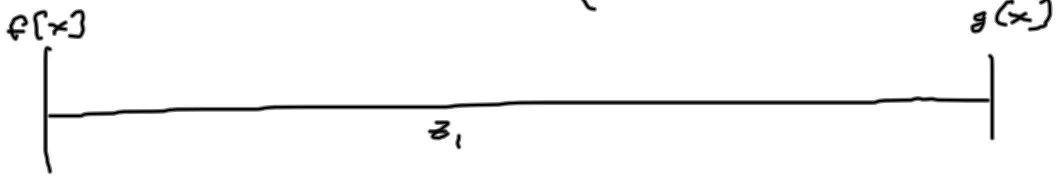
7 DECEMBER 2009

FRESNEL REGION = "NEAR FIELD" (FRAUNHOFER = FAR FIELD)

$$g[x] = \frac{1}{(i\lambda_0) z_1} \underline{e^{2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)}} \left(f(x) * e^{+i\pi \frac{x^2}{\lambda_0 z_1}} \right)$$



MEASURE "IRRADIANCE" — $\langle |g(x,y)|^2 \rangle$

TIME AVERAGE

$$I[x,y] = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} |g(x,y;t)|^2 \, dt \quad ; \quad T_0 = \text{AVERAGING TIME}$$

$$\longrightarrow |g(x,y)|^2$$

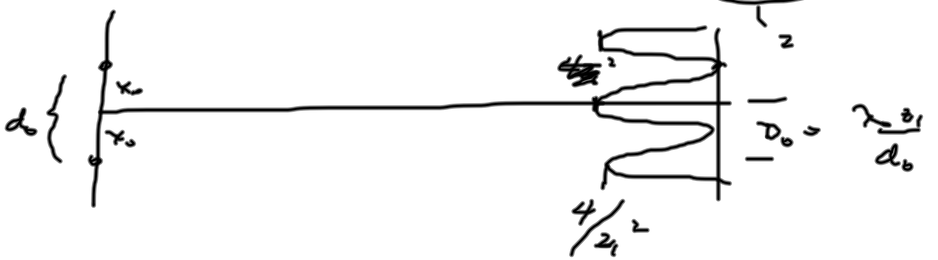
$$f[x,y] = \delta[x-x_0,\, y-y_0] \longrightarrow g[x,y] = \underbrace{\frac{1}{i\lambda z_1}\, e^{+2\pi i\left(\frac{z_0}{\lambda_0} - \nu_0 t\right)}}_{K_0} \; e^{+i\pi \frac{(x-x_0)^2 + (y-y_0)^2}{\lambda_0 z_1}}$$

$$|g[x,y]|^2 = |K_0|^2 \cdot 1[x,y] = \frac{1}{\lambda_0^2 z_1^2} \cdot 1[x,y]$$

INVERSE SQUARE LAW

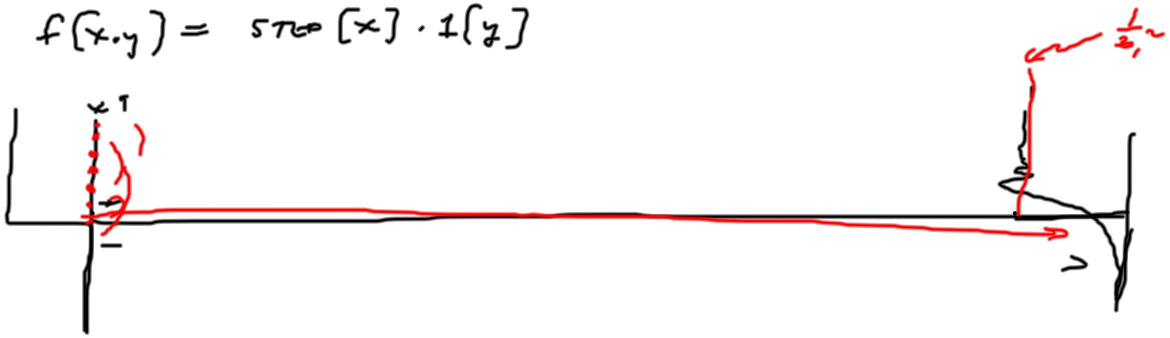
$$\delta[x+x_0] + \delta[x-x_0] \longrightarrow \frac{1}{z_1^2} \cdot \underbrace{4 \cdot \frac{1}{2}}_{2} \left(1 + \cos\left(2\pi \frac{x}{\frac{\lambda_0 z_1}{2x_0}}\right)\right)$$

d_0

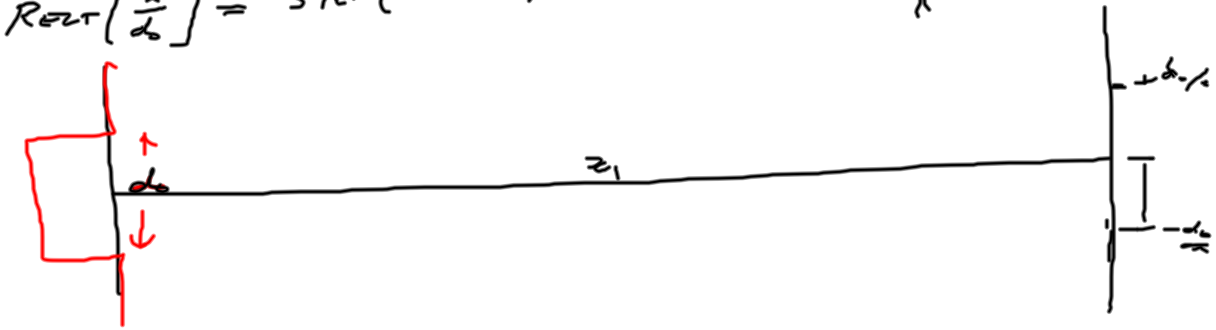


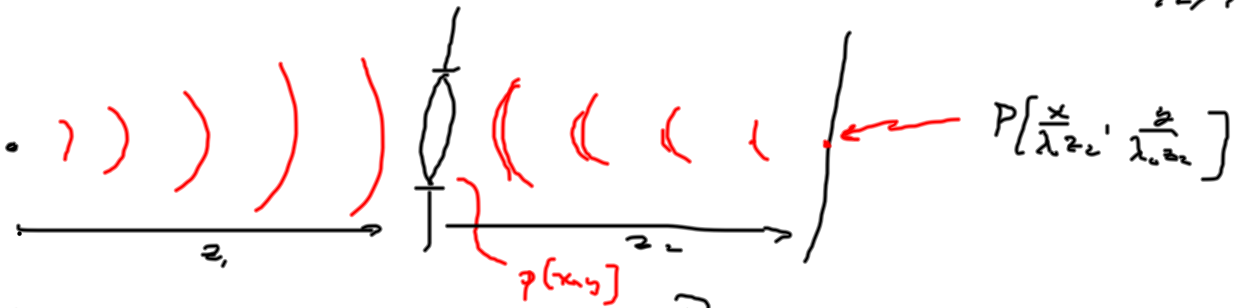
12/3 − 3

$$f[x,y] = \mathrm{STEP}[x] \cdot 1[y]$$



$$\mathrm{RECT}\left[\frac{x}{d_0}\right] = \mathrm{STEP}\left(x + \frac{d_0}{2}\right) - \mathrm{STEP}\left(x - \frac{d_0}{2}\right) \rightarrow \left[\left(\mathrm{STEP}(x) * h(x)\right) * \left(\delta\left(x + \frac{d_0}{2}\right) - \delta\left(x - \frac{d_0}{2}\right)\right)\right]$$



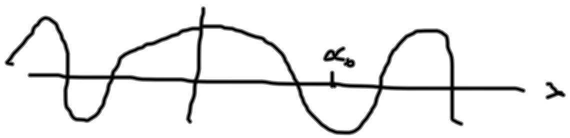


PROPERTIES OF FRESNEL DIFFRACTION PATTERNS

(1) LARGER OBJECT $\Rightarrow$ LARGER "IMAGE"

(2) PATTERN VARIES WITH z_1 $\qquad e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$ $\quad \alpha_0^2$

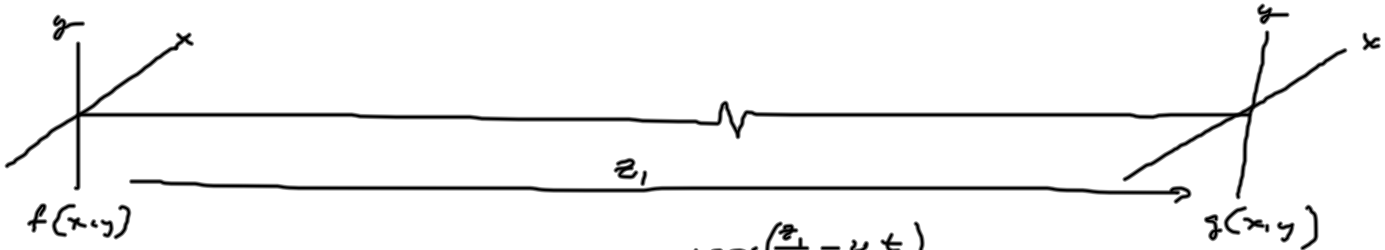
$\alpha_0 = \sqrt{\lambda_0 z_1}$ — CHIRP RATE



INCREASING $z_1 \Rightarrow$ COARSER "OSCILLATIONS" (RINGING)

(3) ALLPASS FILTER W/ QUADRATIC PHASE

"FAR FIELD" – FRAUNHOFER DIFFRACTION



IN NEAR FIELD $\quad \frac{1}{i\lambda_0 z_1}\, e^{+2\pi i\left(\frac{z_1}{\lambda_0} - \nu_0 t\right)}$

$$g(x,y) = K_0 \quad f(x,y) * e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$= K_0 \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, e^{+i\pi \frac{(x-\alpha)^2+(y-\beta)^2}{\lambda_0 z_1}}\, d\alpha\, d\beta$$

$$g(x,y) = K_0 \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, \underbrace{e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}}_{\text{QUADRATIC}}\; \underbrace{e^{+i\pi \frac{\alpha^2+\beta^2}{\lambda_0 z_1}}}_{\text{QUADRATIC}}\; \underbrace{e^{-\frac{2\pi i}{\lambda_0 z_1}(x\alpha+y\beta)}}_{\text{"BILINEAR"}}\, d\alpha\, d\beta$$

12/7 — 6

$$g(x,y) = K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, \cancel{e^{+i\pi \frac{\alpha^2+\beta^2}{\lambda_0 z_1}}}\, e^{-2\pi i\left(\alpha \frac{x}{\lambda_0 z_1} + \beta \frac{y}{\lambda_0 z_1}\right)}\, d\alpha\, d\beta$$

NEW ASSUMPTION, CONSTRAIN SUPPORT OF $f(\alpha,\beta)$, i.e. $|\alpha| \cong 0$, $|\beta|$

IF $\lambda_0 z_1 \gg \alpha^2+\beta^2 \Longrightarrow \frac{\alpha^2+\beta^2}{\lambda_0 z_1} \simeq 0 \Longrightarrow e^{+i\pi \frac{\alpha^2+\beta^2}{\lambda_0 z_1}} \simeq 1$

OBSERVATION PLANE IS FARTHER FROM OBJECT

$$g(x,y) \cong K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \iint_{-\infty}^{+\infty} f(\alpha,\beta)\, e^{-2\pi i\left(\alpha\cdot\left(\frac{x}{\lambda_0 z_1}\right) + \beta\cdot\left(\frac{y}{\lambda_0 z_1}\right)\right)}\, d\alpha\, d\beta$$

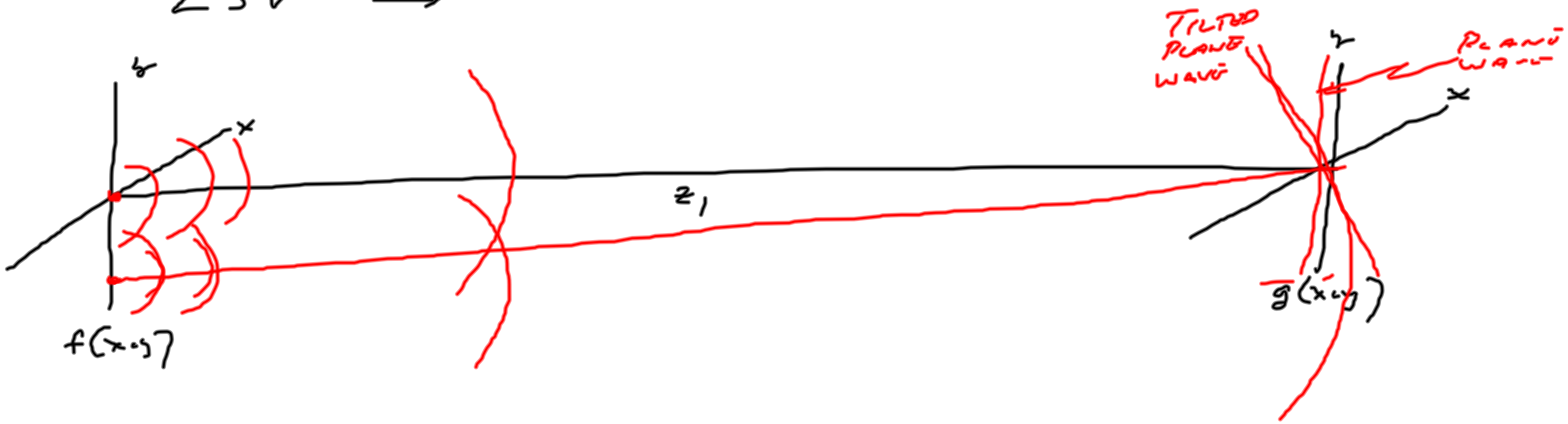
$\frac{x}{\lambda_0 z_1}$, $\frac{y}{\lambda_0 z_1}$ HAVE DIMENSIONS OF LENGTH^{-1}, e.g. $\frac{\text{CYCLES}}{\text{mm}}$

$$\boxed{g(x,y) \cong K_0\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \cdot \underbrace{\mathcal{F}_2\left\{ f(x,y) \right\}}_{F\left[\frac{x}{\lambda_0 z_1},\, \frac{y}{\lambda_0 z_1}\right]}\Bigg|_{\xi \to \frac{x}{\lambda_0 z_1},\ \eta \to \frac{y}{\lambda_0 z_1}}}$$

FRAUNHOFER DIFFRACTION OF $f[x,y]$ 12/? – ⑦

$$g[x,y;z_1,\lambda_0,\nu_0] \cong K_0\, e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_1}}\, F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]$$

$\angle SV \Longrightarrow$ NO IMPULSE RESPONSE



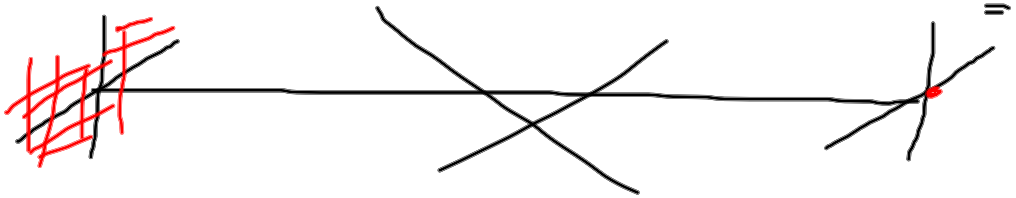
$\circledast$ – ⑧

① $f[x,y] = 1[x,y] \longrightarrow F[\xi,\eta] = \delta(\xi,\eta]$

(NOT SMALL, COMPACT SUPPORT) $F\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] = \delta\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right]$

$\Longrightarrow$ VIOLATES ASSUMPTION

$= (\lambda_0 z_1)^2 \, \delta[x, y]$



---

② $f[x,y] = RECT\left[\frac{x}{b_0}, \frac{y}{d_0}\right] \longrightarrow F[\xi,\eta] = |b_0 d_0| \, SINC(b_0 \xi, d_0 \eta)$

$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] = |b_0 d_0| \, SINC\left(b_0 \frac{x}{\lambda_0 z_1}, \frac{d_0 y}{\lambda_0 z_1}\right)$

$g[x,y] = K_0 \, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \, |b_0 d_0| \, SINC\left(\frac{x}{\left(\frac{\lambda_0 z_1}{b_0}\right)}, \frac{y}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)$

WIDTH PARAMETERS OF SINC ARE $\frac{\lambda_0 z_1}{b_0}, \frac{\lambda_0 z_1}{d_0}$

WIDTH $\uparrow$ IF $\lambda_0 \uparrow$, $z_1 \uparrow$, $b_0 \downarrow$

IRRADIANCE $\propto |g(x,y)|^2 \propto \operatorname{sinc}^2\left(\dfrac{x}{\left(\frac{\lambda_0 z_1}{b_0}\right)}, \dfrac{y}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right)$

3 PAIR OF POINT SOURCES

$$\delta(x,y) \longrightarrow 1(\xi,\eta) \Longrightarrow 1\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = 1(x,y)$$

$$\delta(x+x_0,y) + \delta(x-x_0,y) \longrightarrow \left(e^{-2\pi i x_0 \xi} + e^{+2\pi i x_0 \xi}\right) 1(\eta)$$

$$F(\xi,\eta) = 2\cos(2\pi x_0 \xi)\, 1(\eta)$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = 2\cos\left(2\pi\, x_0 \frac{x}{\lambda_0 z_1}\right) 1\left(\frac{y}{\lambda_0 z_1}\right)$$

$$g(x,y) \propto 2\cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right) 1(y)$$

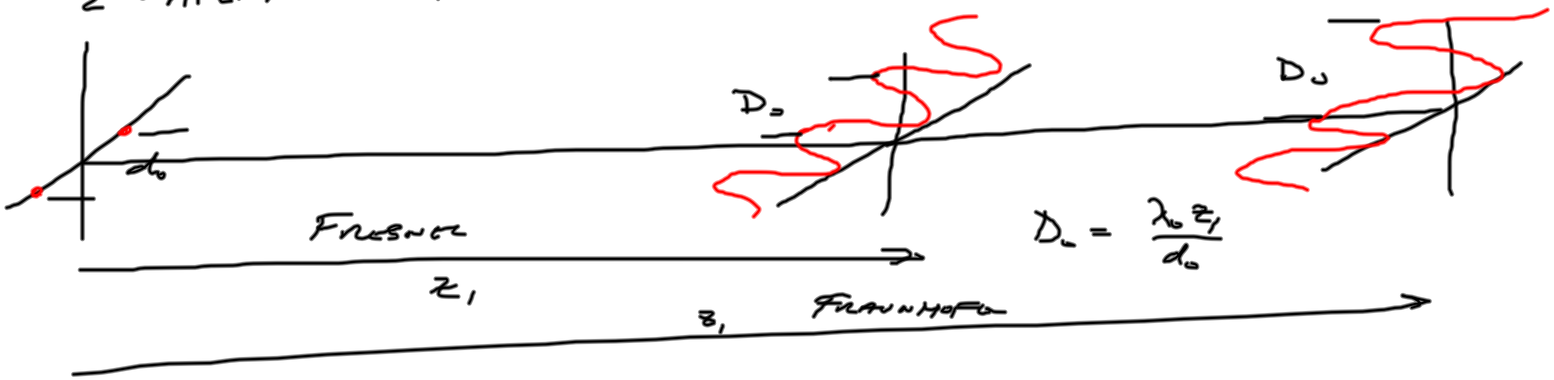
$$|g(x,y)|^2 \propto 4\cos^2\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right) 1(y)$$

$$= 4\left(\frac{1}{2} + \frac{1}{2}\cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{2x_0}\right)}\right)\right) 1(y)$$

$$\frac{\lambda_0 z_1}{2x_0} \equiv D_0 \,,\quad 2x_0 \equiv d_0 \Rightarrow \boxed{\lambda_0 z_1 = d_0 D_0}$$

D_0

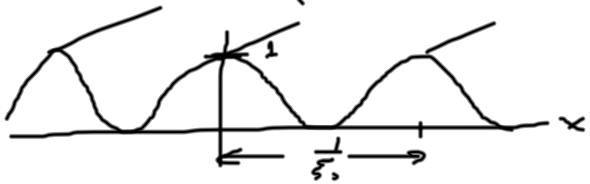
2 - APERTURE EXPERIMENT



$$D_0 = \frac{\lambda_0 z_1}{d_0}$$

PERIODIC "GRATING"

$$f[x,y] = \left(\cos(2\pi \xi_0 x) + 1\right) \cdot \frac{1}{2} \cdot 1[y]$$



$$F[\xi,\eta] = \frac{1}{2}\left(\delta[\xi,\eta] + \frac{1}{2}\,\delta[\xi+\xi_0,\eta] + \frac{1}{2}\,\delta[\xi-\xi_0,\eta] \right)$$

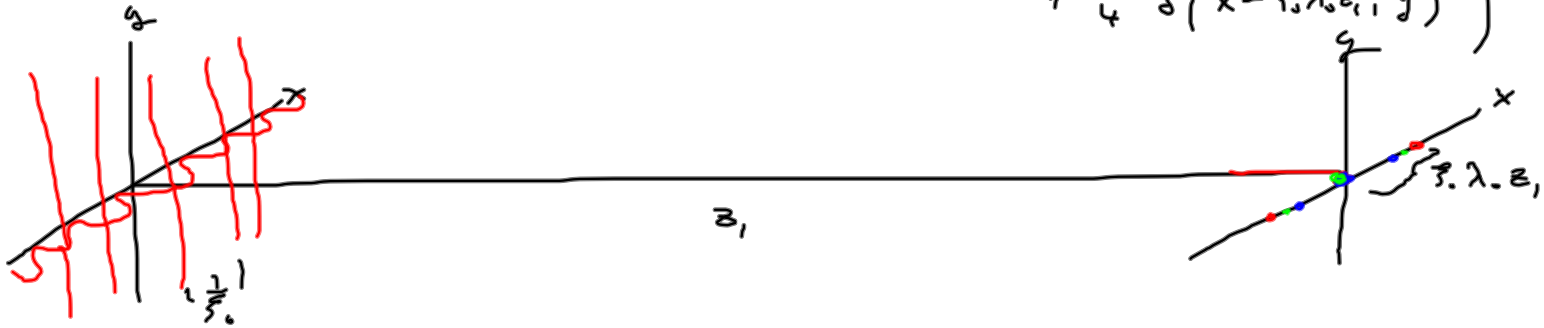
$$= \frac{1}{2}\,\delta(\xi,\eta) + \frac{1}{4}\,\delta[\xi+\xi_0,\eta] + \frac{1}{4}\,\delta[\xi-\xi_0,\eta]$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = \frac{1}{2}\,\delta\left[\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right] + \frac{1}{4}\,\delta\left[\frac{x}{\lambda_0 z_1} + \xi_0, \frac{y}{\lambda_0 z_1}\right] + \frac{1}{4}\,\delta\left(\frac{x}{\lambda_0 z_1} - \xi_0, \frac{y}{\lambda_0 z_1}\right)$$

$$= (\lambda_0 z_1)^2 \left(\frac{1}{2}\,\delta[x,y] + \frac{1}{4}\,\delta[x + \xi_0 \lambda_0 z_1, y] + \frac{1}{4}\,\delta[x - \xi_0 \lambda_0 z_1, y] \right)$$

12 7 - 15

$$g(x,y) = K_0 \, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \cdot (\lambda_0 z_1)^2 \left(\frac{1}{2}\,\delta(x,y) + \frac{1}{4}\,\delta(x+\xi_0\lambda_0 z_1, y) + \frac{1}{4}\,\delta(x-\xi_0\lambda_0 z_1, y) \right)$$



DIFFRACTION GRATING

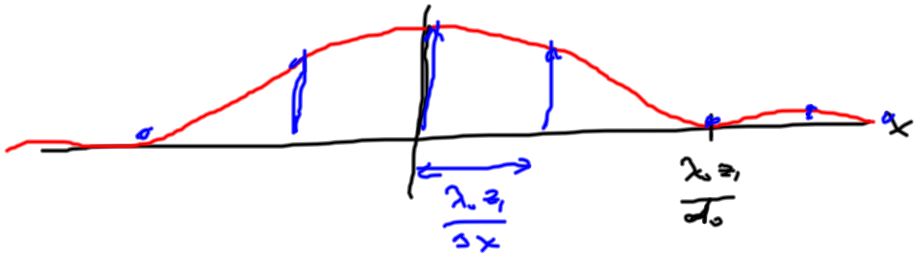
$$f(x,y) = \left(\mathrm{Rect}\left(\frac{x}{d_0}\right) * \frac{1}{\Delta x}\,\mathrm{comb}\left(\frac{x}{\Delta x}\right) \right) 1(y)$$

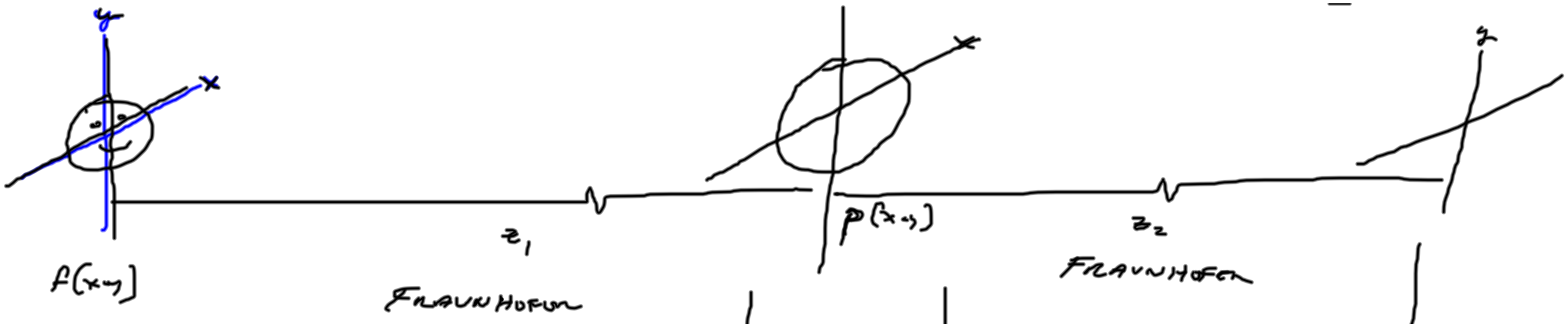
$$F(\xi,\eta) = \left(d_0 \, \mathrm{sinc}(d_0 \xi) \cdot \mathrm{comb}(\Delta x \cdot \xi) \right) \cdot \delta(\eta)$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right) = \left(d_0 \, \mathrm{sinc}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{comb}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)\right) \delta\left(\frac{y}{\lambda_0 z_1}\right)$$

$$= d_0 \lambda_0 z_1 \; \mathrm{sinc}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{comb}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)$$

$$|g(x,y)|^2 \propto \mathrm{sinc}^2\left(\frac{x}{\left(\frac{\lambda_0 z_1}{d_0}\right)}\right) \cdot \mathrm{comb}\left(\frac{x}{\left(\frac{\lambda_0 z_1}{\Delta x}\right)}\right)$$





$$\propto F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right)$$

$$F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right)\cdot p(x,y)$$

$$\mathcal{F}_2\left\{ F\left(\frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1}\right)\cdot p(x,y)\right\} = (\lambda_0 z_1)^2\, f(-\lambda_0 z_1 \xi, -\lambda_0 z_1 \eta) * P(\xi, \eta)$$

$$\xi \to \frac{x}{\lambda_0 z_2}, \quad \eta \to \frac{y}{\lambda_0 z_2}$$

$$g[x,y] \propto (\lambda_0 z_1)^2 \, f\left[-\lambda_0 z_1 \cdot \frac{x}{\lambda_0 z_2}, -\lambda_0 z_1 \cdot \frac{y}{\lambda_0 z_2}\right] * P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]$$

$$\propto f\left[\frac{x}{\left(-\frac{z_2}{z_1}\right)}, \frac{y}{\left(-\frac{z_2}{z_1}\right)}\right] * \underbrace{P\left[\frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}\right]}_{h[x,y]}$$

SCALE FACTOR $= -\dfrac{z_2}{z_1}$

IF $z_2 = z_1 \implies f[-x,-y]$