16 DECEMBER 2009

1

IMAGING IN COHERENT LIGHT $\Rightarrow$ $h[x,y;z_1,f,z_2] \propto P\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right]$

$$H[\xi,\eta;z_1,f,z_2] \propto p[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta]$$

COHERENT $\Rightarrow$ MONOCHROMATIC $\Rightarrow$ "PREDICTABLE" PHASES
"DETERMINISTIC"

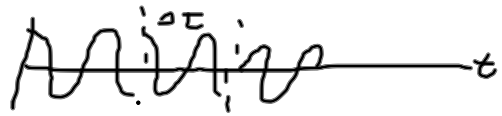
$\Rightarrow$ DESTRUCTIVE INTERFERENCE

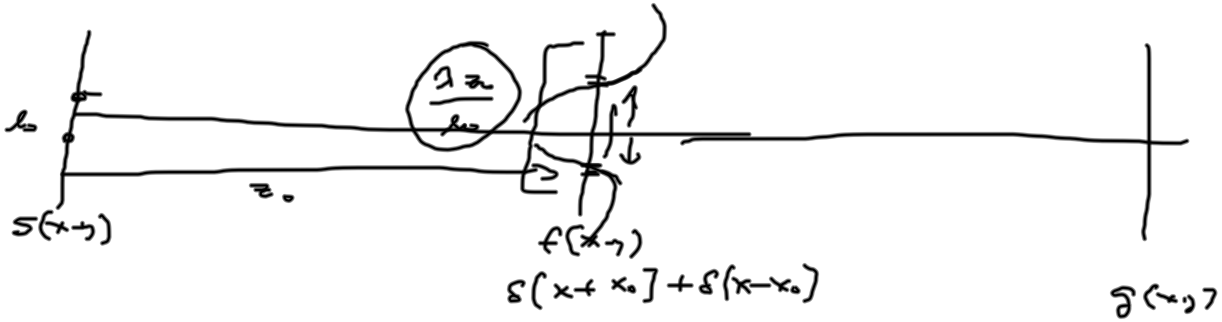
$\Rightarrow$ NEGATIVE AMPLITUDE

$\Rightarrow$ "FRINGE PATTERNS" $\Rightarrow$ AFFECTS ABILITY TO DISTINGUISH ADJACENT POINT OBJECTS

INCOHERENT $\Rightarrow$ UNPREDICTABLE PHASES OVER MEASUREMENT TIME $\Rightarrow$ POLYCHROMATIC LIGHT or MONOCHROMATIC LIGHT ~~WITH~~ RANDOM PHASE

$$h[x,y;z_1,f,z_2] \propto \left|P\left[\frac{x}{\lambda_0 z_2},\frac{y}{\lambda_0 z_2}\right]\right|^2$$

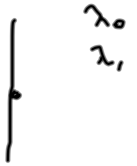




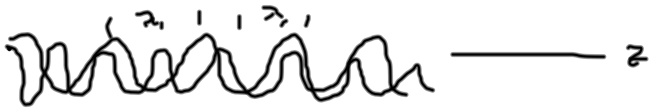
SYSTEM FOR CHARACTERIZING ~~OF A~~ SOURCE COHERENCE

§ 22

COHERENCE "WIDTH" = REGION AT OBJECT WHERE AMPLITUDE IS COHERENT $= \frac{\lambda z_0}{l_0}$



$\tau \neq \frac{1}{\Delta \nu}$



$\nu_0 = \frac{c}{\lambda_0}, \quad \nu_1 = \frac{c}{\lambda_1}$

$\tau = \frac{1}{\Delta \nu}$ = TIME INTERVAL OVER WHICH PHASES ARE CORRELATED = COHERENCE TIME

$\tau = \frac{1}{\Delta \nu}$ $\Rightarrow$ TIME OVER WHICH PHASE IS CORRELATED

$c \cdot \tau = \frac{c}{\Delta \nu}$ $\Rightarrow$ LONGITUDINAL DISTANCE OVER PHASE IS CORRELATED

$\Rightarrow$ COHERENCE LENGTH

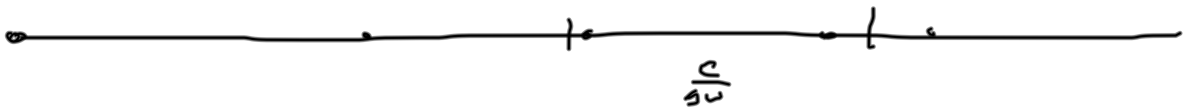


IMAGE IRRADIANCE $\Rightarrow$ TIME AVERAGE OF SQUARED MAGNITUDE

$$f[x,y;t] * h[x,y;z_1,f,z_2] = \iint dx'\,dy'\; f[x',y';t]\; h[x-x';y-y'] \;g$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \left| g(x,y,t;z_1,f,z_2) \right|^2 dt = I[x,y;\ldots]$$

T_0 = MEASUREMENT TIME

$$\cancel{I = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt}$$

$$f[x,y;t] = |f(x,y)|\; \cancel{e^{i\Phi[t]}}\; e^{i\Phi[x,y,t]}$$

$$f^*[x,y,t] = |f(x,y)|\; \cancel{e^{-i\Phi[t]}}\; e^{-i\Phi[x,y,t]}$$

$$f(x,y,t) * h(x,y) = |f(x,y)| e^{i\phi(x,y,t)} * h(x,y)$$

$$\left(f(x,y,t) * h(x,y)\right)^{*} = |f(x,y)| e^{-i\phi(x,y,t)} * h^{*}(x,y)$$

$$\iint_{-\infty}^{+\infty} dx'dy' \; |f(x',y')| \, e^{i\Phi(x',y',t)} \, h(x-x', y-y')$$

$$\iint_{-\infty}^{+\infty} dx''dy'' \; |f(x'',y'')| \, e^{-i\Phi(x'',y'',t)} \, h^{*}(x-x'', y-y'')$$

$$I(x,y;\ldots T_0) = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} \iint dx'dy' \iint dx''dy'' \; |f(x',y')| \, |f(x'',y'')| \, h(x-x', y-y') \, h^{*}(x-x'', y-y'') \, e^{+i\phi(x',y',t)} \, e^{-i\phi(x'',y'',}$$

$$I[x,y,\ldots T_0] = \iint dx'dy' \iint dx''dy''\ |f[x',y']|\,|f[x'',y'']|\ h[x-x',y-y']$$

$$\longrightarrow h^*[x-x'',y-y''] \cdot \underbrace{\frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} e^{i\Phi[x',y';t]}\, e^{-i\Phi[x'',y'';t]}\,dt}_{}$$

MEASUREMENT OF CORRELATION OF PHASES AT TWO LOCATIONS $[x',y']$, $[x'',y'']$

2 CASES

(1) PERFECT CORRELATION

$\Longrightarrow$ TEMPORAL INTEGRAL $= \underline{1}$

COHERENT LIGHT

(2) "PERFECTLY UNCORRELATION"

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{i\Delta\Phi(x',y',x'',y'',t)}\,dt \propto \delta[x'-x'',y'-y'']$$

INCOHERENT

12/16 — 7

INCOHERENT CASE $\quad \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} e^{i\left(\Phi(x',y';t) - \Phi(x'',y'';t)\right)} dt$

MUTUAL COHERENCE FUNCTION $\quad \Gamma[x',y';x'',y'';T_0]$

INCOHERENT MCF $\quad \Gamma(x',y';x'',y'';T_0) \propto \delta(x'-x'';y'-y'')$

$$I[x,y;T_0] = \iint dx'dy' \iint dx''dy'' \, |f(x',y')| \, |f(x'',y'')| \, h(x-x';y-y') \, h^*(x-x'';y-y'')$$

$\delta(x'-x'';y'-y'')$

$$= \iint dx'dy' \, \underbrace{|f(x',y')| \, |f(x',y')|}_{|f(x',y')|^2} \, \underbrace{h(x-x';y-y') \, h^*(x-x';y-y')}_{|h(x-x';y-y')|^2}$$

INCOHERENT IRRADIANCE

$$I[x,y;\tau_0] = \int dx'dy' \, |f[x',y']|^2 \cdot |h(x-x',y-y')|^2$$

$$= \underbrace{|f[x,y]|^2}_{\text{"INPUT"}} * \underbrace{|h[x,y]|^2}_{\text{"IMPULSE RESPONSE"}}$$

$$|h[x,y]|^2 = \mathcal{h}[x,y] =$$ INCOHERENT IMPULSE RESPONSE

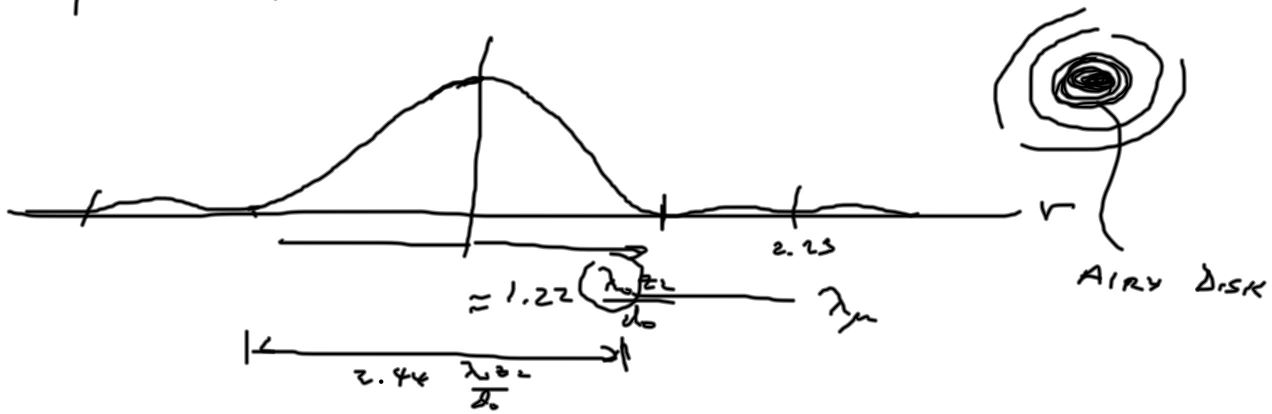
$$\mathcal{h}[x,y] \propto \left| P\left[\frac{x}{\lambda_0 z_0}, \frac{y}{\lambda_0 z_0}\right] \right|^2 ; \quad p(r) = \mathrm{CYL}\left(\frac{r}{d_0}\right)$$

$$P(\rho) = \frac{\pi d_0^2}{4} \, \mathrm{SOMB}(d_0 \rho)$$

$$P\left(\frac{r}{\lambda z}\right) = \frac{\pi d_0^2}{4} \, \mathrm{SOMB}\left(\frac{r}{\lambda z_0 / d_0}\right)$$

$$P\left(\frac{r}{\lambda_0 z_2}\right) = \frac{\pi d_0^2}{4} \operatorname{somb}\left(\frac{r}{\lambda_0 z_2 / d_0}\right)$$

$$\left|P\left(\frac{r}{\lambda_0 z_2}\right)\right|^2 = \left(\frac{\pi d_0^2}{4}\right)^2 \operatorname{somb}^2\left(\frac{r}{\left(\frac{\lambda_0 z_2}{d_0}\right)}\right)$$



INCOHERENT, BUT QUASI MONOCHROMATIC, LIGHT

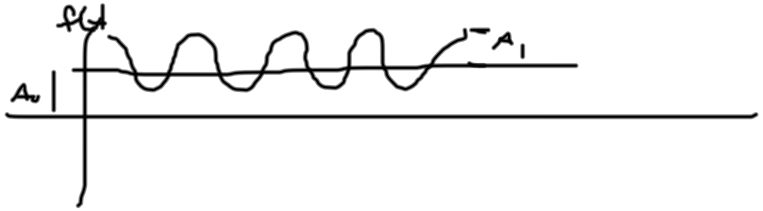
$$h[x,y;\cdots] \propto \left| P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right|^2$$

$$\mathcal{H}[\xi,\eta;\cdots] \propto (\lambda_0 z_2)^2 \cdot \underbrace{\left(P[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta] \star P[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta] \right)}_{\text{AUTOCORRELATION OF PUPIL}}$$

$$\mathcal{H}_{max}[\xi,\eta] = \mathcal{H}[0,0]$$

NORMALIZE $\mathcal{H}$ BY $\mathcal{H}_{max}$ $\Longrightarrow -1 \leq \left(\frac{\mathcal{H}[\xi,\eta]}{\mathcal{H}[0,0]}\right) \leq 1$

$$f[x] = A_0 + A_1 \cos(2\pi\xi_0 x) \geq 0 \qquad A_0 \geq A_1$$



$$m_f = \frac{A_1}{A_0}$$

$$g(x) = A_0 \cdot H[0] + A_1 \cdot H[\xi_0] \cdot \cos(2\pi\xi_0 x)$$

$$= H[0]\left(A_0 + A_1 \frac{H[\xi_0]}{H[0]} \cdot \cos(2\pi\xi_0 x)\right)$$

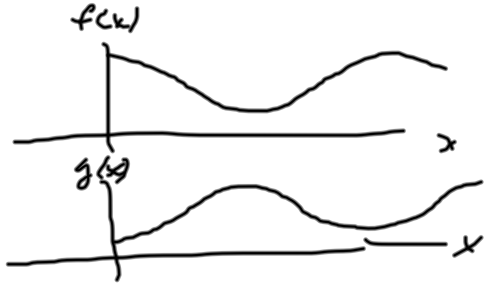
$$m_g = \frac{A_1 \cdot \frac{H[\xi_0]}{H(0)}}{A_0} = m_f \cdot \left(\frac{H[\xi_0]}{H[0]}\right)$$

$MT[\xi_0]$

$$\frac{H[\xi]}{H[0]} = MT[\xi] \equiv MTF \Rightarrow$$ HOW MODULATION OF INPUT IS AFFECTED BY SYSTEM

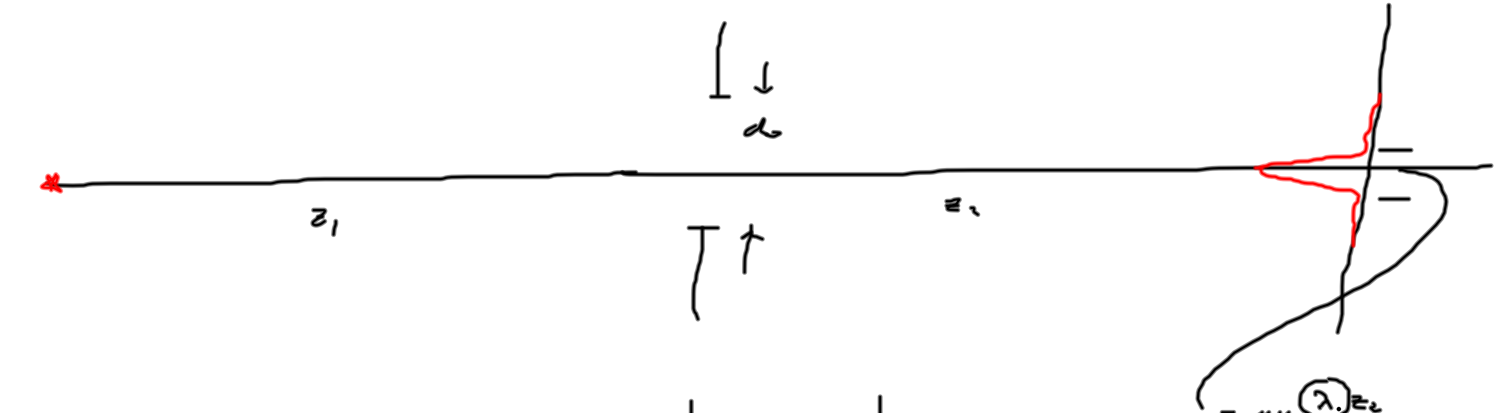
$$-1 \le MT(\xi) \le 1$$

CONTRAST REVERSAL

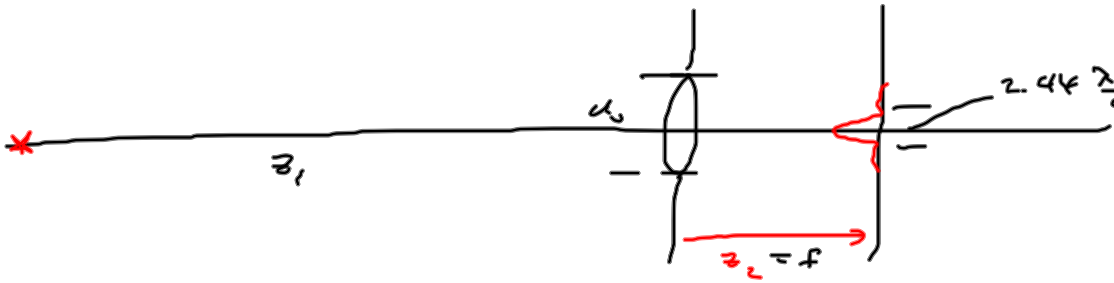


12/6-12

Resolution of a Imaging System
incoherent



$2.44\,\frac{\lambda z_2}{d_0}$



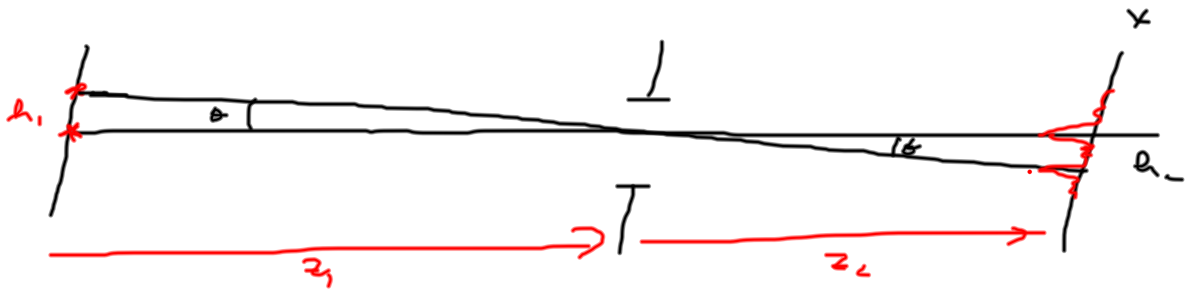
$2.44\,\frac{\lambda z_2}{d_0} \longrightarrow 2.44\,\frac{\lambda f}{d_0}$

$f/d_0 = f/\#$

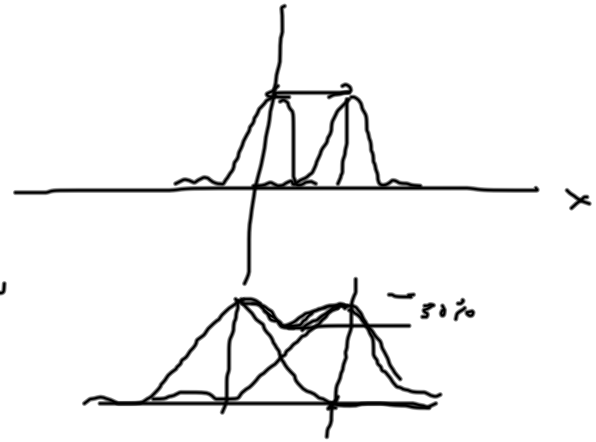
Diameter of Airy Disk $\sim 2.44\,\lambda \cdot f/\#$

$\approx \lambda\, f/\#$ Square aperture

12/12 - 13



$$\theta = \frac{h_1}{z_1} = \frac{-h_2}{z_2}$$



METRICS FOR RESOLUTION

(1) RAYLEIGH'S LIMIT