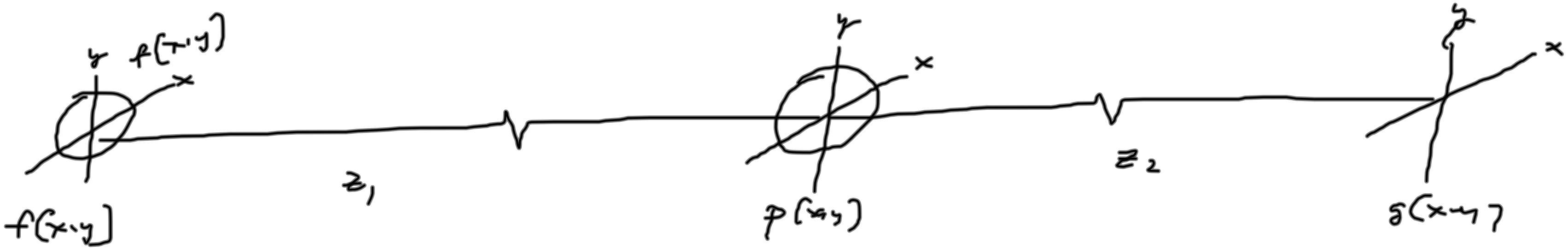


$$\overrightarrow{\left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)} F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]\right)} p[x,y] \quad \text{AFTER APERTURE}$$

$$\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t_1\right)} \frac{1}{i\lambda_0 z_2} e^{+2\pi i\left(\frac{z_2}{\lambda_0} - \nu_0 t_2\right)} \left[f\left[\frac{x}{(-z_2/z_1)}, \frac{y}{(-z_2/z_1)}\right] * P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z}\right.\right.$$

$$g[x,y] \propto f\left[\frac{x}{M_T}, \frac{y}{M_T}\right] * \underbrace{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]}_{h[x,y]}$$

12/9 - ②

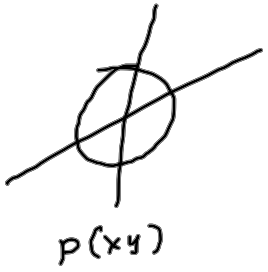
$$h(x,y) \propto P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

$$H(\xi,\eta) \propto \mathcal{F}\left\{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]\right\} \quad (\lambda_0 z_2)^2\, p\left[-\lambda_0 z_2 \xi, -\lambda_0 z_2 \eta\right]$$

SCALED { INVERTED REPLICA OF PUPIL

$$H(\xi,\eta) \propto p\left[\frac{\xi}{\left(-\frac{1}{\lambda_0 z_2}\right)}, \frac{\eta}{\left(-\frac{1}{\lambda_0 z_2}\right)}\right]$$

FOR SINGLE λ_0
(CANCELLATION IF OUT OF PHASE BY π)

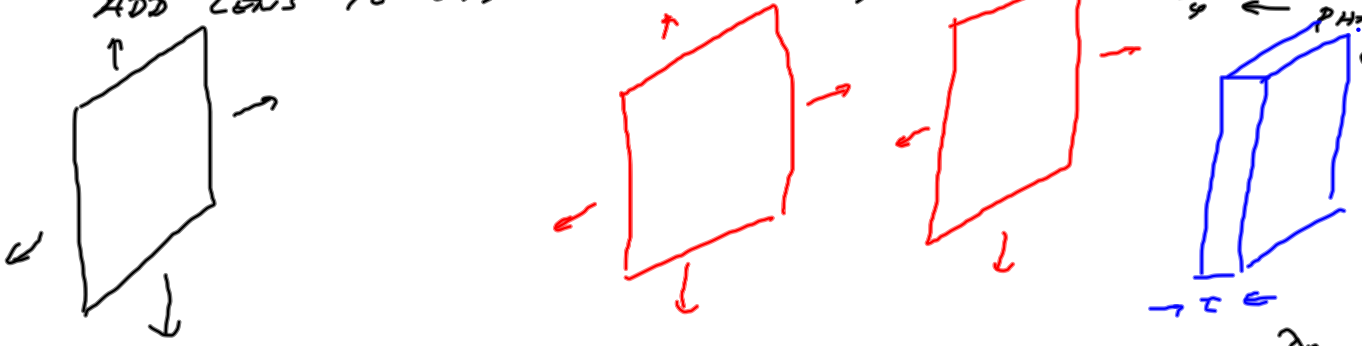


$M_T = -\frac{z_2}{z_1}$

12/9 - ③

RESOLUTION - OVERLAPPING IMPULSE RESPONSES

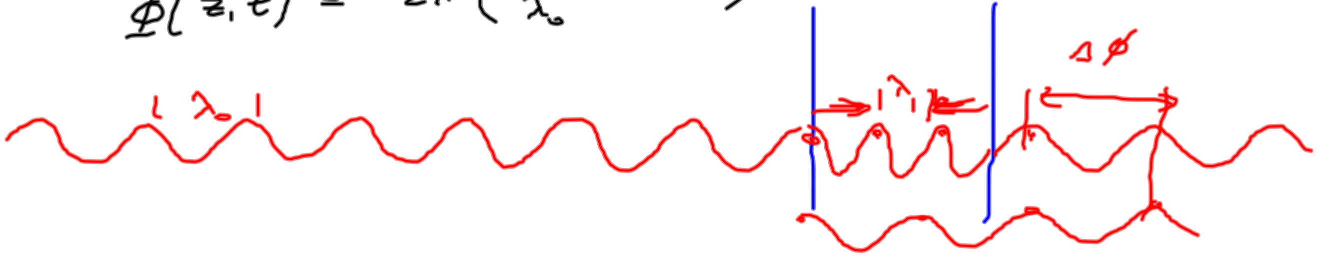
ADD LENS TO SYSTEM $\Longrightarrow$ GLASS, INDEX $n = \frac{c}{v_\phi}$ ← PHASE VELOCITY IN THE GLASS



$\Delta\phi$ = CONSTANT

$$\lambda_1 = \frac{\lambda_0}{n}$$

$$\Phi[z,t] = 2\pi\left(\frac{z}{\lambda_0} - \nu_0 t\right)$$



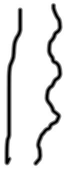
$$\Delta\phi = \phi(\text{GLASS}) - \phi(\text{VACUUM})$$ 12/9 − ④

$$\Delta t = \frac{\tau}{v} \implies \Delta t = \frac{\tau}{c/n} - \frac{\tau}{c}$$

$$\Delta t = (n-1)\cdot\frac{\tau}{c}$$

$$\phi = \frac{2\pi}{\lambda_0}\cdot c\cdot\Delta t$$

$$\Delta\phi = \frac{2\pi}{\lambda_0}\cdot\cancel{c}\,(n-1)\cdot\frac{\tau}{\cancel{c}} = \frac{2\pi}{\lambda_0}\cdot(n-1)\cdot\underset{\uparrow}{\tau}$$



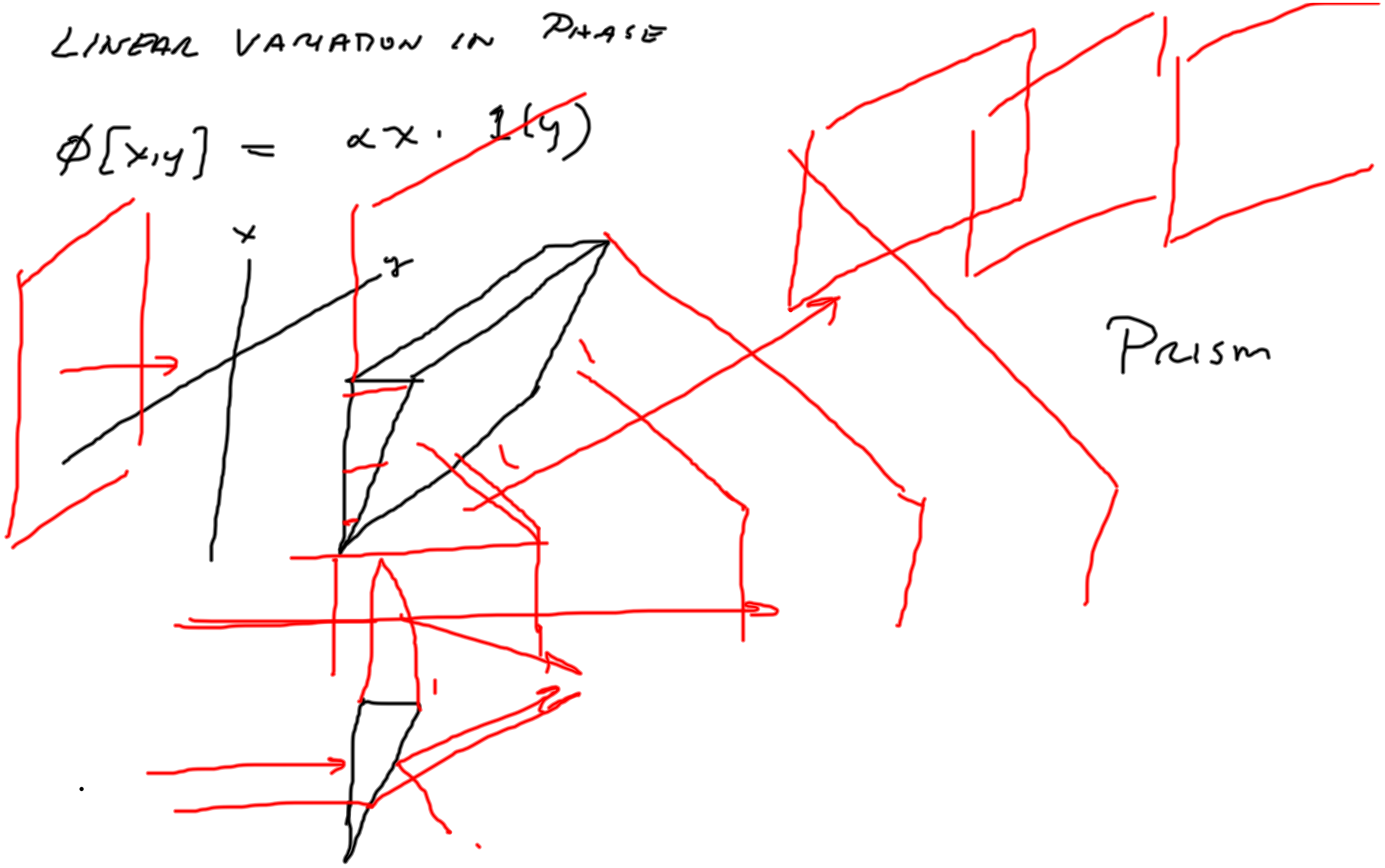
(1) UNIFORM THICKNESS → ~~$\Delta\phi =$~~

$$\Delta\phi[x,y] = \frac{2\pi}{\lambda_0}\cdot(n-1)\cdot\tau_0\cdot 1[x,y]$$

$$e^{i\Delta\phi} = e^{i\frac{2\pi}{\lambda}(n-1)\cdot\tau_0\cdot 1(x,y)} \; ; \; |e^{i\Delta\phi}| = 1$$

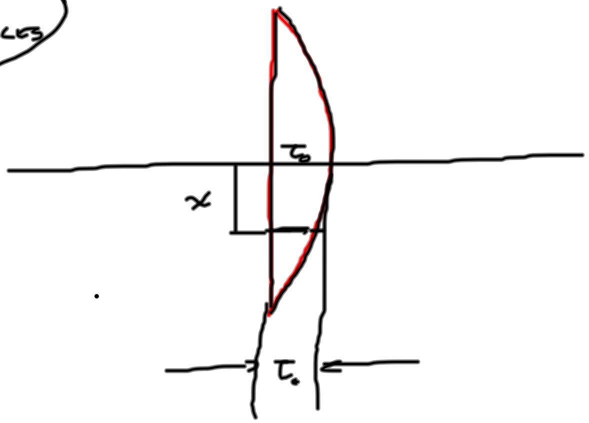
② LINEAR VARIATION IN PHASE

$$\phi[x,y] = \alpha x \cdot 1(y)$$



29-6

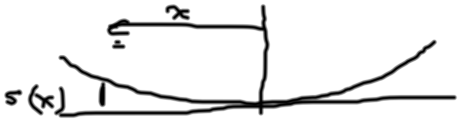
③ SPHERICAL SURFACES



$$\tau_0 = \tau(x) + s(x) \Rightarrow s(x) = \tau_0 - \tau$$

↑ IN GLASS

↑ IN AIR "SAG"

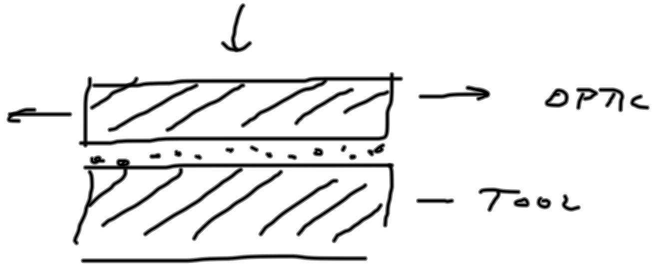


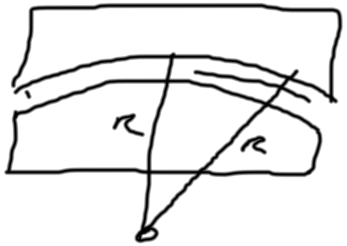
$$\tau(x,y) \rightarrow \tau(x)$$

$$s(x) = \tau_0 - \tau(x)$$

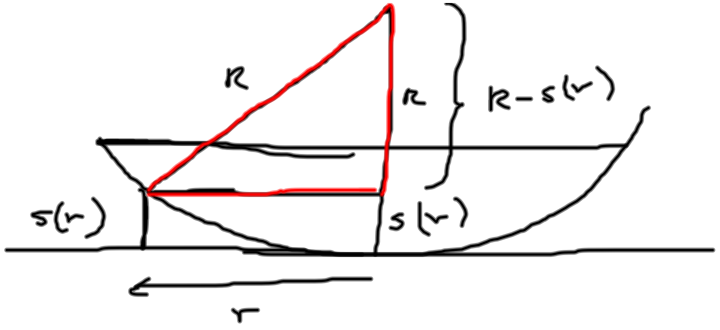
12/9 – ⑦

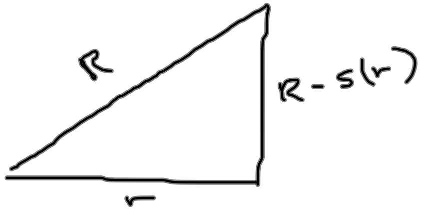
SPHERICAL SURFACES





R S – ⑧





$$R^2 = r^2 + \left(R - s(r)\right)^2$$

$$\cancel{R^2} = r^2 + \cancel{R^2} + s^2(r) - 2Rs(r)$$

$$2Rs(r) = r^2 + \cancel{\left(s(r)\right)^2}$$

IF $R >> s(r) \Longrightarrow R >> r \Longrightarrow \left(s(r)\right)^2 << r^2$

$$\boxed{s(r) \cong \frac{r^2}{2R} \quad \text{SAG FORMULA}}$$

12/9 - ⑨

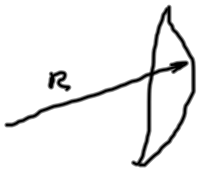
$$s(r) = \frac{r^2}{2R}$$

$$\Delta\phi \propto \tau(r) = \tau_0 - s(r) = \tau_0 - \frac{r^2}{2R}$$

$$\Delta\phi = \frac{2\pi}{\lambda_0}(n-1)\cdot\tau(r) = \frac{2\pi}{\lambda}(n-1)\tau_0 - \frac{2\pi}{\lambda_0}(n-1)\frac{r^2}{2R}$$

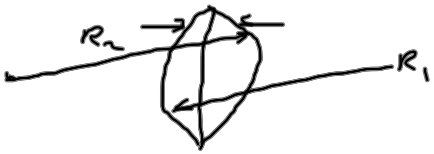
$$= \underset{\text{CONSTANT}}{\frac{2\pi}{\lambda_0}(n-1)\tau_0} - \underset{\text{QUADRATIC}}{\frac{\pi}{\lambda_0 R}(n-1)r^2}$$

$$e^{i\Delta\phi} = \underset{\text{CONSTANT}}{\underline{e^{+\frac{2\pi i}{\lambda_0}(n-1)\tau_0}}}\;\underset{\text{QUADRATIC}}{\underline{e^{-\frac{i\pi}{\lambda_0 R}(n-1)r^2}}}$$



$$\Delta\phi = \pi(n-1)\frac{r^2}{\lambda}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f}$$

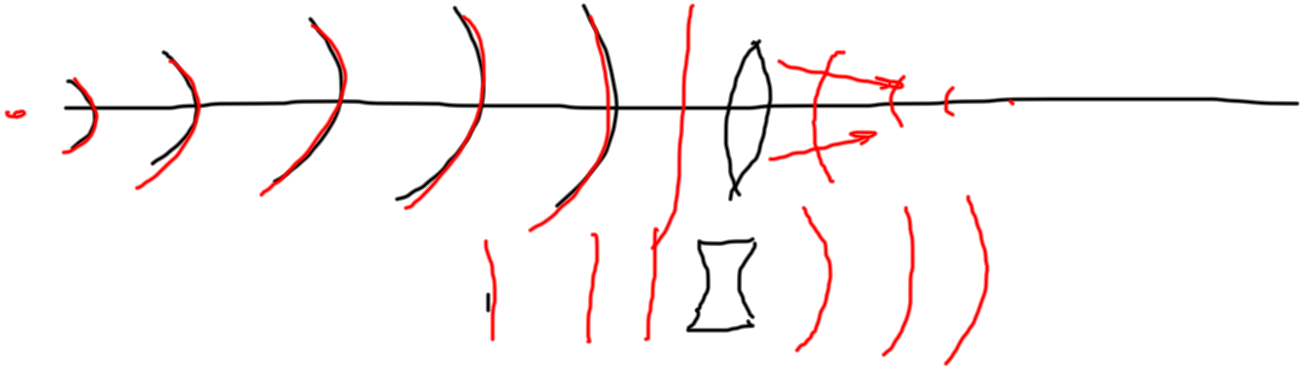


12/9 – ⑩

$$(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \equiv \frac{1}{f}$$; f = "FOCAL LENGTH"

~~Δφ =~~ $$\Delta\phi(r) = -\frac{\pi r^2}{\lambda_0 f}$$

$$e^{-i\pi\frac{r^2}{\lambda_0 f}}$$ — (APPROXIMATE) CHANGE IN PHASE AS FUNCTION r FOR LENS WITH FOCAL LENGTH f



12/9 – ⑪

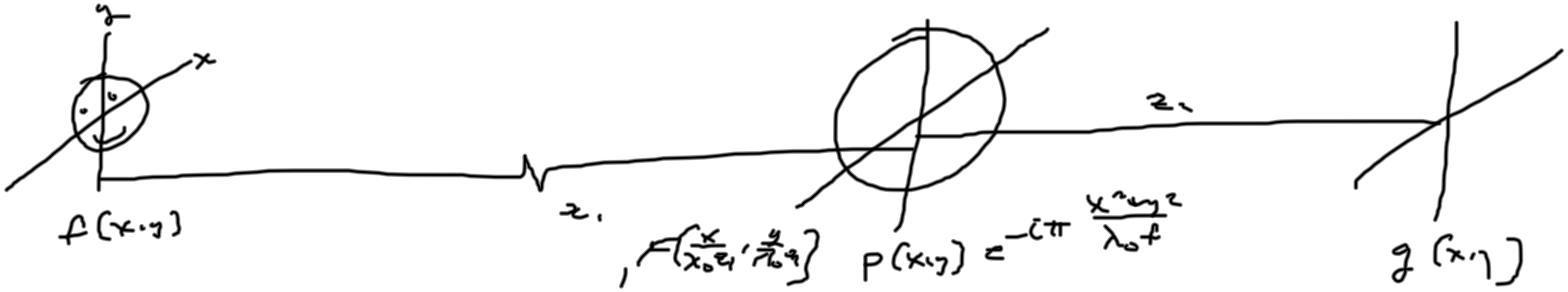
APERTURE $p(r)$

PHASE $e^{-i\pi \frac{r^2}{\lambda_0 f}}$

$$\left.\begin{matrix} p(r) \\ e^{-i\pi \frac{r^2}{\lambda_0 f}} \end{matrix}\right\} \quad t(r) = \underline{\underline{p(r)}}\, \underline{\underline{e^{-i\pi \frac{r^2}{\lambda_0 f}}}}$$

$$= p(x,y)\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

RETURN TO SYSTEM



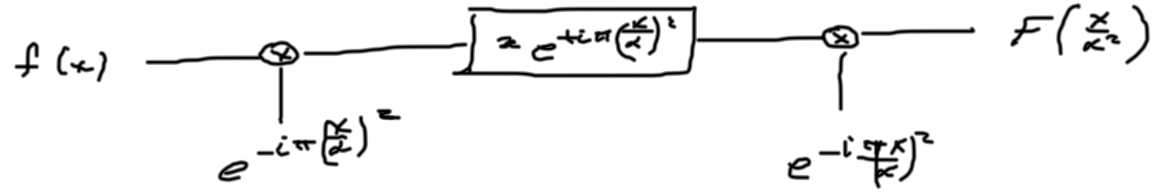
AT BACK OF LENS $\Longrightarrow$ PROPAGATE TO FRESNEL DIFFRACTION z_2

$$(\;\;)\left(F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]\cdot p(x,y)\, e^{-i\pi \frac{x^2+y^2}{\lambda_0 \underset{f}{\cancel{z_1}}}}\right) \ast (\;) e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_2}} = g(x,y)$$

$$(\;\;)\left(\left(F\left(\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right)\cdot p(x,y)\right)\cdot e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}\right) \ast e^{+i\pi \frac{x^2+y^2}{\underset{\substack{\uparrow \\ z_2 = f}}{\lambda_0 z_2}}} = g(x,y)$$

M-C-M CHIRP FOURIER TRANSFORM

$\xi \rightarrow \frac{x}{\alpha^2}$



1-D FOURIER TRANSFORM

$$F(\xi) = \int_{-\infty}^{+\infty} f(x)\, e^{-2\pi i \xi x}\, dx$$

$$\boxed{-2\xi x = \left(\alpha\xi - \frac{x}{\alpha}\right)^2 - (\alpha\xi)^2 - \left(\frac{x}{\alpha}\right)^2}$$

$$= \cancel{(\alpha\xi)^2} + \cancel{\left(\frac{x}{\alpha}\right)^2} - 2\cancel{\alpha}\xi\cdot\frac{x}{\cancel{\alpha}} - \cancel{(\alpha\xi)^2} - \cancel{\left(\frac{x}{\alpha}\right)^2}$$

$$F(\xi) = \int_{-\infty}^{+\infty} \underbrace{\left(f(x)\, e^{-i\pi\left(\frac{x}{\alpha}\right)^2}\right)}_{m\cdot e^{-i\pi\left(\frac{x}{\alpha}\right)^2}} \underbrace{e^{+i\pi\left(\alpha\xi - \frac{x}{\alpha}\right)^2}}_{h\left(\alpha\xi - \frac{x}{\alpha}\right)} \boxed{e^{-i\pi(\alpha\xi)^2}}\, dx$$

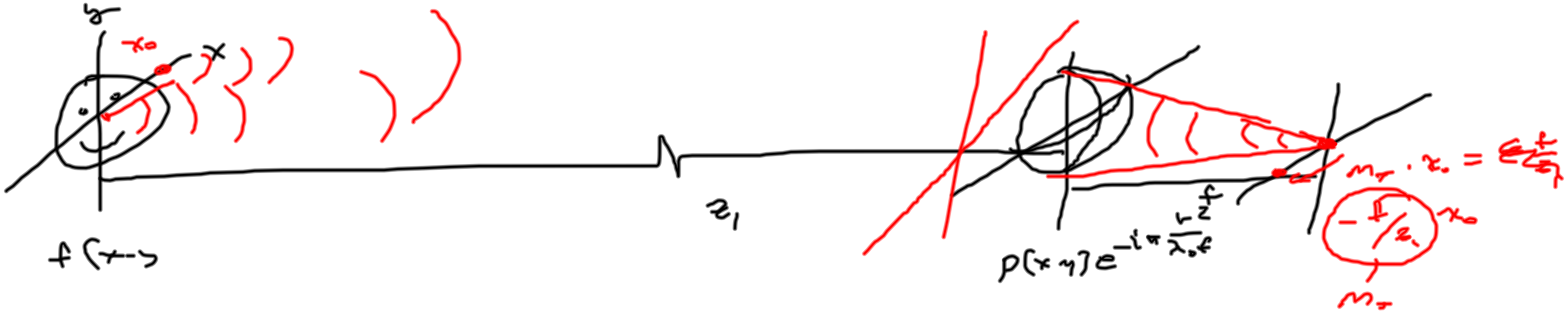
$$F(\xi) = \left(\left(f(x)\, e^{-i\pi\left(\frac{x}{\alpha}\right)^2}\right) \ast e^{+i\pi\left(\frac{x}{\alpha}\right)^2}\right)\Bigg|_{\frac{x}{\alpha}\to\alpha\xi} \cdot e^{-i\pi(\alpha\xi)^2}$$

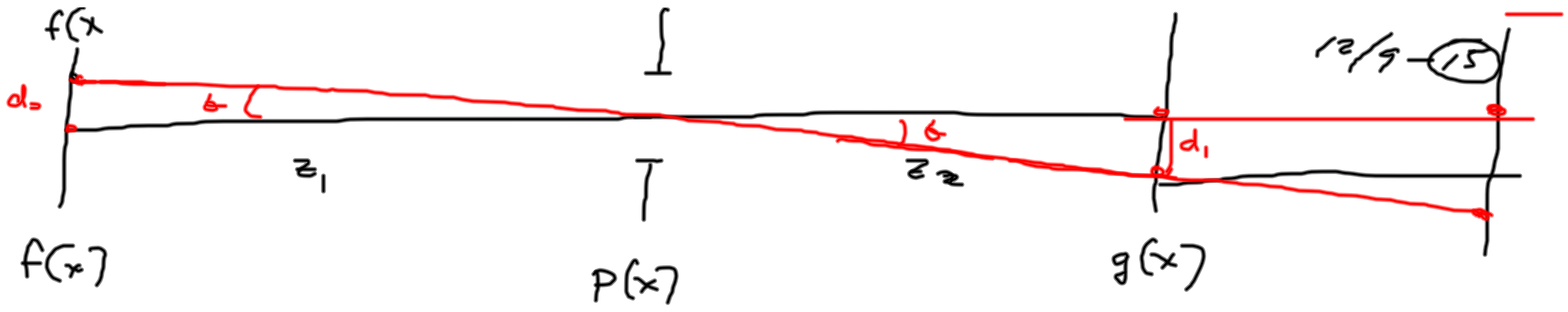
12/9 − ⑭

$$g(x,y) = (\ )\left(\left(F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] p(x,y)\right) \cdot e^{-i\pi \frac{x^2+y^2}{\lambda_0 f}}\right) \times e^{+i\pi \frac{x^2+y^2}{\lambda_0 f}}$$

$$= (\ )\ \mathcal{F}_2\left\{ F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] p(x,y)\right\} \cdot \underline{e^{+i\pi \frac{x^2+y^2}{\lambda_0 f}}}$$

$$|g(x,y)|^2 = |(\ \ )|^2 \ (\lambda_0 z_1)^2 \left|\left(\underline{f\left[\frac{x}{-f/z_1}, \frac{y}{-f/z_1}\right]} \times \underline{p\left(\frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f}\right]}\right)\right|^2 \cdot 1$$



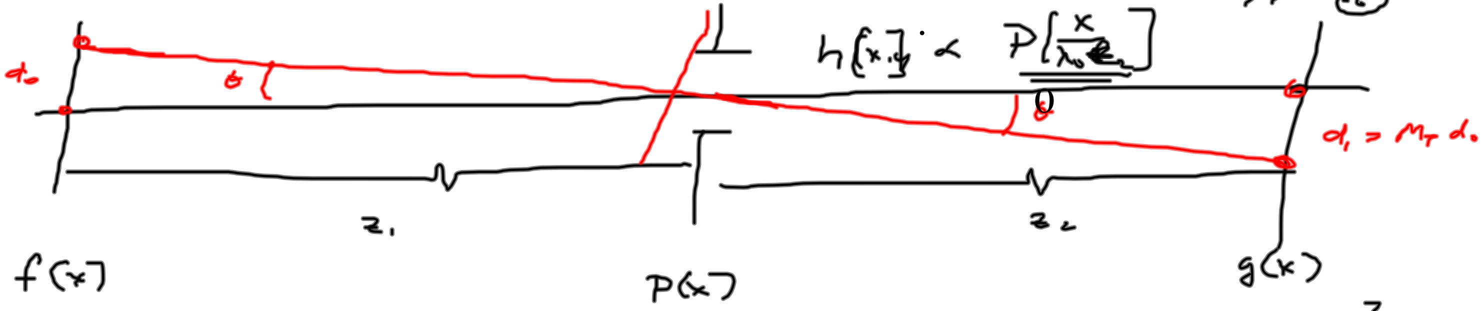


$g(x) =$ ~~$(\quad)\ f\left(\frac{x}{\left(-\frac{z_2}{z_1}\right)}\right) \times P\left[\frac{x}{\lambda_0 z_2}\right]$~~ $(\quad)\ f\left(\frac{x}{\left(-z_2/z_1\right)}\right) \times P\left[\frac{x}{\lambda_0 z_2}\right]$

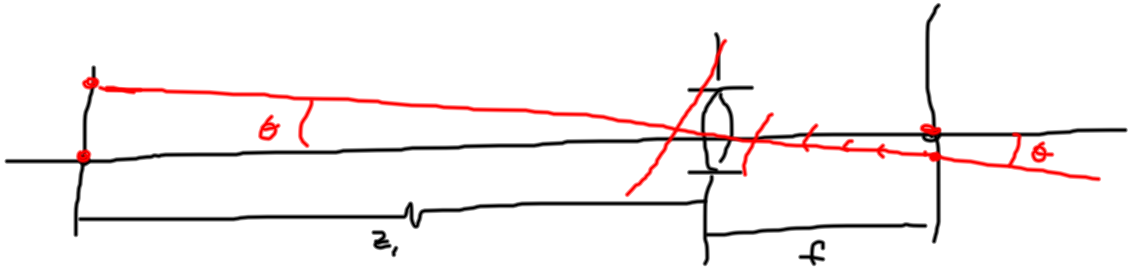
$$\frac{d_1}{d_0} = \left(-\frac{z_2}{z_1}\right)$$

12/9 – ⑯

$h[x] \propto \underline{\underline{P\left[\frac{x}{\lambda_0 z_2}\right]}}$



$M_T = -\frac{z_2}{z_1}$



$M_T = -\frac{f}{z_1}$

$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$

$h[x] \propto \underline{\underline{P\left[\frac{x}{\lambda_0 f}\right]}}$