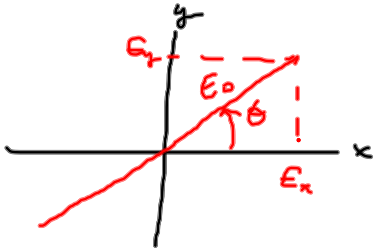
# 1/27/10 —① Polarized Light

$$\underset{\sim}{E}[x,y,z,t] = \left(\hat{\underset{\sim}{x}}\,E_x + \hat{y}\,E_y\right)\cos\left(\frac{2\pi}{\lambda}z - 2\pi\nu_0 t + \varphi_0\right)$$

$e^{-i\delta}$




Linear Polarization

Selective Emission → Radio

Selective Absorption → Polaroid

Reflection → Brewster's Angle TE

Scattering → Blue Sky, Navigation by Birds

## Circular/Elliptical Polarization

CP → $\underset{\sim}{E}$ rotates

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CIRCULAR POLARIZATION

$$E[x,y,z,t] = \left(\underline{E_0}\, \hat{\underline{x}} + \underline{E_0}\, \hat{\underline{y}}\, e^{-i\delta}\right) \cos\left(2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)\right)$$

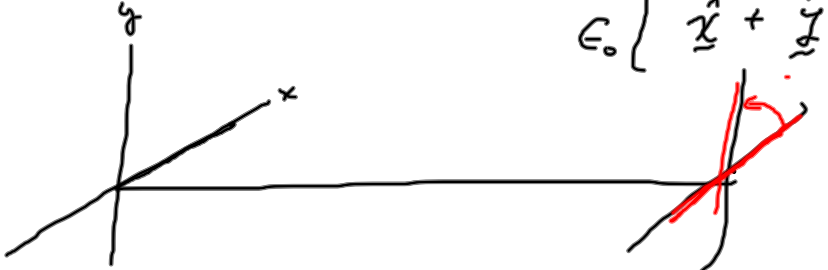
$\delta = \pm \frac{\pi}{2} \Rightarrow E_y$ OSCILLATES IN QUADRATURE w/ $E_x$ AND

$\Delta\phi = \frac{1}{4}$ CYCLE

$\delta = + \frac{\pi}{2}$

$$E_0\left[\hat{\underline{x}} + \hat{\underline{y}}\, e^{-i\frac{\pi}{2}}\right] \cos\left(2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)\right)$$

$\boxed{\text{RHCP}}$



CCW ROTATION IF $\delta = + \frac{\pi}{2}$

$\Rightarrow$

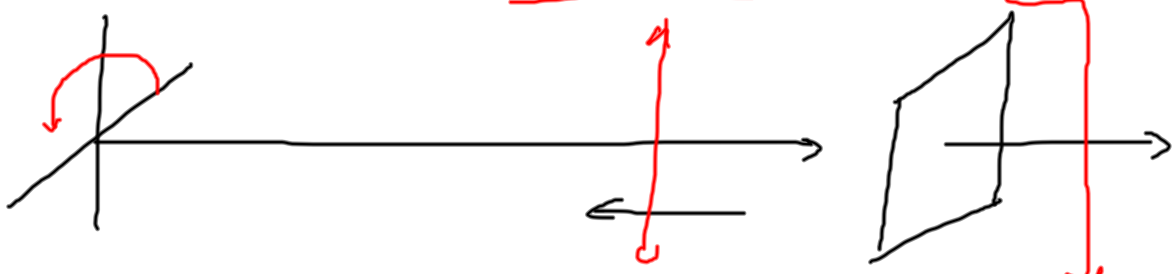
CW ROTATION IF $\delta = - \frac{\pi}{2}$

$\boxed{\text{LHCP}}$

~~OPTICS CONVENTION (SERWAY)~~

ANGULAR MOMENTUM CONVENTION

$E_{IN} = E_0\left(\hat{x} + e^{+i\frac{\pi}{2}}\hat{y}\right)\cos(k_0 z - \omega_0 t)$ 1/27/10 – 5



$E_0\left(\hat{x} + e^{+i\frac{\pi}{2}}\hat{y}\right)\cos(k_0 z + \omega_0 t)$

ANGULAR

$\cos(\theta) + i\sin(\theta)$

RHCP

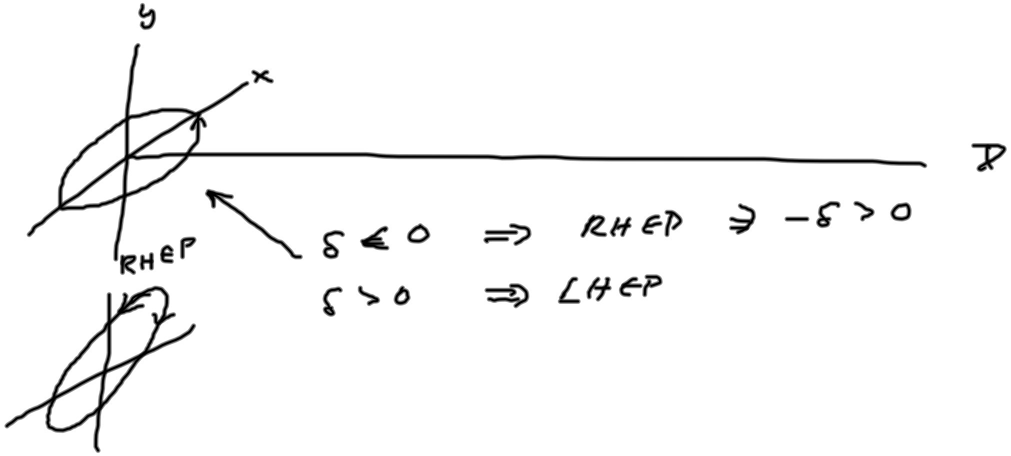
$\cos(\theta - \underbrace{\tfrac{\pi}{2}}_{\delta})$

$-(\delta) = -\left(-\frac{\pi}{2}\right) = +\frac{\pi}{2}$

JONES VECTOR

$\begin{bmatrix}1\\+i\end{bmatrix}$ $e^{-i\delta}$

ELLIPTICAL POLARIZATION 1/27/10 – (4)




$\delta \leq 0 \Rightarrow$ RHEP $\ni -\delta > 0$

$\delta > 0 \Rightarrow$ LHEP

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DESCRIPTION OF POLARIZATION STATES

- JONES VECTOR – 2-D VECTOR  COMPLETELY POLARIZED LIGHT, LP, CP, EP
- COHERENCY MATRIX – 2×2 MATRIX – UNPOLARIZED + POLARIZED

* MUELLER MATRIX – 4×4 MATRIX – UN "

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## Jones Vector

$$\left[\underset{\sim}{\hat{x}}\, E_x + \underset{\sim}{\hat{y}}\, E_y\, e^{-i\delta}\right] \cos(k_0 z - \omega_0 t)$$

$$\underset{\sim}{\mathcal{E}} = \begin{bmatrix} E_x \\ E_y e^{-i\delta} \end{bmatrix}$$

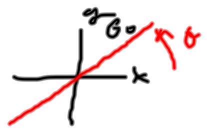
$$\mathcal{O}\{\underset{\sim}{\mathcal{E}}_{IN}\} = \underset{\sim}{\mathcal{E}}_{OUT}$$

$$M \underset{\sim}{\mathcal{E}}_{IN} = \underset{\sim}{\mathcal{E}}_{OUT}$$

OPTIC = 2×2 MATRIX

LP ALONG $\underset{\sim}{\hat{x}} = \underset{\sim}{\mathcal{E}} = \begin{bmatrix} E_x \\ 0 \end{bmatrix} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\underset{\sim}{\hat{y}} \Rightarrow \underset{\sim}{\mathcal{E}} = E_0 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$




$$\underset{\sim}{\mathcal{E}} = E_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

RHCP $\Rightarrow \underline{\mathcal{E}} = E_0 \begin{bmatrix} 1 \\ +i \end{bmatrix}$, LHCP $= E_0 \begin{bmatrix} 1 \\ -i \end{bmatrix}$

IRRADIANCE $= I_x + I_y = \langle \mathcal{E}_x \mathcal{E}_x^* \rangle + \langle \mathcal{E}_y \mathcal{E}_y^* \rangle$

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$$I = \tilde{E}^* \cdot \tilde{E} \qquad \left\langle \begin{bmatrix} E_x \\ E_y e^{-i\delta} \end{bmatrix}^* \cdot \begin{bmatrix} E_x \\ E_y e^{-i\delta} \end{bmatrix} \right\rangle$$

$$= \langle E_x^* E_x \rangle + \langle E_y^* \cancel{e^{+i\delta}} \cdot E_y \cancel{e^{-i\delta}} \rangle$$

$$= \langle E_x^* E_x \rangle + \langle E_y^* E_y \rangle = |E_x|^2 + |E_y|^2$$

NOT A VALID/USEFUL NOTATION FOR UNPOLARIZED/PARTIAL POLARIZED LIGHT

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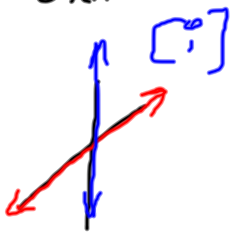
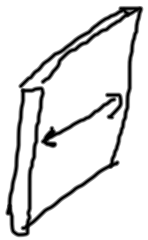
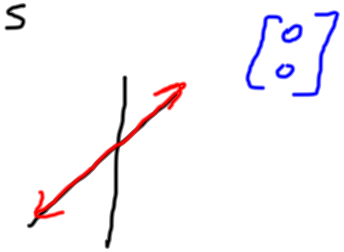
EXAMPLES OF OPTICS




$E_0 \begin{bmatrix}1\\0\end{bmatrix}$






$E_0 \begin{bmatrix}1\\0\end{bmatrix}$

$$\tilde{M}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \left[ \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \right] = \left[ \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0\\0\end{pmatrix} \right]$$

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b d

$$\underline{\underline{\begin{bmatrix} a & c \\ b & d \end{bmatrix}}} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right]$$

$$\underbrace{\begin{bmatrix} a & c \\ b & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$\underset{\sim}{M}_x = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$ FOR LP ALONG $\underset{\sim}{\hat{x}}$

$$\underset{\sim}{M}_x \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$$

LP @ $\theta$

IRRADIANCE $\langle E_{out}^{*} \cdot E_{out} \rangle = E_0^2 \begin{bmatrix} \cos\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix} = \cos^2\theta \cdot E_0^2 = \cos^2\theta \cdot I_0$

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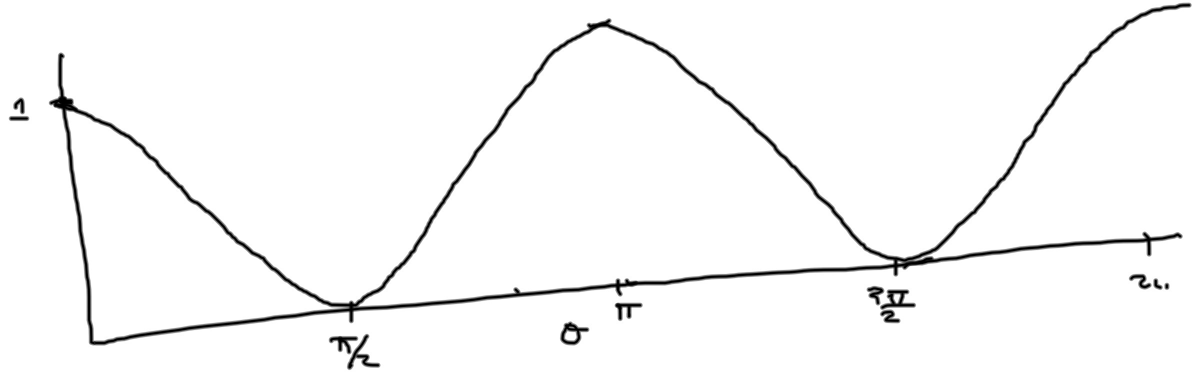
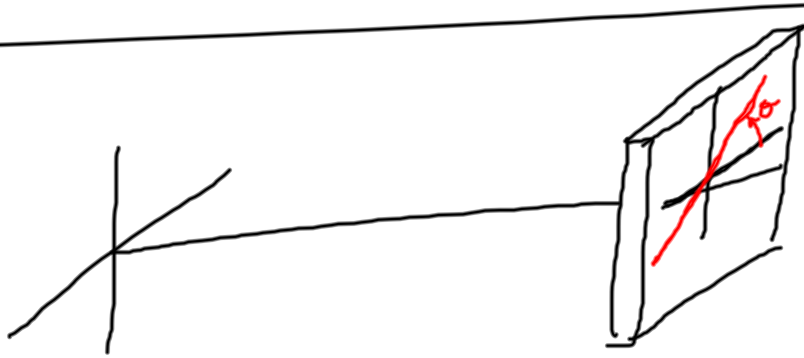
MALUS' LAW $\quad I_{OUT} = I_0 \cos^2\theta$




$$M_y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$




$$\overset{M}{\begin{bmatrix} a & c \\ b & d \end{bmatrix}} \begin{bmatrix} \cos\theta & \cos(\theta+\frac{\pi}{2}) \\ \sin\theta & \sin(\theta+\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}$$

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$$\underset{M}{\begin{bmatrix} a & c \\ b & d \end{bmatrix}} \underset{E_{IN}}{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} = \underset{E_{OUT}}{\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}}$$

$$\det \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \cos^2\theta - (-\sin^2\theta) = 1$$

$$(E_{IN})^{-1} = \frac{1}{\det} \begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

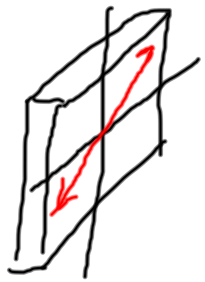
$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \cdot \cancel{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta \;\; \cancel{\cos\theta\sin\theta} & \sin^2\theta \end{bmatrix}$$

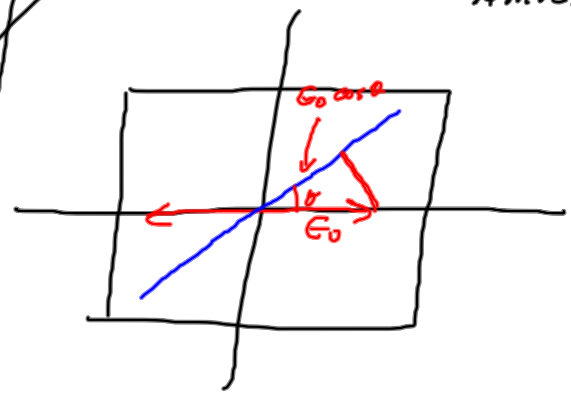
LP ORIENTED ALONG $\theta$ 1/27/10 - 10

$$\underset{\sim}{M} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$\underset{\sim}{E}_{IN} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow E_{OUT} = E_0 \begin{bmatrix} \cos^2\theta \\ \cos\theta\sin\theta \end{bmatrix}$$



$$E_{OUT} = \underbrace{(E_0 \cos\theta)}_{\text{AMPLITUDE}} \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}_{\text{STATE OF POLARIZATION}}$$



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$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ +i \end{bmatrix} \quad \text{RHCP}$$

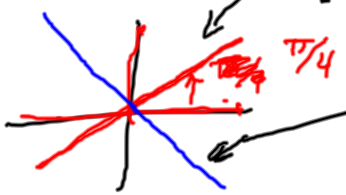
LP @ $\theta = \frac{\pi}{4}$

$$E_0 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad \theta = \frac{\pi}{4}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix} = \begin{bmatrix} 1 \\ +i \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$




$a + c = 1$

$b + d = i$

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \quad \text{RHCP}$$

QUARTER WAVE PLATE

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{LHCP}$$

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COHERENCY MATRIX $\underset{\sim}{J}$

$$\underset{\sim}{J} = \left\langle \underset{\sim}{\mathcal{E}}\, \underline{\mathcal{E}}^{\dagger} \right\rangle = \left\langle \underset{\sim}{\mathcal{E}} \left(\underset{\sim}{\mathcal{E}}^{T}\right)^{*} \right\rangle = \left\langle \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{bmatrix} \begin{bmatrix} \mathcal{E}_x^* & \mathcal{E}_y^* \end{bmatrix} \right\rangle$$

$\underbrace{\text{OUTER PRODUCT}}_{\text{MATRIX}}$ ( $\underbrace{\text{INNER PRODUCT}}_{\text{SCALAR}}$ )

$$= \left\langle \begin{bmatrix} \mathcal{E}_x \mathcal{E}_x^* & \mathcal{E}_x \mathcal{E}_y^* \\ \mathcal{E}_y \mathcal{E}_x^* & \mathcal{E}_y \mathcal{E}_y^* \end{bmatrix} \right\rangle = \begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y^* \rangle \\ \langle \mathcal{E}_x^* \mathcal{E}_y \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y e^{i\delta} \rangle \\ \langle \mathcal{E}_x \mathcal{E}_y e^{-i\delta} \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

$\delta = 0 \Rightarrow LP$

$$\begin{bmatrix} \langle |\mathcal{E}_x|^2 \rangle & \langle \mathcal{E}_x \mathcal{E}_y \rangle \\ \langle \mathcal{E}_x \mathcal{E}_y \rangle & \langle |\mathcal{E}_y|^2 \rangle \end{bmatrix}$$

$J$ FOR LP ALONG $x$ $\Rightarrow$ $\mathcal{E}_y = 0$

$$J_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \underset{\sim}{\varepsilon} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{\varepsilon}\,\underset{\sim}{\varepsilon}^{+} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_y \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} ; \qquad LP @ \theta \quad \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix}$$

$$J_\theta = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} = M_\theta$$