11 January 2000

(7)

HW 1, 2 Returned to folders, Solutions posted

HW 3 Due W 1/13

Midterm W 1/20

No class M 1/18

Problem session F 1/15 ?

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psf & MTF of optics

Physical properties that affect imaging systems

e.g. Refractive index
Polarization
Reflectance & Transmittance @ interface (Fresnel Equations)

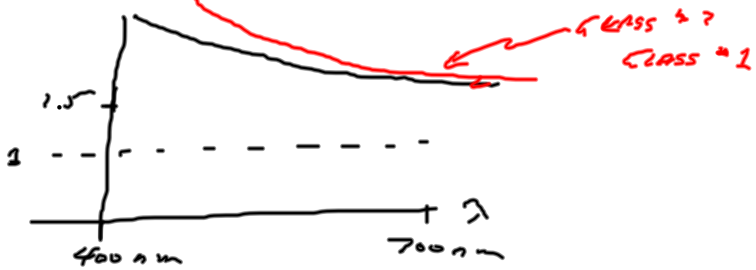
→ Interference
Geometrical optics

1/11/10 – 2

INDEX OF REFRACTION $\rightarrow$ DISPERSION, · VARIATION IN n WITH λ

$$n = \frac{c}{v} \geq 1$$

$$n(\lambda) = \frac{c}{v(\lambda)} \longrightarrow$$



$n(\lambda) \downarrow$ AS $\lambda \uparrow$ NORMAL DISPERSION

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow f \uparrow \text{ AS } \lambda \uparrow$$

$\Rightarrow z_2 \uparrow$ (IMAGE DISTANCE) AS $\lambda \uparrow$

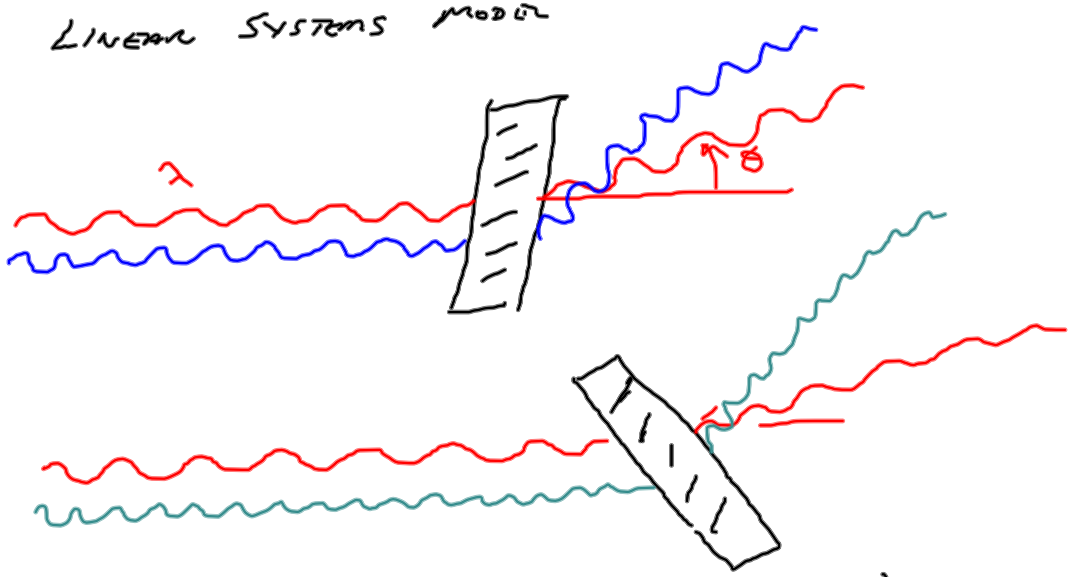
IMAGES FOR DIFFERENT λ AT DIFFERENT z_2 $\Rightarrow$ CHROMATIC ABERRATION

$\Rightarrow$ DEGRADATION OF RECORDED IMAGE OF COLORED OBJECTS

MATCH FOCAL LENGTH @ DIFFERENT λ $\Rightarrow$ VIA MULTIPLE ELEMENTS $\Rightarrow$ ACHROMATIC LENS (2λ), APOCHROMATIC LENS (3λ)

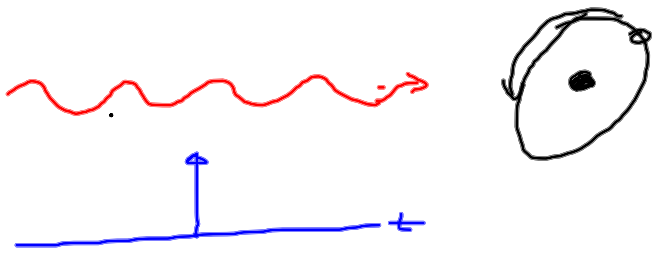
(1) PHYSICAL "PROPERTIES" $\Longrightarrow n(\lambda)$ 1/11/10 (3)

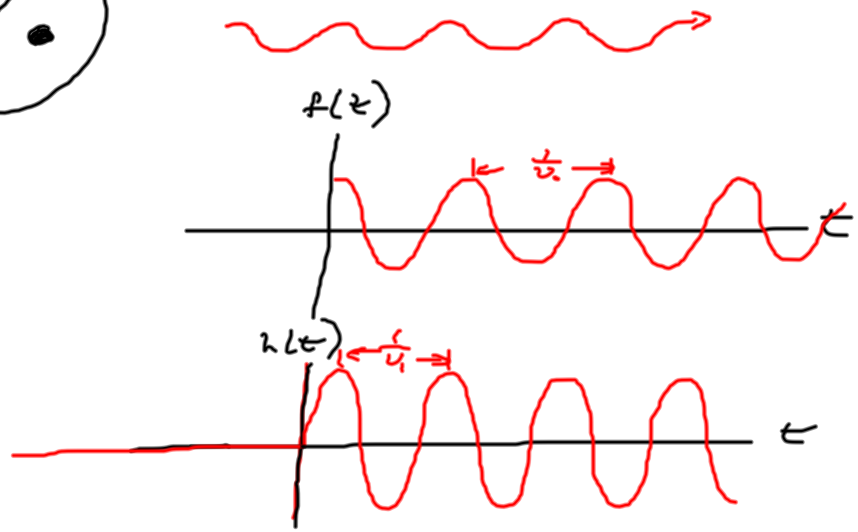
(2) LINEAR SYSTEMS MODEL



LIGHT INTERACTS W/ ATOMS (ELECTRONS) IN MATERIAL

ELECTRONS ABSORB & RE-EMIT LIGHT (SCATTERING)

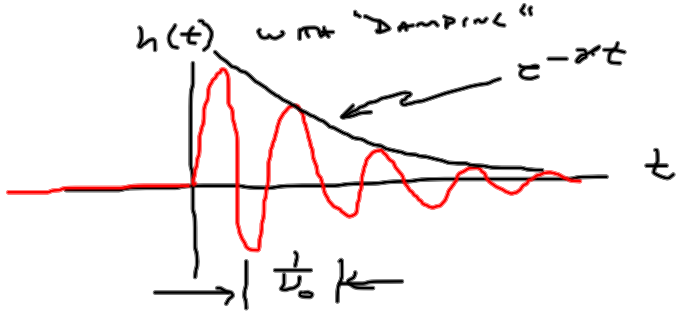




$$h(t) \propto \sin(2\pi \nu_0 t) \cdot \mathrm{STEP}(t) \Longrightarrow \text{CAUSAL SYSTEM}$$

$h(t) = 0$ FOR $t < 0$

$$H(\nu) \propto i\left(\delta(\nu+\nu_0) - \delta(\nu-\nu_0)\right) * \left(\frac{1}{2}\delta(\nu) + \frac{1}{2\pi i \nu}\right)$$

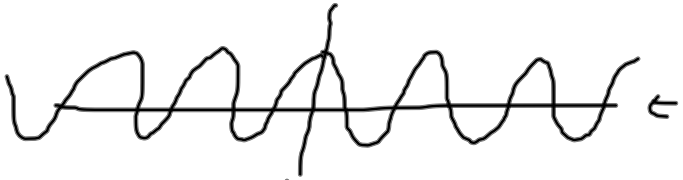


1/11/10 (5)

$\frac{1}{\gamma} = t_0 \Rightarrow$ Time to decay to $\frac{1}{e}$

What is form of $f(t)$? (Driving force – incoming light)

$f(t) = \cos(2\pi\nu t)$



Not physical

$f(t) = \text{step}(t)\cos(2\pi\nu t - \frac{\pi}{2}) = \text{step}(t)\sin(2\pi\nu t)$

$h(t) = \text{step}(t)\cos(2\pi\nu_0 t - \frac{\pi}{2}) = \text{step}(t)\sin(2\pi\nu_0 t)$

$g(t) = f(t) * h(t)$

1/11/10 — 6

$n = \frac{c}{v}$ $\qquad c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$ $\qquad v = \sqrt{\frac{1}{\mu \epsilon}}$

μ_0 = PERMEABILITY — MAGNETIC

ϵ_0 = PERMITTIVITY — ELECTRIC

$\mu = \mu_0$ IN MOST MATERIALS OF INTEREST

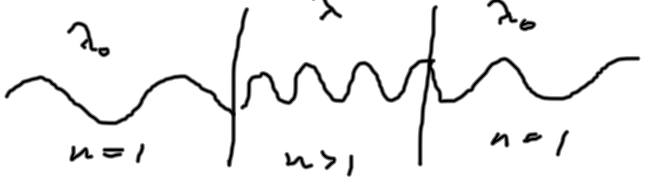
$n = \sqrt{\frac{\cancel{\mu}\epsilon}{\cancel{\mu_0}\epsilon_0}}$

$n^2 = \frac{\epsilon}{\epsilon_0}$

$E = h\nu = h\frac{c}{\lambda} \propto \frac{1}{\lambda}$ $\Rightarrow$ ν IN VACUUM = ν IN MEDIUM

$\Rightarrow$ WAVELENGTH IN MEDIUM $\neq$ WAVELENGTH IN VACUUM

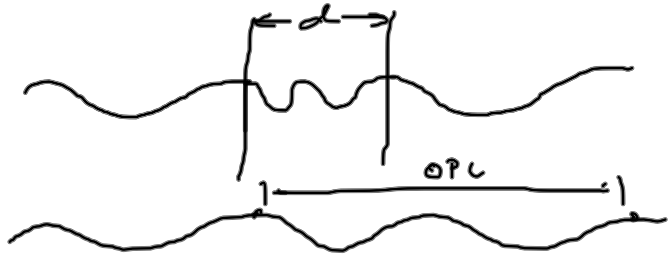
$\boxed{\lambda' = \frac{\lambda}{n}}$ $\qquad \lambda' \neq \lambda_0$



$n = \frac{c}{v} = \frac{\lambda_0 \cancel{\nu_0}}{\lambda' \cancel{\nu_0}} = \frac{\lambda_0}{\lambda'}$

OPTICAL PATH LENGTH OPL 1/11/10 – 7

LENGTH IN VACUUM THAT LIGHT TRAVERSES <u>IN SAME TIME</u> AS WOULD TRAVERSE THICKNESS d IN GLASS



$\boxed{OPL = nd} > d$

OPL > PHYSICAL PATH LENGTH

$n + iK = \tilde{n}$ COMPLEX REFRACTIVE INDEX

$\tilde{n}^2 = \frac{\epsilon}{\epsilon_0}$

k_0 – PROPAGATION CONSTANT

$|k_0| = \frac{2\pi}{\lambda_0} \longrightarrow \frac{2\pi}{\lambda'} = \frac{2\pi}{\lambda_0/n} = \frac{2\pi n}{\lambda_0}$

IN MEDIUM

1/21/10 – 8

COMPLEX n & PHASE

$$\frac{2\pi n}{\lambda_0} \longrightarrow \frac{2\pi \tilde{n}}{\lambda_0} = \frac{2\pi}{\lambda_0}(n + i\kappa)$$

$$e^{+i\frac{2\pi n}{\lambda_0}\cdot z} = e^{+i\frac{2\pi}{\lambda_0}z(n+i\kappa)} = e^{+i\frac{2\pi}{\lambda_0}nz} \cdot e^{+i\frac{2\pi}{\lambda_0}z\,i\kappa}$$

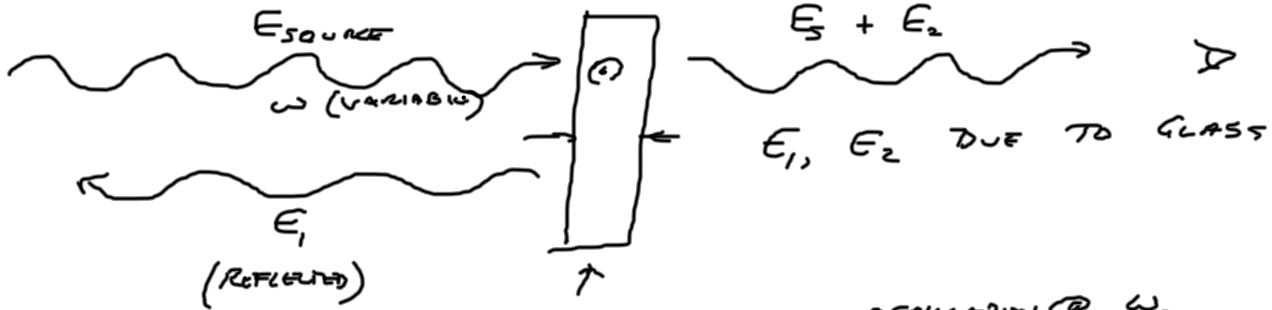
$$= \underbrace{e^{+i\frac{2\pi}{\lambda_0}nz}}_{\text{PROPAGATING (OSCILLATION)}} \cdot \underbrace{e^{-\frac{2\pi}{\lambda_0}z\kappa}}_{\text{ATTENUATION}}$$

IMAGINARY PART OF $\tilde{n}$ $\Longrightarrow$ ABSORPTION OF LIGHT BY MEDIUM

$$\begin{cases} n(\lambda)\downarrow \text{ AS } \lambda\uparrow & \text{NORMAL DISPERSION} \\ n(\lambda)\uparrow \text{ AS } \lambda\uparrow & \text{ANOMALOUS DISPERSION} \end{cases}$$

PHASE VELOCITY & GROUP VELOCITY (MODULATION VELOCITY)

PHYSICAL MODEL FOR n (FEYNMAN LECTURES ON PHYSICS) 1/11/10 – 9
VOL I §31, §32 II §32



E_1, E_2 DUE TO GLASS

MEDIUM, NATURAL OSCILLATION @ ω_0

(1) IN TERMS OF PHASE
(2) IN TERMS OF FORCES } n

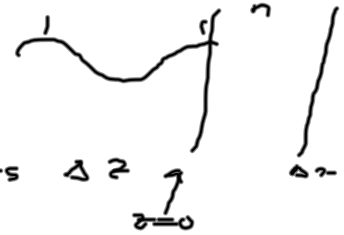
(A) UNDAMPED
(B) DAMPED $\longrightarrow$ RESONANCE "WIDTH" IN FREQ.

1/11/10 – 10

$$E_s = E_0\, e^{+i(k_0 z - \omega t)} = E_0\, e^{i\omega\left(\frac{k}{\omega} z - t\right)}$$

VARIABLE

$$= E_0\, e^{i\omega\left(\frac{z}{c} - t\right)}$$



ASSUME MEDIUM "STARTS" AT $z=0$, THICKNESS IS Δz

FRONT OF GLASS

@ $z=0$ $\Longrightarrow$ $E_s = E_0\, e^{-i\omega t}$

@ $z=\Delta z$ $\Longrightarrow$ $E_0\, e^{i\omega\left(\frac{\Delta z}{c} - t\right)} = E_0\, e^{i\frac{\omega \Delta z}{c}}\, e^{-i\omega t}$ PHASE @ $z=\Delta z$ IF NO GLASS

IF NO GLASS

$$\Phi_{NO}[\Delta z, t] = \frac{\omega \Delta z}{c} - \omega t$$

@ $z=\Delta z$ WITH GLASS

$$E_0\, e^{i\omega\left(\frac{\Delta z}{c} \cdot n - t\right)}$$

INDEX

$$\Phi_{GLASS}[\Delta z, t] = n\omega \frac{\Delta z}{c} - \omega t$$

1 11/10 - 21

PHASE CHANGE DUE TO GLASS

$$\Delta\Phi = \Phi[\Delta z, \text{VACUUM}] - \Phi[\Delta z, t, \text{GLASS}]$$

$$= \omega\left(\frac{\Delta z}{c} - t\right) - \omega\left(\frac{\Delta z}{c} n - t\right)$$

$$\Delta\Phi = \omega\frac{\Delta z}{c} - \omega n \frac{\Delta z}{c} = \boxed{-\omega(n-1)\frac{\Delta z}{c} = \Delta\Phi}$$

$$E[\Delta z, t] = \overbrace{\boxed{E_0\, e^{i\omega\left(\frac{\Delta z}{c} - t\right)}}}^{E_s} \cdot \underbrace{e^{-i\omega(n-1)\frac{\Delta z}{c}}}_{\Delta z \text{ IS SMALL}} \Longrightarrow E_s + E_2$$

FIELD @ BACK OF GLASS $= E_s + E_2$

$$e^A = \frac{1}{0!} + \frac{A^1}{1!} + \frac{A^2}{2!} + \cdots$$

$$e^{-i\omega(n-1)\frac{\Delta z}{c}} \cong 1 - i\omega(n-1)\frac{\Delta z}{c} \cancel{+ \cdots}$$

1/11/10 — 12

$$E_s + E_2 \cong E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}\left(1 - i\omega(n-1)\frac{\Delta z}{c}\right)$$

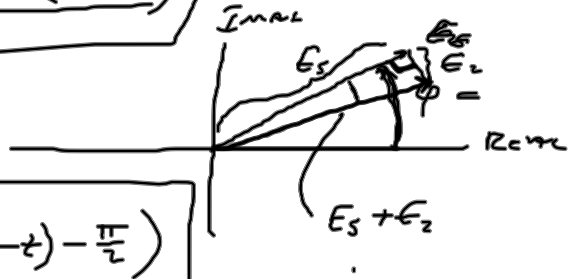
$$= \underbrace{E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}_{E_s} \underbrace{- i\omega(n-1)\frac{\Delta z}{c} E_0 e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}_{E_2}$$

$$\boxed{E_2 = \left(-i\omega(n-1)\frac{\Delta z}{c} E_0\right) e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right)\right)}}$$

PHASE → AMPLITUDE

$$\Rightarrow -i = e^{-i\frac{\pi}{2}}$$

$$\boxed{E_2 = \underbrace{\omega(n-1)\frac{\Delta z}{c} E_0} \; e^{i\left(\omega\left(\frac{\Delta z}{c} - t\right) - \frac{\pi}{2}\right)}}$$



FORCES

$$F = ma = m\frac{d^2x}{dt^2} = m\ddot{x}\;; \qquad \ddot{x} \equiv \frac{d^2x}{dt^2}$$

$$\dot{x} = \frac{dx}{dt}$$

FOR FIELD E_s DRIVING ELECTRON

$$E_s[z=0,t]\cdot e = F$$

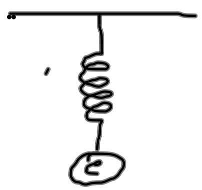
$$F = eE_0\, e^{+i\omega\left(\frac{0}{c} - t\right)} = eE_0\, e^{-i\omega t}$$ DRIVING FORCE

HOOKE'S LAW

$$F = k(x - x_0)\cdot(-1)$$

SPRING CONSTANT

RESTORING FORCE



$$\frac{k}{m_e} = \left(\frac{1}{\text{sec}}\right)^2$$

$$F = -k(x - x_0)$$

$\sqrt{\frac{k}{m}}$ IS A FREQUENCY $\Rightarrow \omega_0$

$$\boxed{\omega_0 = \sqrt{\frac{k}{m_e}}}$$