2 DECEMBER 2009

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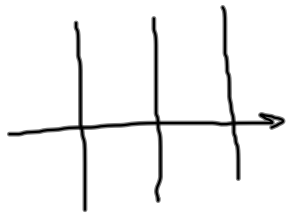
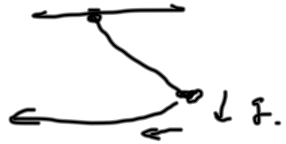


MAXWELL'S EQUATION – ELECTROMAGNETIC WAVES = EM WAVES

ELECTRIC
FIELDS
MAGNETIC

WAVE = TRAVELING OSCILLATION

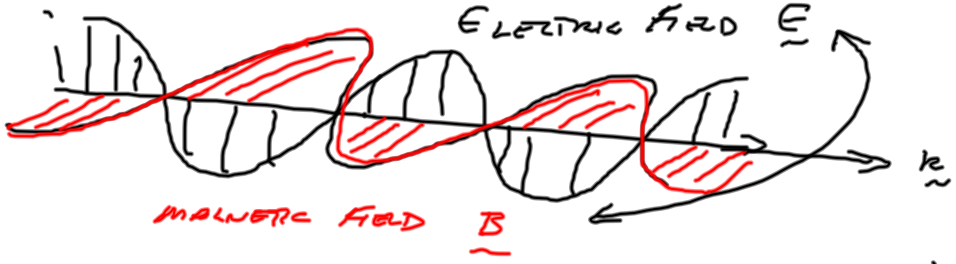



(1) INERTIA

(2) RESTORING   MEDIUM

AETHER

LIGHT IS EM WAVE




$$F = q\left(\underset{\sim}{E} + \frac{\underset{\sim}{v}}{c} \times \underset{\sim}{B}\right)$$

$\Rightarrow$ ELECTRIC EFFECTS DOMINATE

MICHELSON – MORLEY $\rightarrow$ NO MEDIUM (AETHER)

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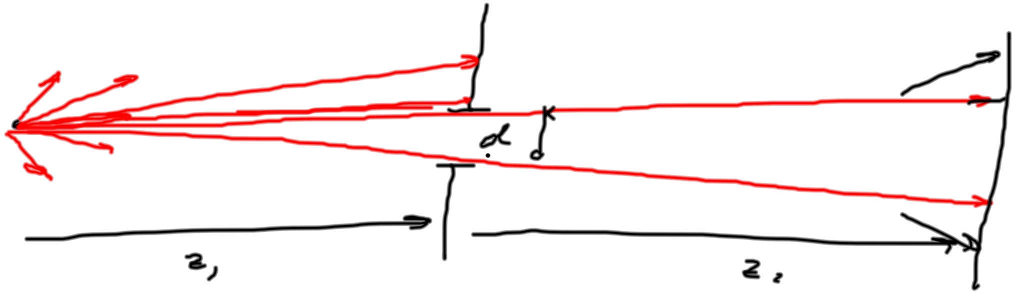
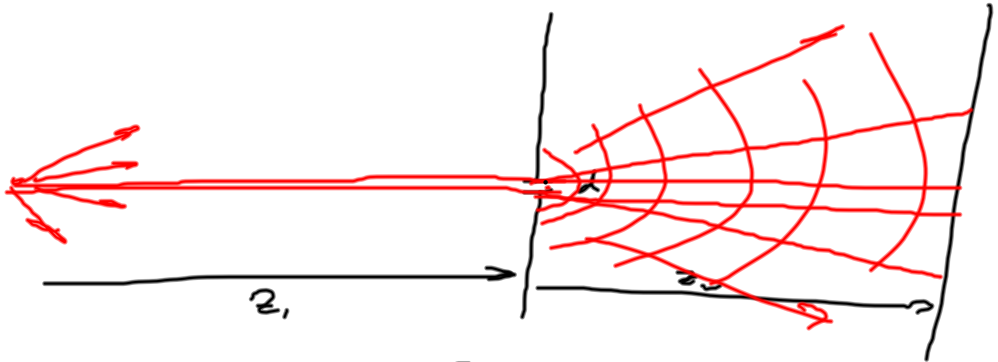
RAY

WAVE




$\boxed{\underset{\sim}{D} \propto d}$

$D \propto d \, \frac{z_2}{z_1 +}$




$D$

$D \not\propto d$

SMALL APERTURE

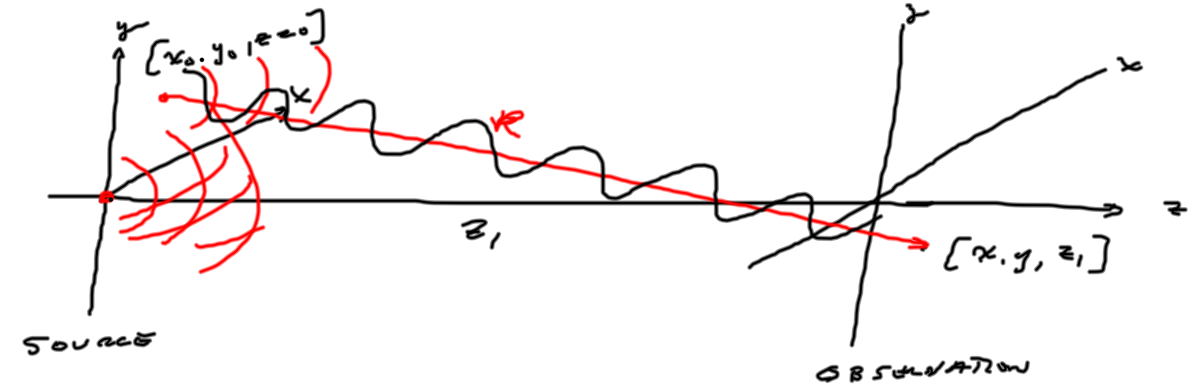
MEASUREMENT $D \propto \frac{1}{d}$

$d \sim \lambda_0$

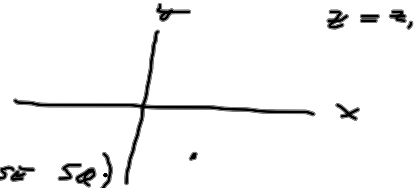
SCALING THEOREM

DIFFRACTION OF LIGHT

12/2 — 4




$\delta[x - x_0, y - y_0, z]$




AMPLITUDE AT $[x, y, z_1] \propto \frac{1}{R}$ (INVERSE SQ.)

PHASE $\propto$ # OF ~~[illegible]~~ WAVELENGTHS IN R $\quad 2\pi \frac{R}{\lambda_0}$

$$g[x, y, z_1] \propto \frac{1}{R} \exp\left[2\pi i \left(\frac{R}{\lambda_0} - \nu_0 t\right) + i\varphi_0\right]$$

STRICT CALCULATION → SPHERICAL WAVES → RAYLEIGH - SOMMERFELD DIFFRACTION
LSV

1ST - ORDER APPROXIMATION → PARABOLOIDAL WAVES → FRESNEL DIFFRACTION
LSI → CONVOLUTION

→ FRAUNHOFER DIFFRACTION
2ND - ORDER APPROXIMATION → PLANE WAVES
LSV

INPUT PLANE

$z_1 - 0$

$$R^2 = (x - x_0)^2 + (y - y_0)^2 + (z_1)^2$$

OUTPUT PLANE

$$R = \sqrt{z_1^2 + (x - x_0)^2 + (y - y_0)^2}$$

$$= \sqrt{z_1^2}\sqrt{1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2}} = z_1\left(1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2}\right)^{1/2}$$

AMPLITUDE PHASE 12/2 — 6

$$\frac{1}{R} = R^{-1} \qquad e^{i\left(2\pi \frac{R}{\lambda_0}\right)}$$

$\lambda_0 \sim 0.5\ \mu m$

$\sim 0.5 \cdot 10^{-6}\ m$

SMALL ERROR IN R IN PHASE $\Rightarrow$ BIG ERROR IN PHASE

SMALL ERROR IN AMPLITUDE $\Rightarrow$ NOT MUCH EFFECT

$$\frac{R}{R+\Delta r} = \frac{1}{1+\frac{\Delta r}{R}} = \Big(1+\underbrace{\frac{\Delta r}{R}}_{\text{small}}\Big)^{-1} = 1 + \frac{\Delta R}{R} \Big| + \frac{1}{2}\left(\frac{\Delta r}{R}\right)^2 - \cdots$$

DIFFERENT APPROXIMATIONS FOR R

(1) COARSE APPROX. IN AMPLITUDE

(2) "FINE" APPROX. IN PHASE

$n = 1/2$ — 7

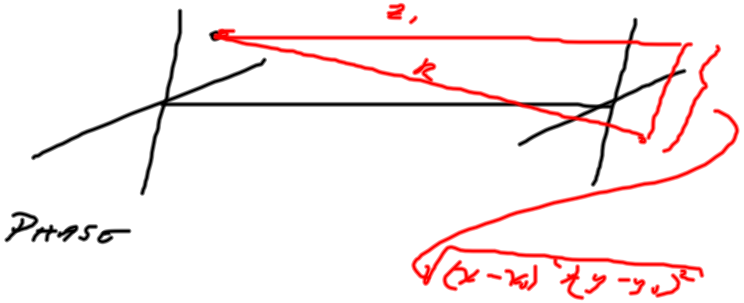
$$R = z_1 \left( 1 + \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right)^{1/2} \quad ; \quad (1+\alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!}\alpha^2 + \frac{n(n-1)(n-2)}{3!}\alpha^3 + \cdots$$

IF $|\alpha| < 1$

$$= z_1 \left( 1 + \frac{1}{2}\left( \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right) + \left(-\frac{1}{8}\right)\left( \frac{(x-x_0)^2 + (y-y_0)^2}{z_1^2} \right)^2 + \cdots \right)$$

$$R = z_1 + \frac{(x-x_0)^2 + (y-y_0)^2}{2 z_1} + \cdots$$

"DOWNSTREAM PROPAGATION" — "OFF-AXIS CORRECTION"




USE TWO TERMS IN SUM FOR PHASE

1ST TERM FOR AMPLITUDE

12/2 — 8

OBSERVED AMPLITUDE AT $[x, y, z_1]$ DUE TO SOURCE AT $[x_0, y_0, z=0]$

$$g[x, y; z=z_1] \propto \underbrace{\frac{1}{z_1}}_{\text{COARSE APPROXIMATION}} \exp\left[2\pi i \left( \underbrace{\frac{z_1 + \frac{(x-x_0)^2+(y-y_0)^2}{2z_1}}{\lambda_0} - \nu_0 t}_{\text{FINE APPROXIMATION}} \right) \cancel{+ i\phi_0} \right] \rightarrow 0$$

FRESNEL APPROXIMATION FOR DIFFRACTION

$$\underbrace{\frac{1}{z_1}}_{\text{INVERSE SQUARE LAW}} \underbrace{e^{+2\pi i \frac{z_1}{\lambda_0}}}_{\text{CONSTANT PHASE DOWN AXIS}} \underbrace{e^{+i\cancel{2}\pi \frac{(x-x_0)^2+(y-y_0)^2}{\cancel{2}\lambda_0 z_1}}}_{\text{QUADRATIC PHASE OFF-AXIS PROPAGATION}} \underbrace{e^{-2\pi i \nu_0 t}}_{\text{TEMPORAL PART}}$$

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$$\boxed{g[\underbrace{x,y}_{\text{OBSERVATION}};\ z=z_1,\ \underbrace{x_0,y_0}_{\text{SOURCE}}] = \underbrace{\left(\frac{1}{i\lambda_0}\right)}_{\text{CONSTANT}} \frac{1}{z_1}\ \underline{e^{+2\pi i \frac{z_1}{\lambda_0}}}\ \underline{e^{+i\pi \frac{(x-x_0)^2+(y-y_0)^2}{\left(\sqrt{\lambda_0 z_1}\right)^2}}} \cdot e^{+2\pi i \nu_1 t}}$$

$$e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}$$

RATE

CHIRP RATE IS $\sqrt{\lambda_0 z_1}$


$$\delta[x-x_0,\ y-y_0]\ \delta[z] \longrightarrow g[x,y;\ x_0,y_0;\ z_1]$$

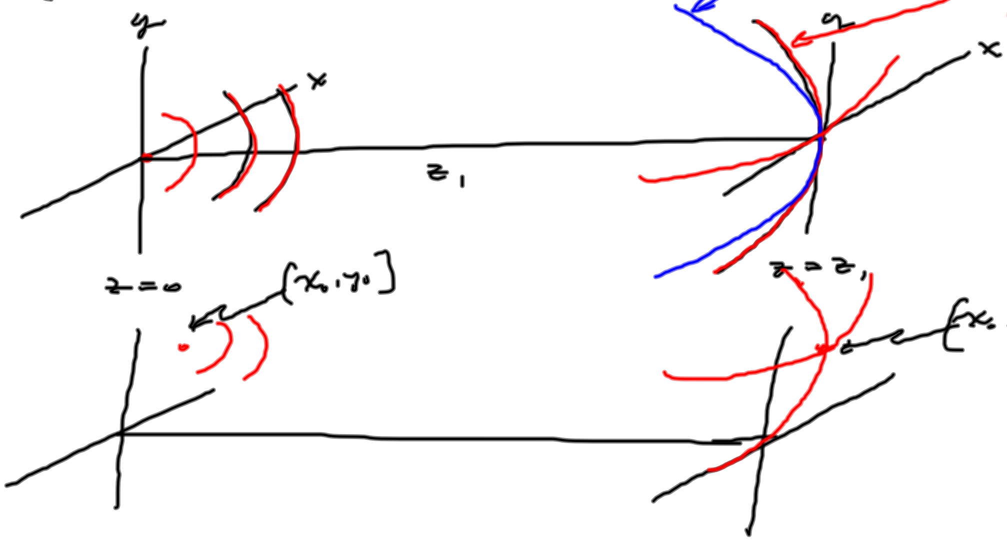
$$\delta[x,y]\ \delta[z] \longrightarrow g[x,y;\ 0,0,z_1] = \left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i \frac{z_1}{\lambda_0}}\right) e^{+i\pi \frac{x^2+y^2}{\left(\sqrt{\lambda z_1}\right)^2}} \cdot \text{TIME}$$

LSI SYSTEM

$$h[x,y\,;\,z_1] = \underbrace{\frac{1}{i\lambda_0 z_1}\, e^{+2\pi i \frac{z_1}{\lambda_0}}\, e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}}_{\text{SPATIAL PART}}\, e^{-2\pi i \nu t}$$

CONSTANT PHASE

(1) SINGLE POINT SOURCE



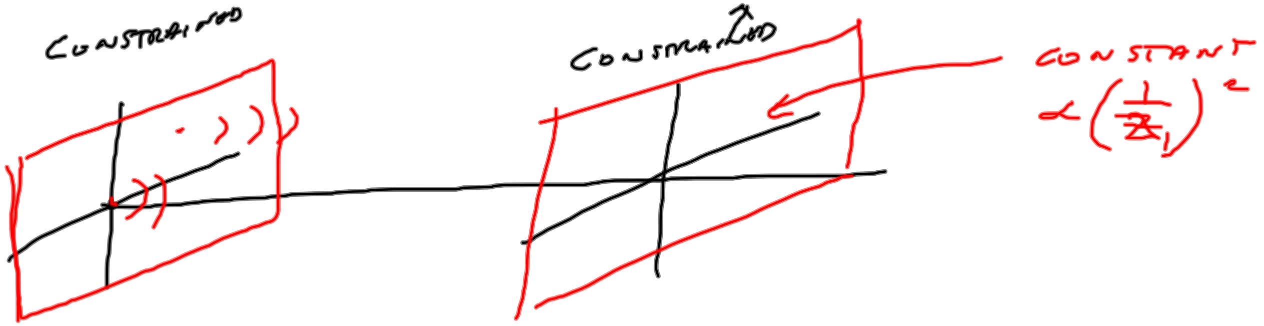

$$\frac{1}{i} = \frac{1}{e^{+i\frac{\pi}{2}}} = e^{-i\frac{\pi}{2}}$$

OBSERVE "POWER", NOT THE AMPLITUDE

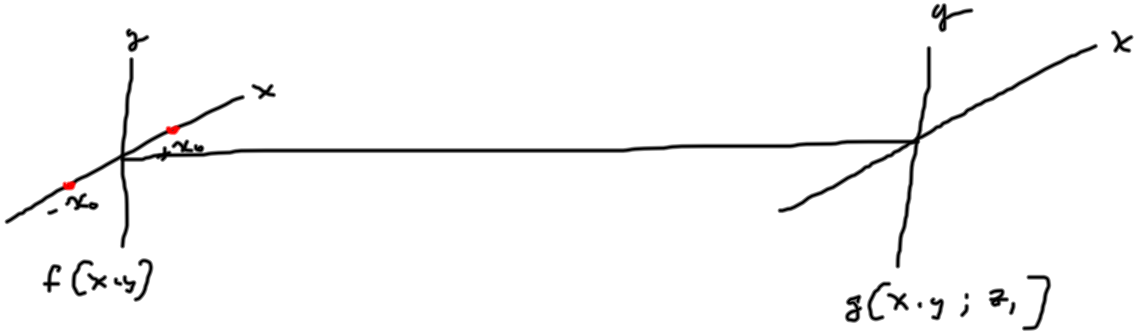
TIME AVERAGE OF SQUARED MAGNITUDE

$$\left\langle \left| h[x,y;z_1,t] \right|^2 \right\rangle = \frac{1}{|\tau|^2}\left(\frac{1}{\lambda_0 z_1}\right)^2 \underbrace{\cancel{\left| e^{+i2\pi \frac{z_1}{\lambda}} \right|^2}}_{1} \underbrace{\cancel{\left| e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}} \right|^2}}_{1}$$

$$\propto \frac{1}{z_1^2} \qquad \text{INVERSE SQUARE LAW}$$



$$f[x,y] = \delta(x+x_0, y] + \delta(x-x_0, y)$$




$$g(x,y;z_1) = f(x,y) * h(x,y;z_1]$$

$$= \left[\delta[x+x_0] + \delta(x-x_0)\right]\delta(y] * \underbrace{\left(\frac{1}{i\lambda_0 z_1} e^{+2\pi i\left(\frac{z_1}{\lambda_0}\right)}\right)}_{K_0} e^{+i\pi\frac{x^2+y^2}{\lambda_0 z_1}}$$

$$= K_0\left(\left(\delta[x+x_0] + \delta(x-x_0)\right) * e^{+i\pi\frac{x^2}{\lambda_0 z_1}}\right)\left(\delta(y) * e^{+i\pi\frac{y^2}{\lambda_0 z_1}}\right)$$

$$g[x,y;z_1] = \underline{K_0}\left( e^{+i\pi \frac{(x+x_0)^2}{\lambda_0 z_1}} + e^{+i\pi \frac{(x-x_0)^2}{\lambda_0 z_1}} \right)\left( \underline{e^{+i\pi \frac{y^2}{\lambda_0 z_1}}} \right)^{1/2/2} \quad (13)$$

$$= \left(K_0\, e^{+i\pi \frac{y^2}{\lambda_0 z_1}}\right)\left( e^{+i\pi \frac{x^2+x_0^2+2xx_0}{\lambda_0 z_1}} + e^{+i\pi \frac{x^2+x_0^2-2xx_0}{\lambda_0 z_1}} \right)$$

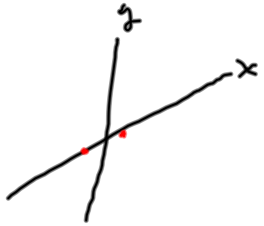
$$= \left(K_0\, e^{+i\pi \frac{y^2}{\lambda_0 z_1}}\right)\left(e^{+i\pi \frac{x^2+x_0^2}{\lambda_0 z_1}}\right)\left( \underbrace{e^{+i\pi \frac{2xx_0}{\lambda_0 z_1}} + e^{-i\pi \frac{2xx_0}{\lambda_0 z_1}}}\right)$$

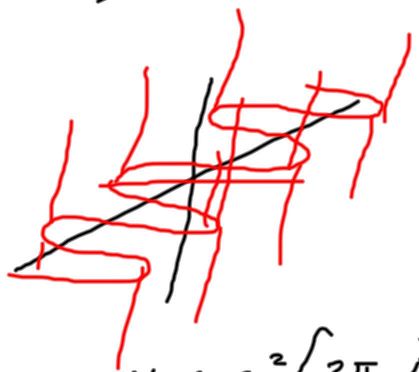
$$= \underbrace{\left(\underline{K_0}\, \underbrace{e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}}_{\text{PARABOLOIDAL WAVE}}\; \underbrace{e^{+i\pi \frac{x_0^2}{\lambda_0 z_1}}}_{\text{CONSTANT PHASE}}\right)} \cdot 2\cos\left(2\pi\, \frac{x\, x_0}{\lambda_0 z_1}\right) \quad \xi_0$$

$$= \left( \qquad \right) \boxed{2\cos 2\pi\left(\frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right)}$$

PERIOD

$$g[x,y;\, z_1, x_0, \lambda_0] = \left(\frac{1}{i\lambda_0 z_1}\, e^{+2\pi i \frac{z_1}{\lambda_0}}\right) e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}\, e^{+i\pi \frac{x_0^2}{\lambda_0 z_1}} \cdot 2\cos\left[2\pi \frac{x}{\underbrace{\left(\frac{\lambda_0 z_1}{x_0}\right)}_{\text{PERIOD}}}\right]$$



$$\left|g[x,y;\, z_1, \lambda_0, x_0]\right|^2 = \underbrace{\frac{1}{\lambda_0^2 z_1^2}}_{\text{IGNORE}} \cdot 4\cos^2\left[2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right]$$

$$\propto \frac{1}{z_1^2} \cdot 4\cos^2\left[2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{x_0}\right)}\right] \quad ; \quad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$= \frac{2}{z_1^2}\left(1 + \cos\left(2\pi \frac{x}{\left(\frac{\lambda_0 z_1}{2x_0}\right)}\right)\right)$$

PERIOD $\frac{\lambda_0 z_1}{2x_0}$

12-2-15

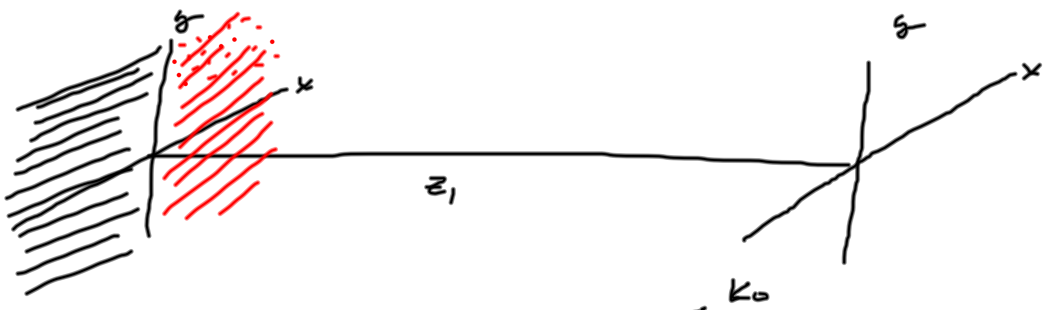
YOUNG'S TWO-~~APERTURE~~ SOURCE EXPERIMENT IN FRESNEL REGION

PERIOD $D_0 = \dfrac{\lambda_0 z_1}{2x_0}$ IRRADIANCE

$$\underbrace{(2x_0)}_{\text{SEPARATION OF SOURCES}} D_0 = \lambda_0 z_1$$

$2x_0 \equiv d_0 \implies$ $\boxed{\underbrace{d_0}_{\text{TRANSVERSE}} D_0 = \underbrace{\lambda_0 z_1}_{\text{"LONGITUDINAL"}}}$

KNIFE EDGE




$$f[x,y] = \mathrm{STEP}[x]\cdot 1[y]$$

$$h[x,y;z_1,\lambda_0] = \boxed{\frac{1}{i\lambda_0 z_1}\, e^{+2\pi i \frac{z_1}{\lambda_0}}}\; e^{+i\pi \frac{x^2+y^2}{\lambda_0 z_1}}$$

$$g[x,y;z_1,\lambda_0] = f[x,y] * h[x,y;z_1,\lambda_0]$$

$$= (K_0)\left(\mathrm{STEP}[x] * e^{+i\pi\frac{x^2}{\lambda_0 z_1}}\right)\left(\underbrace{1[y] * e^{+i\pi\frac{y^2}{\lambda_0 z_1}}}_{\delta[\cancel{x,y}]\cdot}\right)$$

12/2 17

$$1(y) \propto e^{+i\pi \frac{y^2}{(\lambda_0 z_1)^2}} \xrightarrow{\mathcal{F}_1} \delta(\eta) \cdot \left|\sqrt{\lambda_0 z_1}\right| e^{+i\frac{\pi}{4}} e^{-i\pi \lambda_0 z_1 \eta^2}$$

$$= \sqrt{\lambda_0 z_1}\, e^{+i\frac{\pi}{4}} \quad \delta(\eta)\, e^{-i\pi \lambda_0 z_1 \eta^2}$$

$$= \left(\sqrt{\lambda_0 z_1}\; e^{+i\frac{\pi}{4}}\right) \delta(\eta - 0)\, \cancel{e^{-i\pi \lambda_0 z_1 \cdot 0^2}}^{\,1}$$

$$= \sqrt{\lambda_0 z_1}\; e^{+i\frac{\pi}{4}}\; \delta(\eta)$$

$$\sqrt{\lambda_0 z_1}\; e^{+i\frac{\pi}{4}}\, 1(y) \xleftarrow{\mathcal{F}_1^{-1}}$$

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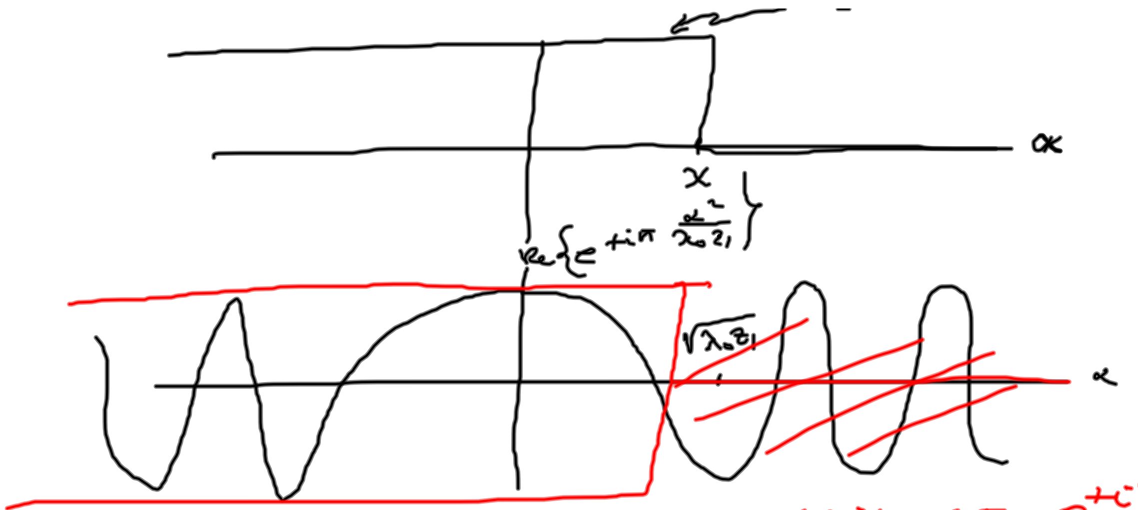
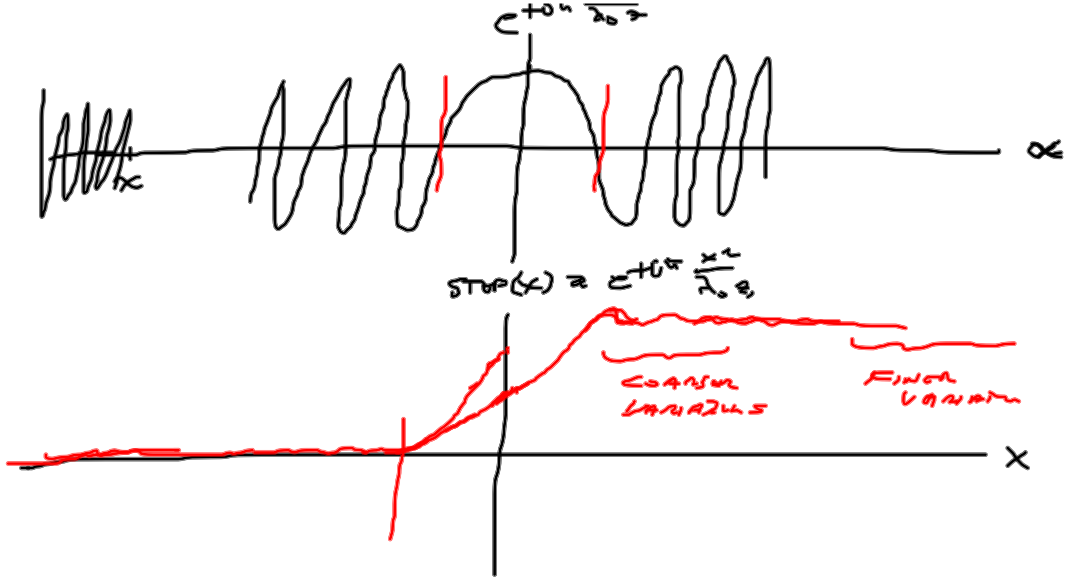
$$\mathrm{STEP}(x) \propto e^{+i\pi \frac{x^2}{\lambda_0 z_1}} = \int_{-\infty}^{+\infty} \mathrm{STEP}(\alpha)\, e^{+i\pi \frac{(x-\alpha)^2}{\lambda_0 z_1}}\, d\alpha$$

$$= \int_{-\infty}^{+\infty} \mathrm{STEP}(x-\alpha)\, e^{+i\pi \frac{\alpha^2}{\lambda_0 z_1}}\, d\alpha$$




AREA FROM $\alpha = -\infty$ to $\alpha = x$ OF $e^{+i\pi \frac{x^2}{\lambda_0 z_1}}$

$$\mathrm{STEP}(x) * e^{+i\pi \frac{x^2}{\lambda_0 z_1}} = \int_{-\infty}^{x} e^{+i\pi \frac{\alpha^2}{\lambda_0 z_1}}\, d\alpha$$



COARSER VARIATIONS
FINER VARIATIONS