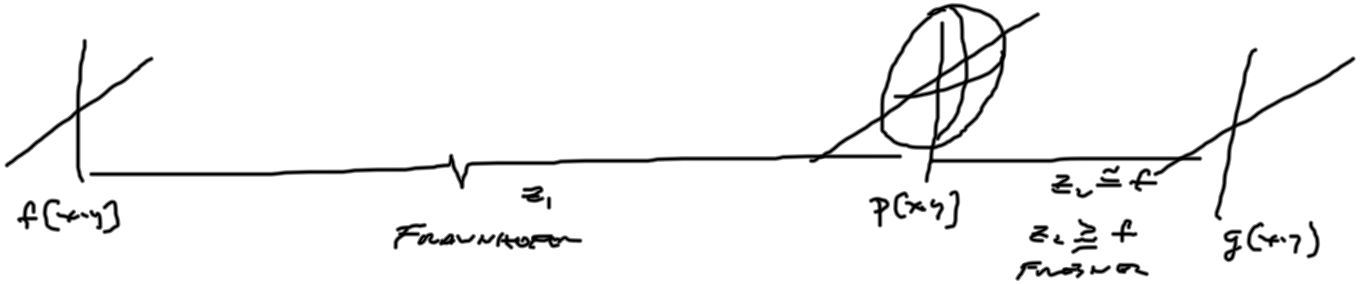
14 Dezember 2009

1

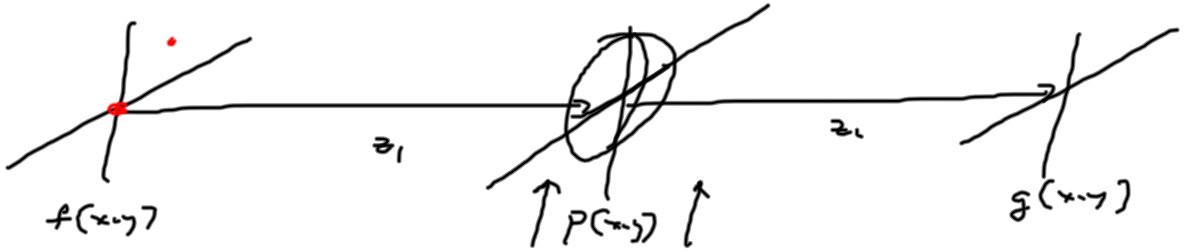
$z_1 >> f$



$$g[x,y] \propto f\left[\frac{x}{\left(-\frac{f}{z_1}\right)}, \frac{y}{\left(-\frac{f}{z_1}\right)}\right] * P\left[\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right]$$

$$h[x,y] \propto P\left[\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right]$$

FRONNEL + LENS + FRESNEL



$$f(x,y) = \delta(x,y)\,\delta(z)\,\underline{\delta(\lambda - \lambda_0)}$$

AT FRONT OF LENS $\quad f(x,y) * h(x,y;\, z=z_1,\, \lambda_0)$ (FRESNEL ~~CO PHASE~~ IMPULSE RESPO

$$h(x,y;\, z=z_1, \lambda_0) = \left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)} \right) e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$p(x,y;f) \equiv P(x,y)\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

APODIZED PUPIL ≠ BINARY

AT BACK OF LENS

$$(K_0)\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}\, p[x,y]\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}} = K_0\, p[x,y]\, e^{+i\pi \frac{x^2+y^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)}$$

PROPAGATE z_2 TO OBSERVATION PLANE

$$\left(\frac{-1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t_1\right)} \frac{1}{i\lambda_0 z_2} e^{+2\pi i\left(\frac{z_2}{\lambda_0} - \nu_0 t_2\right)}\right)\left[p[x,y]\, e^{+i\pi \frac{x^2+y^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} * e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}}\right]$$

EVALUATE CONVOLUTION IN SPACE DOMAIN

$$\text{1-D: } p[x]\, e^{+i\pi \frac{x^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} * e^{+i\pi \frac{x^2}{\lambda_0 z_2}} = \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)} \cdot e^{+i\pi \frac{(x-u)^2}{\lambda_0 z_2}}\, du$$

$$= \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0}\left(\frac{1}{z_1} - \frac{1}{f}\right)}\, e^{+i\pi \frac{x^2}{\lambda_0 z_2}}\, e^{+i\pi \frac{u^2}{\lambda_0 z_2}}\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \int_{-\infty}^{+\infty} p[u]\, e^{+i\pi \frac{u^2}{\lambda_0}\underbrace{\left(\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2}\right)}_{=0}}\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

IF $\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2} = 0$, THEN

$$e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \int_{-\infty}^{+\infty} p[u]\, e^{-2\pi i \frac{xu}{\lambda_0 z_2}}\, du$$

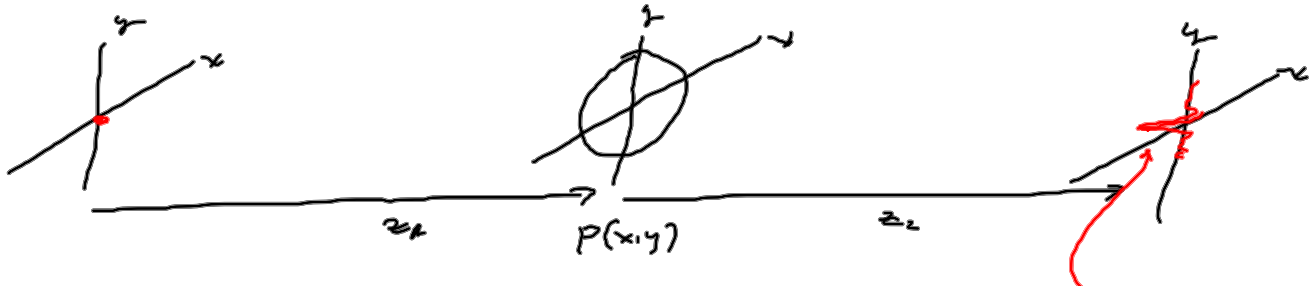
$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \mathcal{F}_1 \{ p[u] \} \Big|_{\xi = \frac{x}{\lambda_0 z_2}}$$

$$= e^{+i\pi \frac{x^2}{\lambda_0 z_2}} \cdot P\left[\frac{x}{\lambda_0 z_2}\right]$$

GENERALIZE TO 2-D CASE $e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}} P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$

IF $\frac{1}{z_1} - \frac{1}{f} + \frac{1}{z_2} = 0 \Longrightarrow \boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}}$

12/14 – (5)



$g(x,y) = P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$

$|g(x,y)|^2 = \left|P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right|^2$

$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$

$-\frac{z_2}{z_1} = M_T$

$z_1 = 2f = z_2$

$M_T = -1$

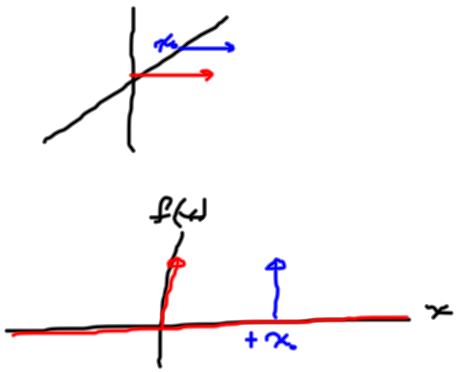
$e^{+i\pi \frac{x^2+y^2}{\lambda_0(2f)}}$ – PROPAGATION

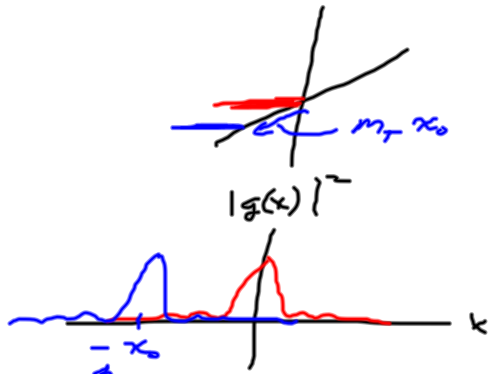
$e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}$ – LENS

$\alpha_1 = \sqrt{2\lambda_0 f}$

$\alpha_{LENS} = \sqrt{\lambda_0 f}$

12 14 ~ 6





$z_1 = z_2 \Rightarrow m_T = -1$

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$$\cancel{\left|g[x,y]\right|^2 = \left|f\left[\frac{x}{-m_T}, \frac{y}{-m_T}\right] * h[x,y;\lambda_0,z_1,z_2]\right|^2}$$

$$h[x,y;\lambda;z_1,z_2] = e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_2}}\, P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

LSI IF CORRECTLY INTERPRETED

$$\left| f\left[\frac{x}{+m_T}, \frac{y}{+m_T}\right] * h[x,y;\ldots]\right|^2 = g[x,y]$$

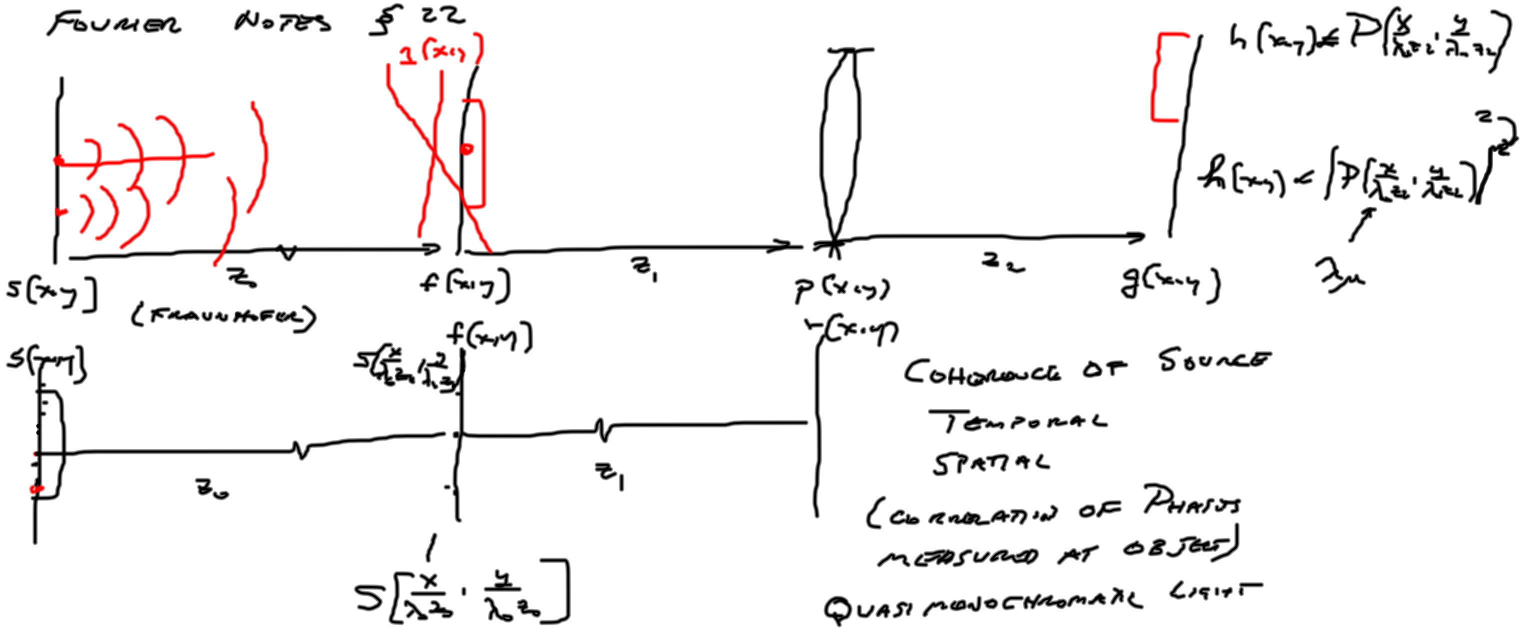
$$m_T = -\frac{z_2}{z_1}$$

WHAT IF MULTIPLE WAVELENGTHS?

WHAT IF PHASE IS NOT DETERMINISTIC?

OPTICS NOTES § 3.8

FOURIER NOTES § 22



COHERENCE OF SOURCE

TEMPORAL

SPATIAL

(CORRELATION OF PHASES MEASURED AT OBJECT)

QUASI MONOCHROMATIC LIGHT

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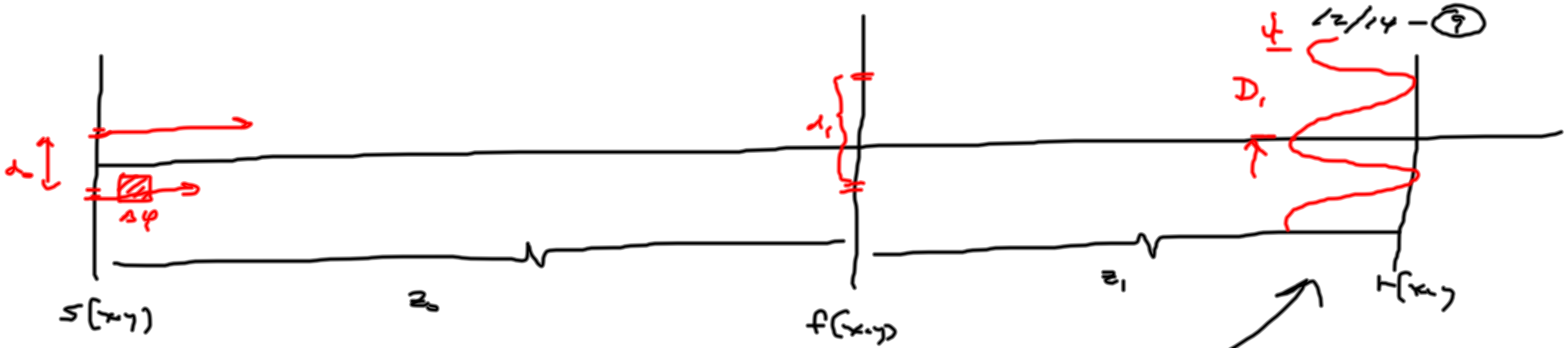
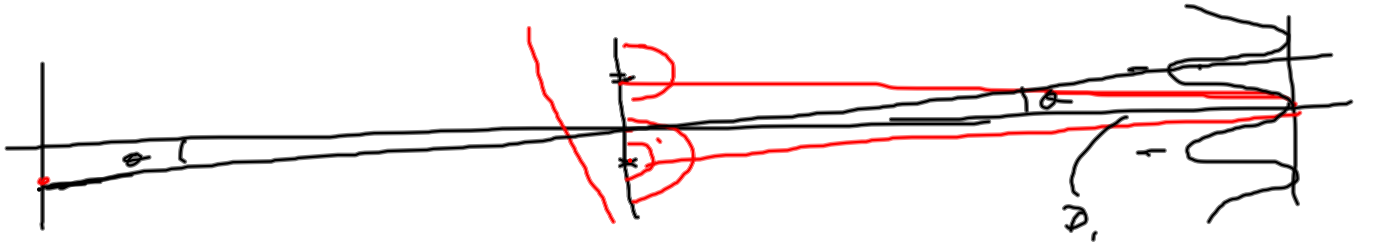
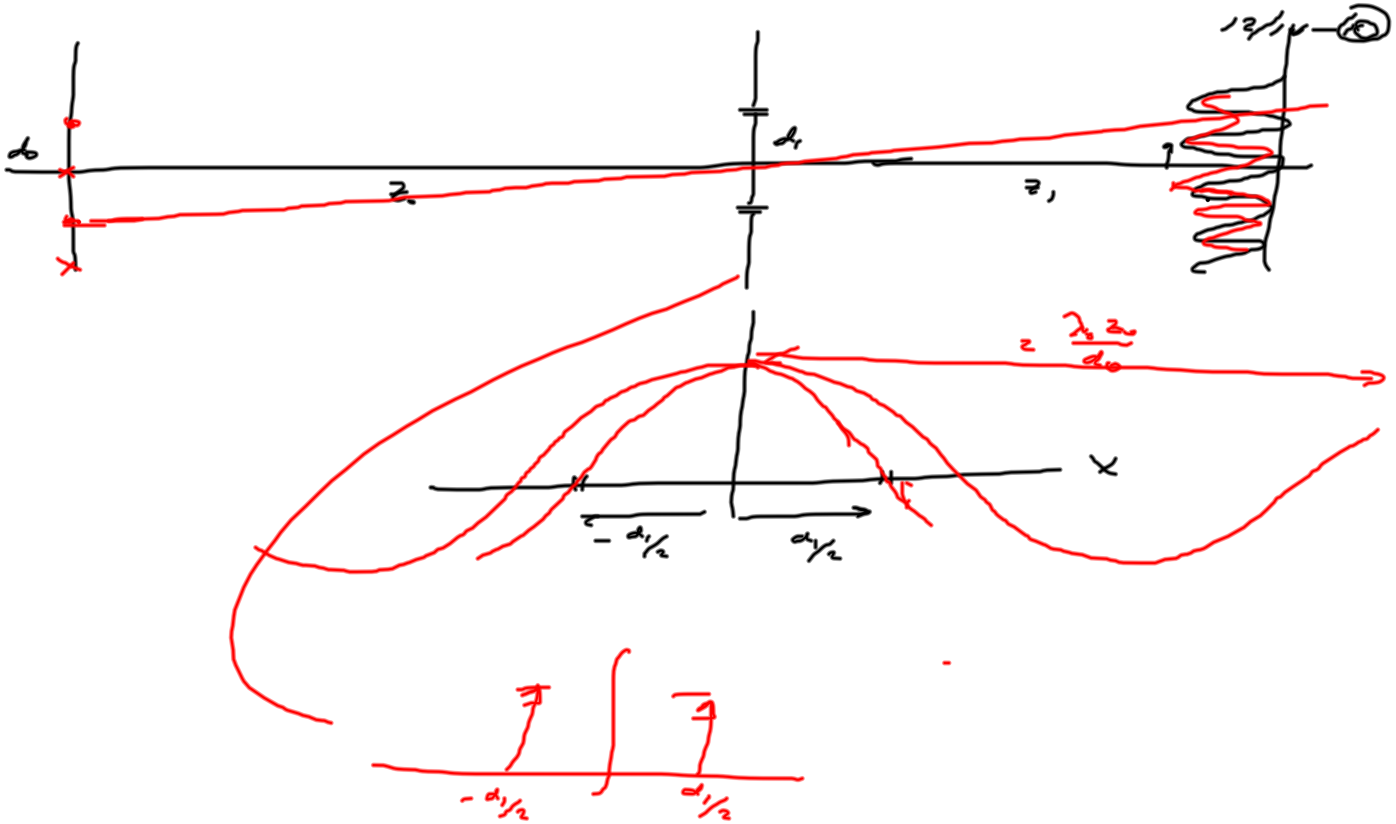


$$D_1 = \frac{\lambda_0 z_1}{d_1}$$

INTERFERENCE

IF INTERFERENCE FRINGES ARE VISIBLE ⇒ "COHERENCE" OF LIGHT AT OBJECT




12/14
d_0
d_1
z_0
z_1
$= \frac{\lambda_0 z_0}{d_{10}}$
x
$-d_1/2$
$d_1/2$
$-d_1/2$
$d_1/2$