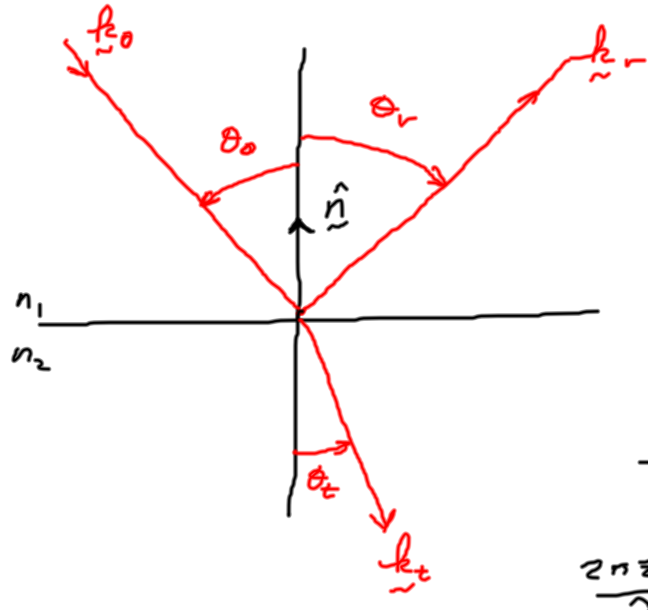
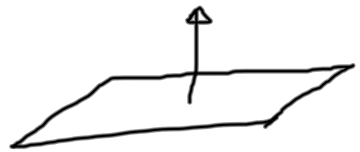
1/22/10 POLARIZATION AND FRESNEL EQUATIONS (1)

INTERFACE BETWEEN TRANSPARENT MEDIA WITH DIFFERENT n





k_0 – PROPAGATION VECTOR

LOCATION

$$k_0 \cdot r \pm \omega_0 t + \varphi_0$$

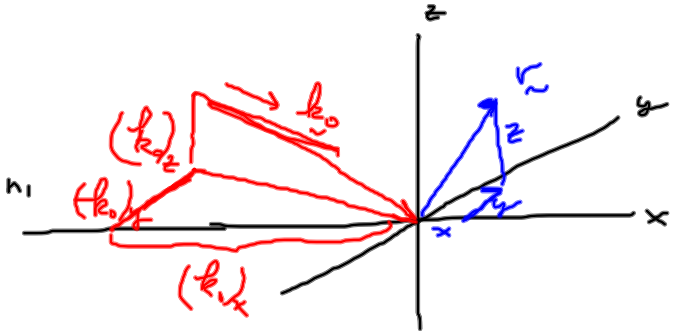
$$\frac{2\pi z_0}{\lambda_0} \; ; \; |k_0| = \frac{2\pi}{\lambda'} = \frac{2\pi}{\lambda_0/n_1} = \frac{2\pi n_1}{\lambda_0}$$

(1) MACROSCOPIC EFFECTS

SNELL'S LAW

(2) MICROSCOPIC EFFECTS $\Longrightarrow$ REFLECTANCE & TRANSMISSION COEFFICIENTS

FRESNEL'S EQUATIONS (ORIENTATION OF E FIELD)

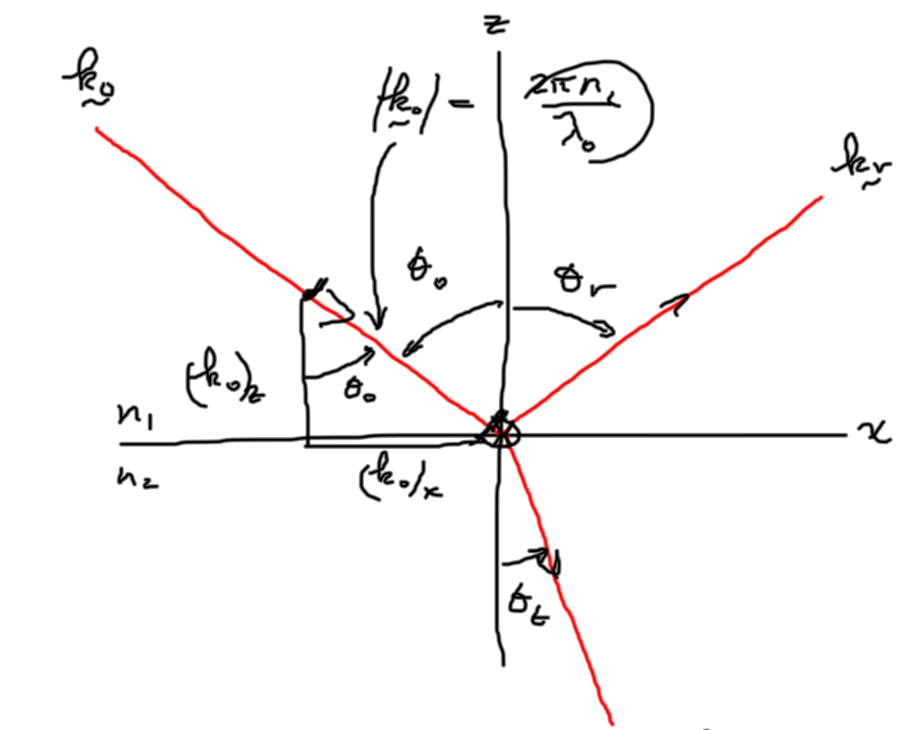


$$\left|\underset{\sim}{k_0}\right| = \frac{2\pi}{\lambda_0/n_1} = \frac{2\pi n_1}{\lambda_0}$$

PHASE EVALUATED AT $\underset{\sim}{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ IS $\underset{\sim}{k_0} \cdot \underset{\sim}{r} - \omega_0 t + \varphi_0$

$$\begin{bmatrix} (k_0)_x \\ (k_0)_y \\ (k_0)_z \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \omega_0 t + \varphi_0 = (k_0)_x x + (k_0)_y y + (k_0)_z z - \omega_0 t + \varphi_0$$

1/22 – 3



$$\underset{\sim}{k}_0 = \begin{bmatrix} (k_0)_x \\ 0 \\ (k_0)_z \end{bmatrix}$$

$$(\underset{\sim}{k}_0)_x = (\underset{\sim}{k}_r)_x = (\underset{\sim}{k}_t)_x$$

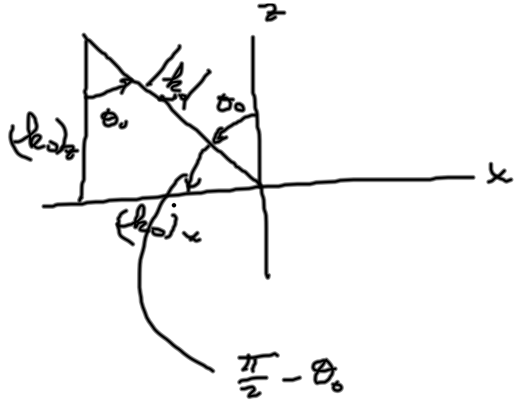
$$(\underset{\sim}{k}_0)_y = 0$$

MATCH PHASES AT $z=0$ $\Rightarrow$ $\left.\left(\underset{\sim}{k}_0 \cdot \underset{\sim}{r} - \omega_0 t\right)\right|_{z=0} = \left.\left(\underset{\sim}{k}_r \cdot \underset{\sim}{r} - \omega_0 t + \varphi_r\right)\right|_{z=0}$

$$= \left.\left(\underset{\sim}{k}_t \cdot \underset{\sim}{r} - \omega_0 t + \varphi_t\right)\right|_{z=0}$$

$$\left.\left(\underset{\sim}{k}_0 \cdot \underset{\sim}{r}\right)\right|_{z=0} = \left.\left(\underset{\sim}{k}_r \cdot \underset{\sim}{r} + \varphi_r\right)\right|_{z=0} = \left.\left(\underset{\sim}{k}_t \cdot \underset{\sim}{r} + \varphi_t\right)\right|_{z=0}$$

NUMBER OF WAVES PER UNIT LENGTH MUST BE EQUAL FOR ALL THREE



$$(\underset{\sim}{k}_0)_x = |\underset{\sim}{k}_0| \sin\theta_0 = \frac{2\pi n_1}{\lambda_0} \sin\theta_0$$

$$(\underset{\sim}{k}_0)_z = |\underset{\sim}{k}_0| \cos\theta_0 = \frac{2\pi n_1}{\lambda_0} \cos\theta_0$$

$$(\underset{\sim}{k}_r)_x = |\underset{\sim}{k}_r| \sin\theta_r = \frac{2\pi n_1}{\lambda_0} \sin\theta_r$$

$$(\underset{\sim}{k}_r)_z = \frac{2\pi n_1}{\lambda_0} \cos\theta_r$$

$$(\underset{\sim}{k}_t)_x = |\underset{\sim}{k}_t| \sin\theta_t = \frac{2\pi n_2}{\lambda_0} \sin\theta_t$$

$$(\underset{\sim}{k}_t)_z = |\underset{\sim}{k}_t| \cos\theta_t = \frac{2\pi n_2}{\lambda_0} \cos\theta_t$$

$$(\underset{\sim}{k}_0)_x = (\underset{\sim}{k}_r)_x \Longrightarrow \cancel{\frac{2\pi n_1}{\lambda_0}} \sin\theta_0 = \cancel{\frac{2\pi n_1}{\lambda_0}} \sin\theta_r \quad - \quad |\theta_0| = |+\theta_r|$$

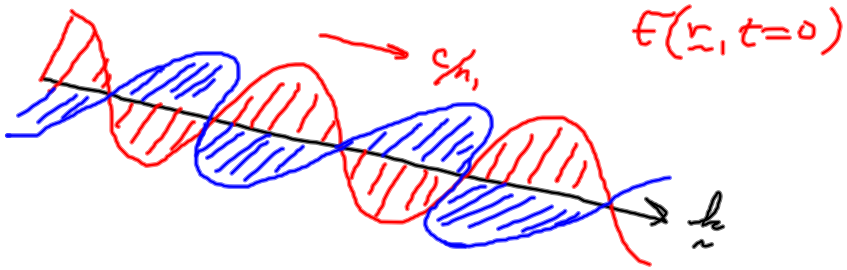
$\theta_0 = -\theta_r$ SNELL'S LAW FOR REFLECTION

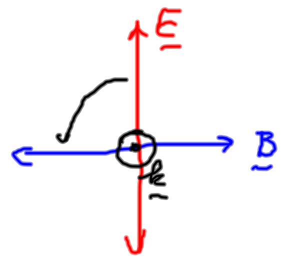
$$(\underset{\sim}{k}_0)_x = (\underset{\sim}{k}_t)_x \Rightarrow \frac{2\pi n_1}{\lambda_0}\sin\theta_0 = \frac{2\pi n_2}{\lambda_0}\sin\theta_t$$

$$\Rightarrow n_1 \sin\theta_0 = n_2 \sin\theta_t$$

SNELL'S LAW FOR REFLECTION

ORIENTATION OF INCIDENT FIELDS



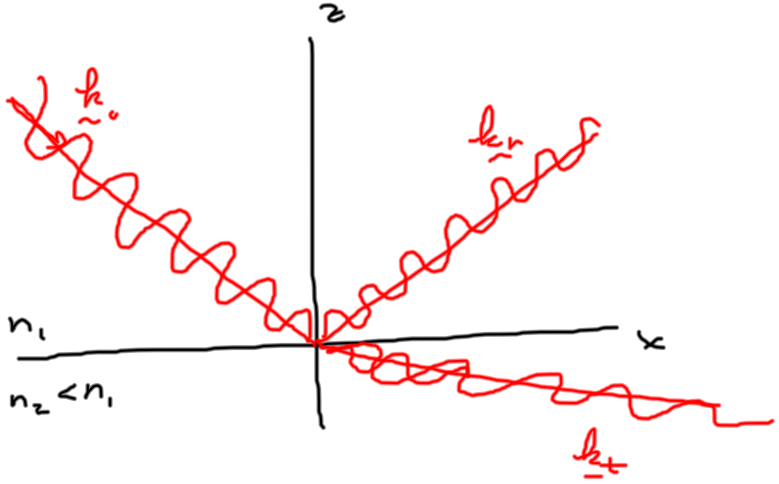


POLARIZATION DETERMINED BY $\underset{\sim}{E}$

$$\underset{\sim}{F} = q\underset{\sim}{E} + \frac{q}{c}(\underset{\sim}{v} \times \underset{\sim}{B})$$ LORENTZ FORCE

$$|q\underset{\sim}{E}| \gg \left|\frac{q}{c}(\underset{\sim}{v} \times \underset{\sim}{B})\right|$$

1/22/10



$\underset{\sim}{E}$ OSCILLATES

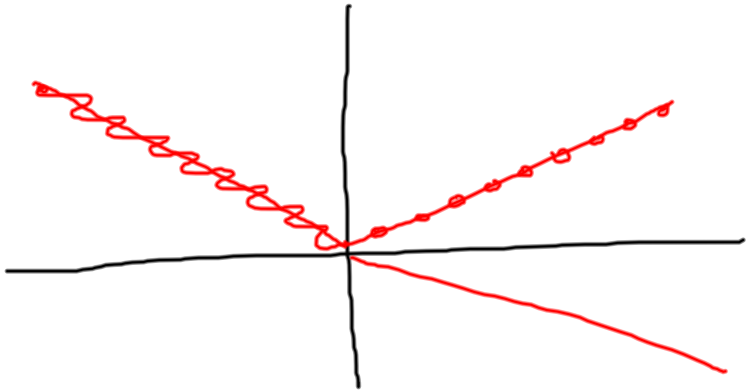
IN PLANE OF PAPER

IN PLANE OF INCIDENCE.

TRANSVERSE MAGNETIC TM (JACKSON)

$\parallel$ $E_\parallel$

P POLARIZATION E_p



$\underset{\sim}{E}$ OSCILLATES PERPENDICULAR TO PLANE OF INCIDENCE

TE TRANSVERSE ELECTRIC, E_{TE}

$\perp$ $E_\perp$

S POLARIZATION E_s

MATCH BOUNDARY CONDITIONS ON $\underset{\sim}{E}$ AND $\underset{\sim}{B}$ AT INTERFACE

MAXWELL'S EQUATIONS $\underbrace{\underset{\sim}{E},\ \underset{\sim}{B}}_{\text{VACUUM}},\ \underbrace{\underset{\sim}{D},\ \underset{\sim}{H}}_{\text{MATERIALS}}$

$\underset{\sim}{D} = \epsilon \underset{\sim}{E}$ PERMITTIVITY

$\underset{\sim}{B} = \mu \underset{\sim}{H}$ PERMEABILITY

CHARGE DENSITY

(1) $\nabla \cdot \underset{\sim}{D} = \nabla \cdot \epsilon \underset{\sim}{E} = \rho$

(2) $\nabla \cdot \underset{\sim}{B} = 0$

(3) $\nabla \times \underset{\sim}{E} \propto -\frac{\partial B}{\partial t}$ FARADAY'S

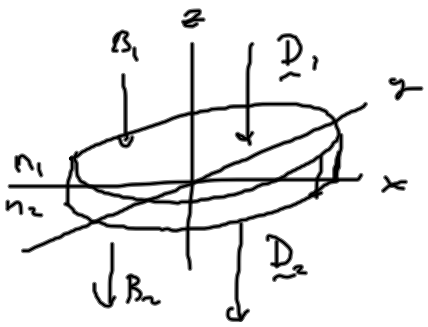
(4) $\nabla \times \underset{\sim}{B} \propto + \frac{\partial E}{\partial t}$ AMPERE'S LAW

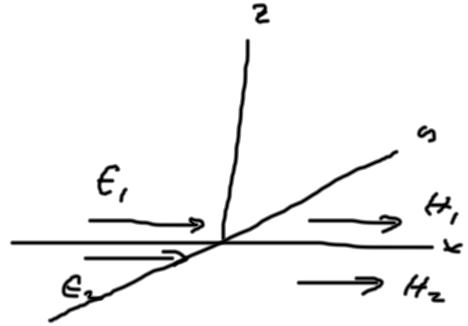
$$\nabla \cdot \underset{\sim}{D} \equiv \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

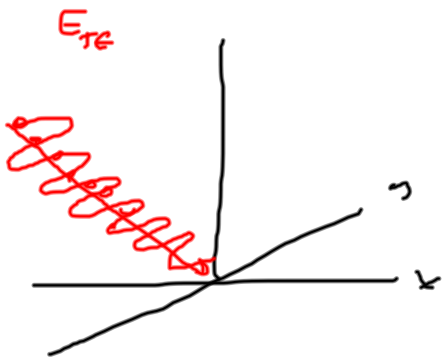
$$\nabla \times \underset{\sim}{E} = \begin{vmatrix} \hat{\underset{\sim}{x}} & \hat{\underset{\sim}{y}} & \hat{\underset{\sim}{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{\underset{\sim}{x}} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \cdots$$

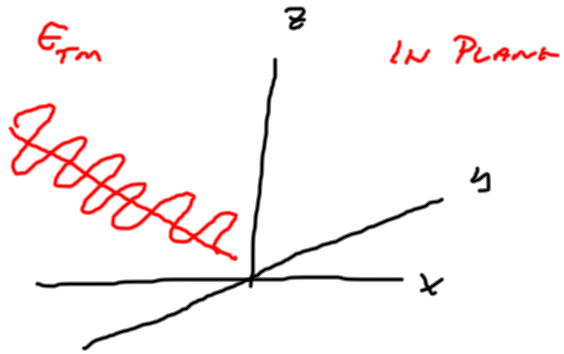
MATCH NORMAL COMPONENTS OF $\underset{\sim}{D} = \epsilon \underset{\sim}{E}$ AND $\underset{\sim}{B}$

TANGENTIAL COMPONENTS OF $\underset{\sim}{E}$ AND $\underset{\sim}{H} = \frac{\underset{\sim}{B}}{\mu}$

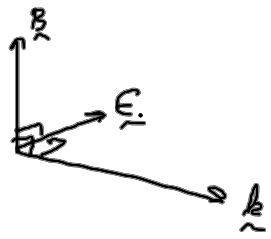








$\underset{\sim}{E} \perp \underset{\sim}{B} \perp \underset{\sim}{k} \perp \underset{\sim}{E}$



$\underset{\sim}{E} \times \underset{\sim}{B} = \underset{\sim}{k}$

$\frac{n}{c} \underset{\sim}{k} \times \underset{\sim}{E} = \underset{\sim}{B}$

$$r_{TE}(\theta = 0) = \frac{n_1 - n_2}{n_1 + n_2}$$

NORMAL

$$r_{TE}(\theta) = \frac{n_1 \cos\theta_0 - n_2 \cos\theta_t}{n_1 \cos\theta_0 + n_2 \cos\theta_t}$$

$$t_{TE} = 1 - r_{TE}$$

R_{TE} = REFLECTANCE FOR INCIDENT POWER $= \left(\frac{1-1.5}{1+1.5}\right)^2 = (-0.2)^2$

$= 0.04$