset{\sim}{M} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right]$$

1/27/20 — 7

$$\underline{\underline{\begin{bmatrix} a & c \\ b & d \end{bmatrix}}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right]$$

$$\underbrace{\begin{bmatrix} a & c \\ b & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$\underset{\sim}{M}_x = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ FOR LP ALONG } \underset{\sim}{\hat{x}}$$

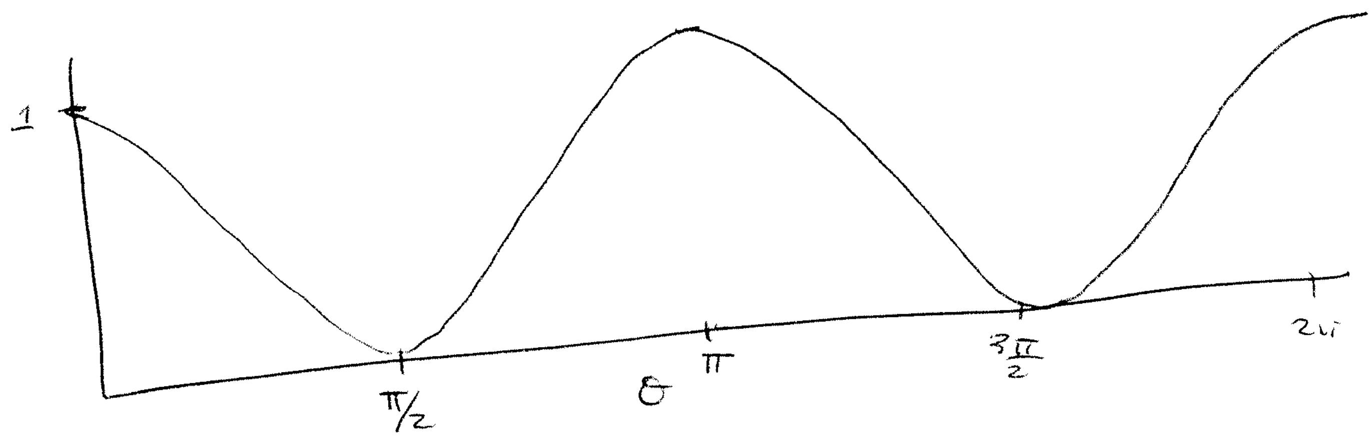
$$\underset{\sim}{M}_x \cdot \underset{\text{LP @ } \theta}{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$$

IRRADIANCE $\langle E_{OUT}^{*} \cdot E_{OUT} \rangle = |E_0|^2 \begin{bmatrix} \cos\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix} = \cos^2\theta \cdot E_0^2$

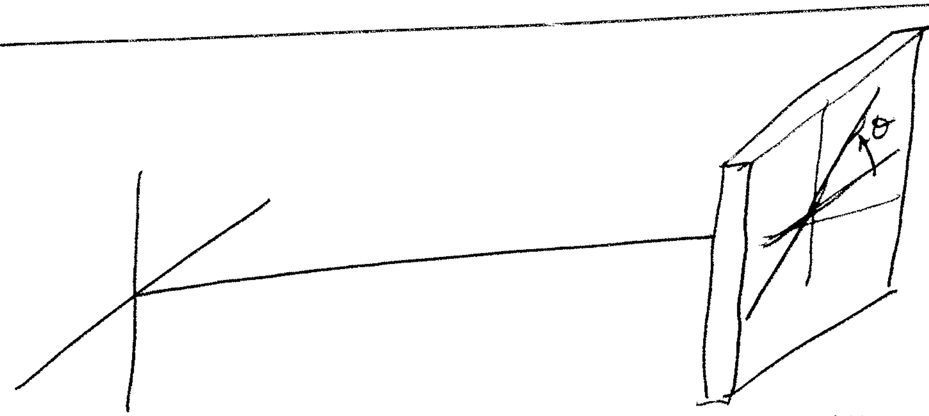
$= \cos^2\theta \cdot I_0$

1/27/10 - 8

MALUS' LAW $\quad I_{OUT} = I_0 \cos^2\theta$



$$M_y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$



M

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} \cos\theta & \cos(\theta+\frac{\pi}{2}) \\ \sin\theta & \sin(\theta+\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}$$

1/27/10 — 9

$$\underset{\tilde{M}}{\begin{bmatrix} a & c \\ b & d \end{bmatrix}} \underset{\tilde{E}_{IN}}{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} = \underset{\tilde{E}_{OUT}}{\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}}$$

$$\det \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \cos^2\theta - (-\sin^2\theta) = 1$$

$$(\tilde{E}_{IN})^{-1} = \frac{1}{\det} \begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

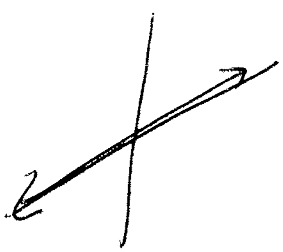
$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta \; \cancel{\cos\theta\sin\theta} & \sin^2\theta \end{bmatrix}$$

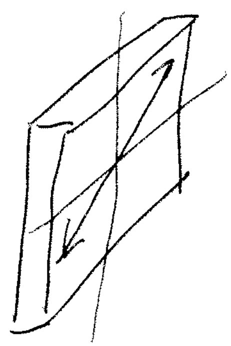
1/27/10 - 10

LP ORIENTED ALONG θ

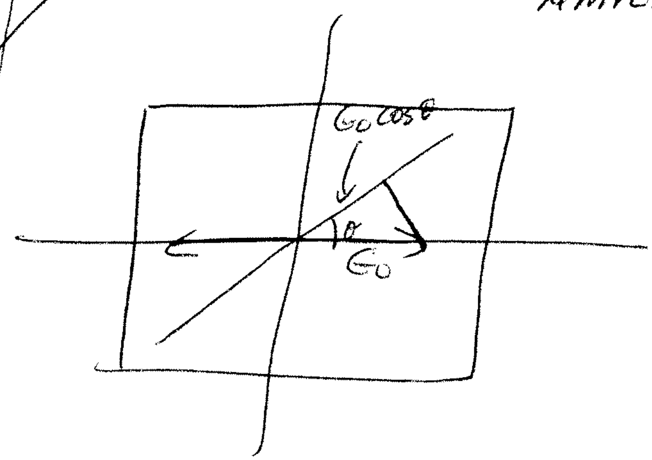
$$\underset{\sim}{M} = \begin{bmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{bmatrix}$$

$$\underset{\sim}{E}_{IN} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow E_{OUT} = E_0 \begin{bmatrix} \cos^2\theta \\ \cos\theta \sin\theta \end{bmatrix}$$





$$E_{OUT} = \underbrace{\left(E_0 \cos\theta\right)}_{\text{AMPLITUDE}} \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}_{\text{STATE OF POLARIZATION}}$$



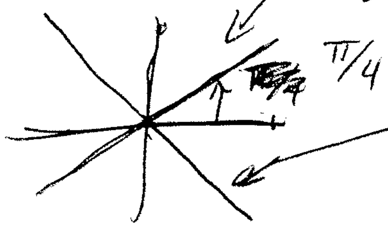
$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ +i \end{bmatrix} \quad \text{RHCP}$$

LP @ $\theta = \frac{\pi}{4}$

$$E_0 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad \theta = \frac{\pi}{4}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix} = \begin{bmatrix} 1 \\ +i \end{bmatrix}$$

$$a + c = 1$$
$$b + d = i$$

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \quad \text{RHCP}$$

QUARTER WAVE PLATE

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{LHCP}$$

COHERENCY MATRIX $\underset{\sim}{J}$

$$\underset{\sim}{J} = \left\langle \underset{\sim}{\mathcal{E}}\, \underset{\sim}{\mathcal{E}}^{\dagger} \right\rangle = \left\langle \underset{\sim}{\mathcal{E}} \left(\underset{\sim}{\mathcal{E}}^{T}\right)^{*} \right\rangle = \left\langle \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{bmatrix} \begin{bmatrix} \mathcal{E}_x^{*} & \mathcal{E}_y^{*} \end{bmatrix} \right\rangle$$

$\underbrace{\text{OUTER PRODUCT}}_{\text{MATRIX}}$ (INNER PRODUCT SCALAR)

$$= \left\langle \begin{bmatrix} \mathcal{E}_x \mathcal{E}_x^{*} & \mathcal{E}_x \mathcal{E}_y^{*} \\ \mathcal{E}_y \mathcal{E}_x^{*} & \mathcal{E}_y \mathcal{E}_y^{*} \end{bmatrix} \right\rangle = \begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y^{*} \rangle \\ \langle \mathcal{E}_x^{*} \mathcal{E}_y \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y e^{+i\delta} \rangle \\ \langle \mathcal{E}_x \mathcal{E}_y e^{-i\delta} \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

$\delta = 0 \Rightarrow LP$

$$\begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y \rangle \\ \langle \mathcal{E}_x \mathcal{E}_y \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

J FOR LP ALONG $x \Rightarrow \mathcal{E}_y = 0$

$$J_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \underset{\sim}{\mathcal{E}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{\mathcal{E}}\,\underset{\sim}{\mathcal{E}}^{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$J_y \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix};$$

LP @ θ $\quad \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}$

$$J_\theta = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} = M_\theta$$