IMGS-616 20141

Solutions to

HW #5.

1).

a. $f(x) \Rightarrow F(\xi) = 2\,SINC(2\xi) + 4\,SINC(4\xi)$

b. $h(x) \Rightarrow H(\xi) = SINC(2\xi)\cdot e^{-i2\pi\xi} + i\cdot 2\cdot SINC(2\xi)\cdot e^{i2\pi\xi}$

c. $p(x) \Rightarrow P(\xi) = e^{+i\frac{\pi}{4}}\cdot e^{-i4\pi\xi^2} + e^{-i\frac{\pi}{4}}\, e^{i4\pi\xi^2}$

$$= 2\cdot\cos\left(\pi\left(4\xi^2 - \tfrac{1}{4}\right)\right).$$

d. $r(x) \Rightarrow R(\xi) = e^{-i\frac{\pi}{4}}\, e^{-i4\pi\xi^2} + e^{i\frac{\pi}{4}}\, e^{i4\pi\xi^2}$

$$= -2\sin\left(\pi\left(4\xi^2 - \tfrac{1}{4}\right)\right).$$

e. $u(x) \Rightarrow U(\xi) = 2e^{i\frac{\pi}{4}}\, e^{-i4\pi\xi^2}.$

2).

a). $R(\xi) = \frac{1}{10}\cdot\left(\delta\left(\frac{\xi}{10}\right) * \delta(\xi+10) + \delta\left(\frac{\xi}{10}\right) * \delta(\xi-10)\right).$

$\mathcal{F}^{-1} \Downarrow$

$$r(x) = \frac{1}{10}\cdot 10\cdot\left(e^{-i2\pi\cdot 10\cdot x} + e^{i2\pi\cdot(10)\cdot x}\right)$$

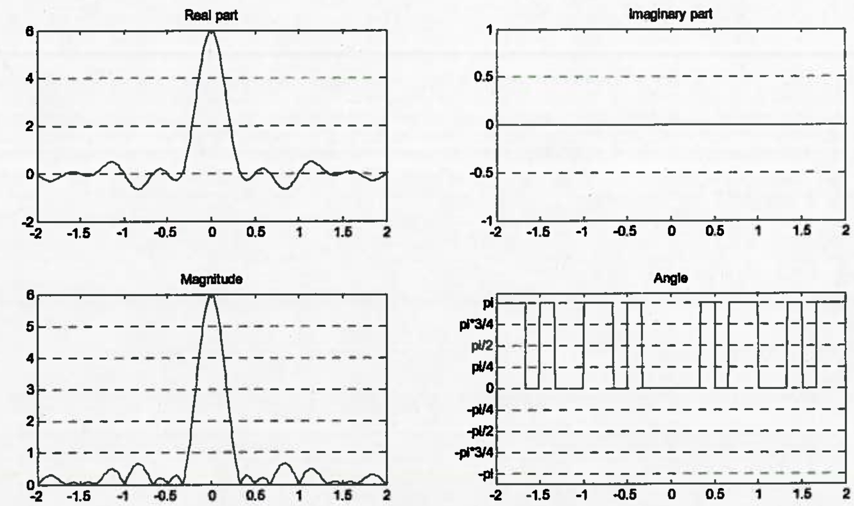
$$= 2\cdot\cos(2\pi\cdot 10\cdot x).$$

b): $g(x) = 2\,TRI(2x)\cdot e^{i2\pi x}.$

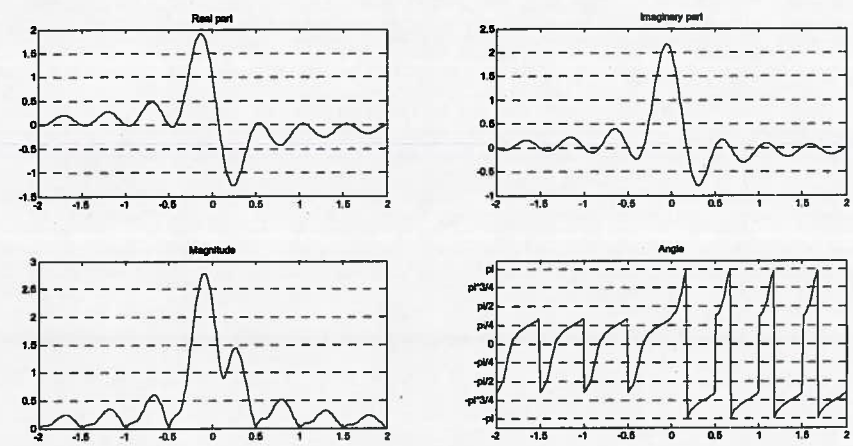
c): $s(x) = SINC^2(x)\cdot e^{-i2\pi x(-1)} + SINC^2(x)\cdot e^{i2\pi\cdot x\cdot 1}$

$$= 2\cdot\cos(2\pi x)\cdot SINC^2(x).$$

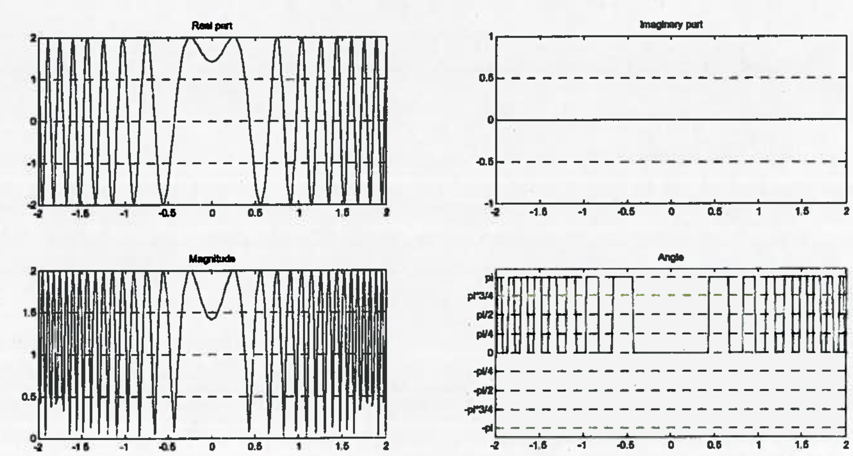
1.(a)


Real part
Imaginary part
Magnitude
Angle

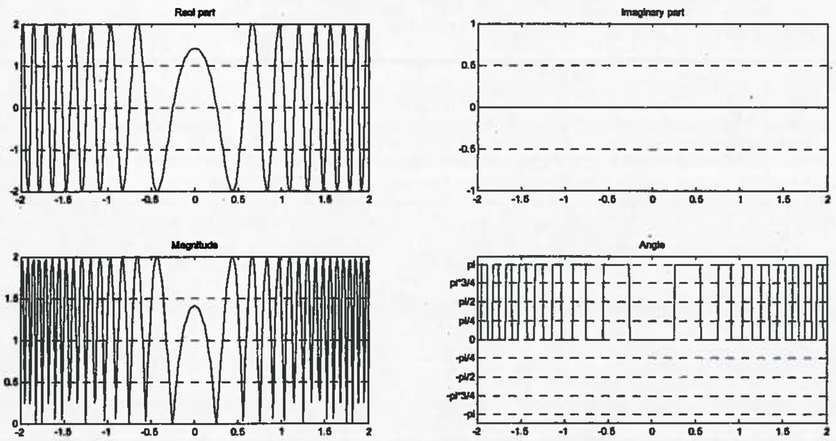
(b)


Real part
Imaginary part
Magnitude
Angle

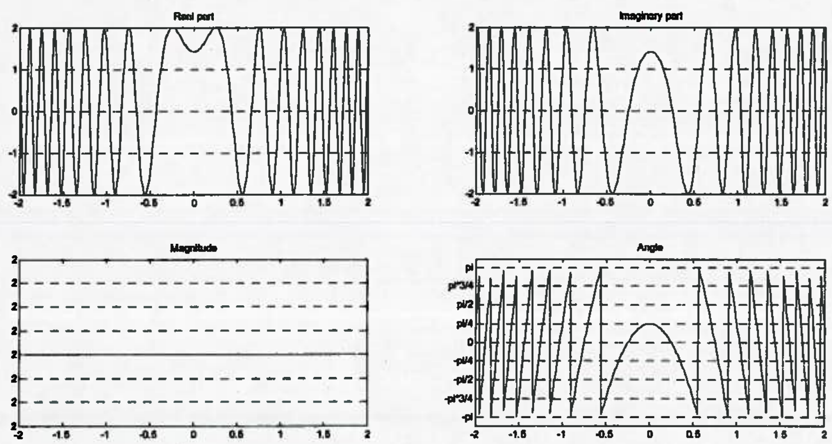
(c)


Real part
Imaginary part
Magnitude
Angle

(d)

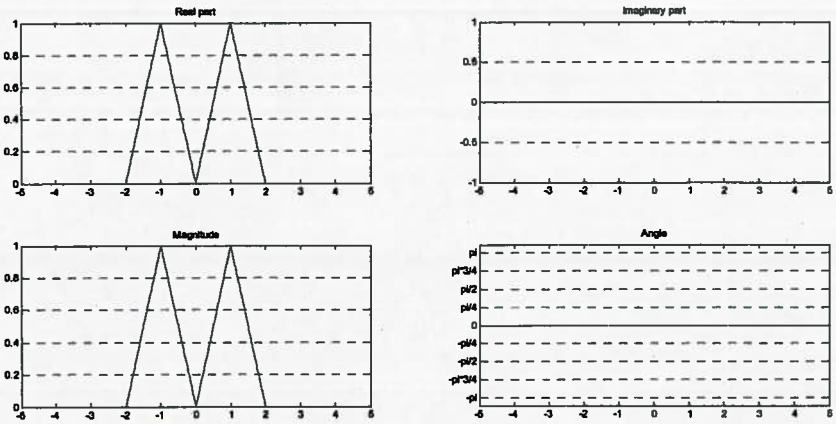

Real part
Imaginary part
Magnitude
Angle

(e)

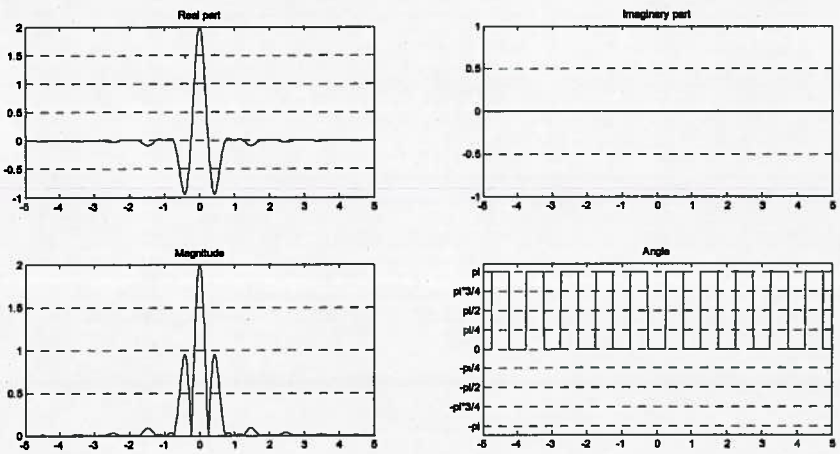

Real part
Imaginary part
Magnitude
Angle

2(c)

S[ξ]


Real part
Imaginary part
Magnitude
Angle

s[x]


Real part
Imaginary part
Magnitude
Angle

3).

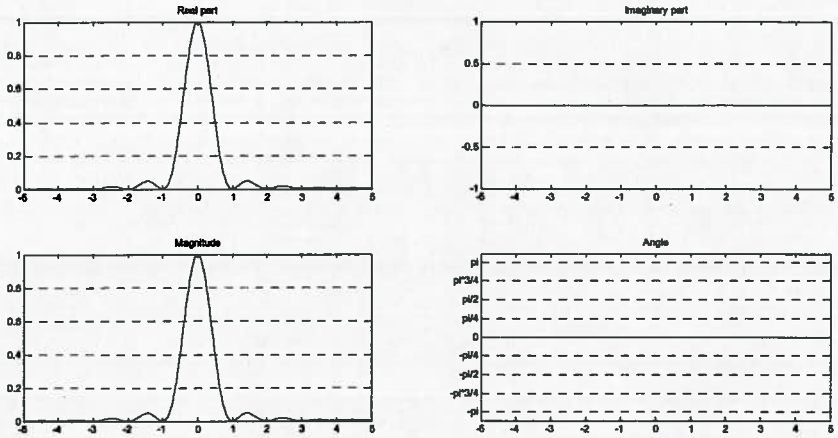
a. $$\mathcal{F}(\text{RECT}(x) * \text{RECT}(x)) = \mathcal{F}(\text{RECT}(x)) \cdot \mathcal{F}(\text{RECT}(x))$$
$$= \text{SINC}(\xi) \cdot \text{SINC}(\xi)$$
$$= \text{SINC}^2(\xi)$$

b. $$\mathcal{F}(\text{RECT}(x-1) * \text{RECT}(x)) = \mathcal{F}(\text{RECT}(x-1)) \cdot \mathcal{F}(\text{RECT}(x))$$
$$= \text{SINC}(\xi) \cdot e^{-i2\pi\xi} \cdot \text{SINC}(\xi)$$
$$= \text{SINC}^2(\xi) \cdot e^{-i2\pi\xi}.$$

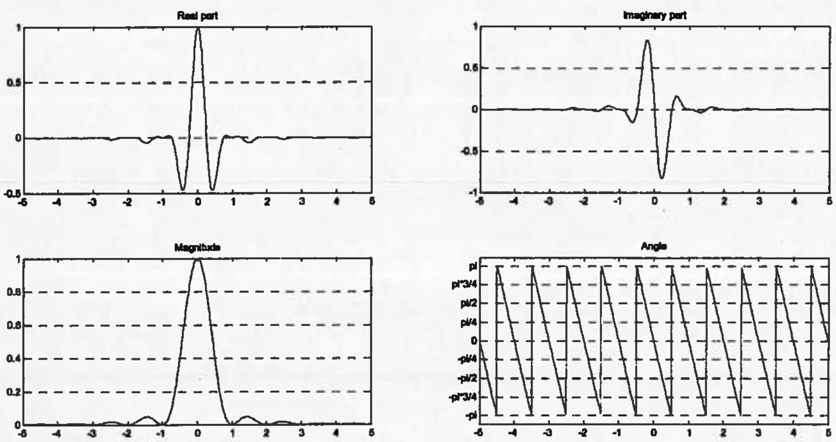
c. $$\mathcal{F}(\text{RECT}(x-1) * \text{RECT}(x+1)) = \mathcal{F}(\text{RECT}(x) * \delta(x-1) * \text{RECT}(x) * \delta(x+1))$$
$$= \mathcal{F}(\text{RECT}(x) * \text{RECT}(x))$$
$$= \text{SINC}^2(\xi)$$

d. $$\mathcal{F}(\text{RECT}(x-1) \star \text{RECT}(x+1)) = \mathcal{F}(\text{RECT}(x-1)) \cdot \mathcal{F}^*(\text{RECT}(x+1))$$
$$= \text{SINC}(\xi) \cdot e^{-i2\pi\xi} \cdot \left(\text{SINC}(\xi) \cdot e^{i2\pi\xi}\right)^*$$
$$= \text{SINC}^2(\xi) \cdot e^{-i4\pi\xi}.$$

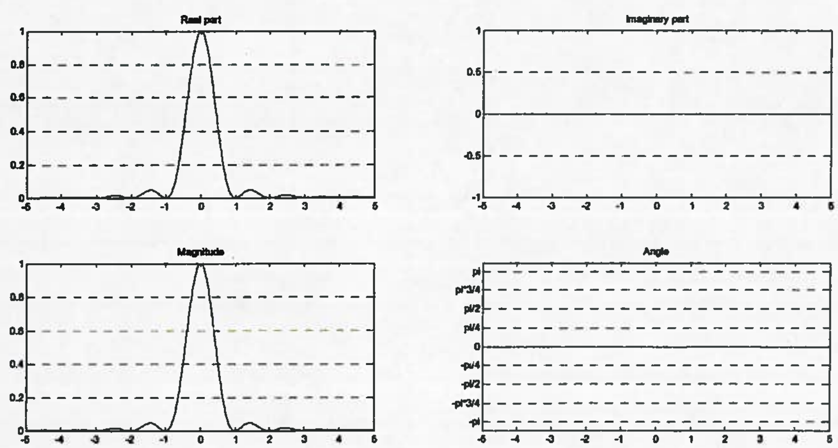
3.(a)


Real part
Imaginary part
Magnitude
Angle
pi
pi*3/4
pi/2
pi/4
0
-pi/4
-pi/2
-pi*3/4
-pi

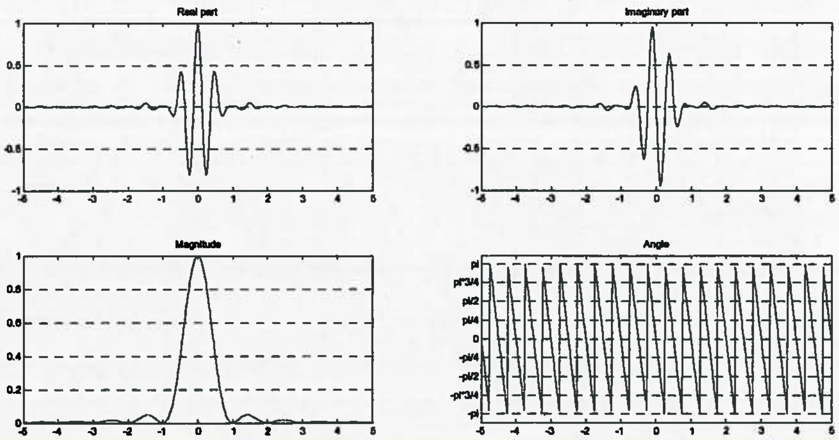
(b)


Real part
Imaginary part
Magnitude
Angle
pi
pi*3/4
pi/2
pi/4
0
-pi/4
-pi/2
-pi*3/4
-pi

(c)


Real part
Imaginary part
Magnitude
Angle
pi
pi*3/4
pi/2
pi/4
0
-pi/4
-pi/2
-pi*3/4
-pi

(d)



Real part
Imaginary part
Magnitude
Angle
pi
pi*3/4
pi/2
pi/4
0
-pi/4
-pi/2
-pi*3/4
-pi

4.

a. $F(\xi) = \text{COMB}(\xi) * 4\,\text{SINC}(4\xi)$

$= \sum_{-\infty}^{+\infty} 4\,\text{SINC}(4(\xi-n))$.

or $F(\xi) = 1 + \cos(4\pi\xi) + 2\cdot\cos(2\pi\xi)$.

b. $G(\xi) = \frac{1}{2}\cdot\text{SINC}(\xi)\cdot\left(\sum_{-\infty}^{+\infty} 4\,\text{SINC}(4(\xi-n))\right)$.

or $= \frac{1}{2}\,\text{SINC}(\xi)\cdot\left(1 + \cos(4\pi\xi) + 2\cos(2\pi\xi)\right)$.

c. $R(\xi) = \text{COMB}(\xi) * 4\,\text{RECT}(4\xi)$

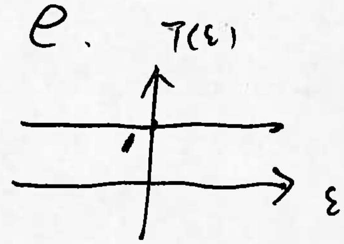
$= \sum_{-\infty}^{+\infty} 4\,\text{RECT}(4(\xi-n))$.

d. $S(\xi) = \sum_{-\infty}^{+\infty} 2\,\text{RECT}(2(\xi-n))$.

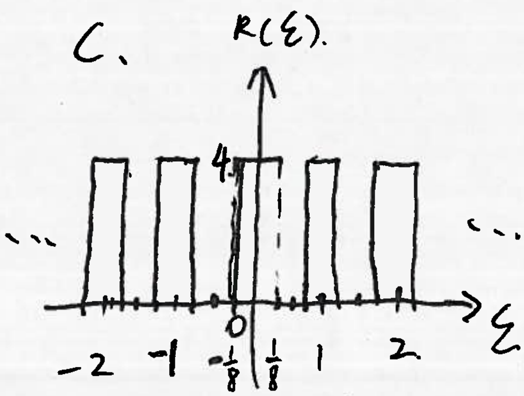
e. $T(\xi) = \sum_{-\infty}^{+\infty} \text{RECT}(\xi-n) = 1\ [\xi]$

f. $U(\xi) = \sum_{-\infty}^{+\infty} \frac{1}{2}\,\text{RECT}(\frac{1}{2}(\xi-n)) = 1\ [\xi]$.

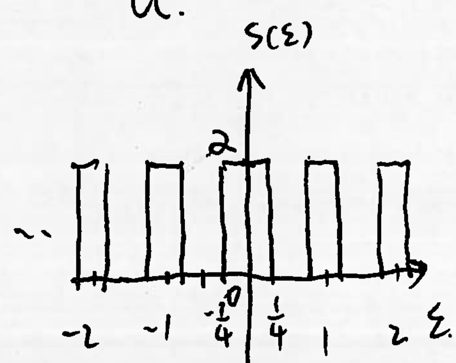
e.



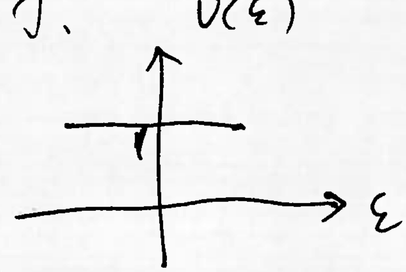
c.



d.

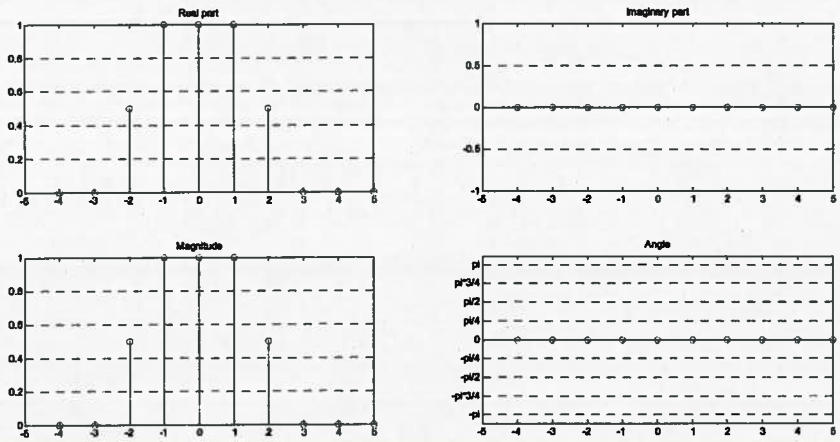


f.

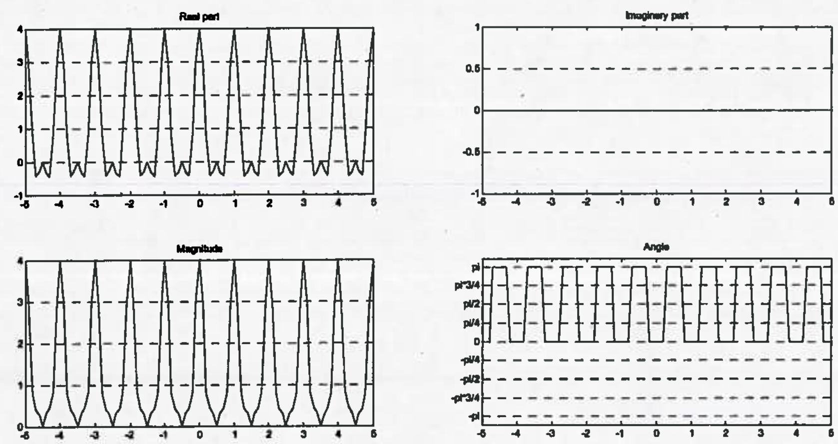


4(a)

f(x)

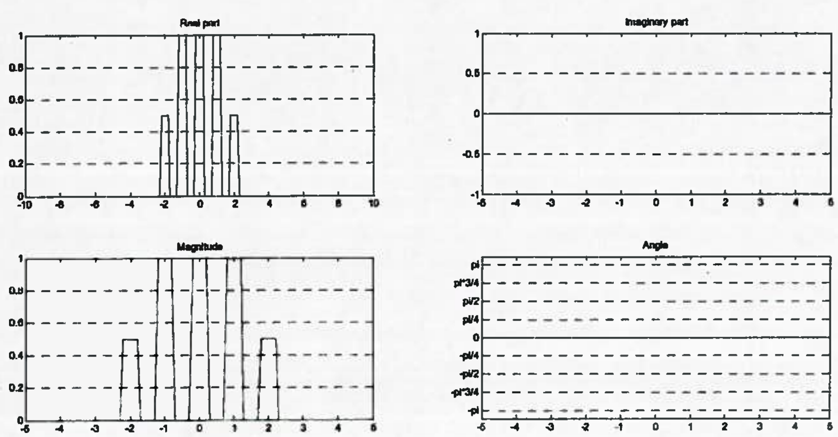

Real part
Imaginary part
Magnitude
Angle

F[ξ]

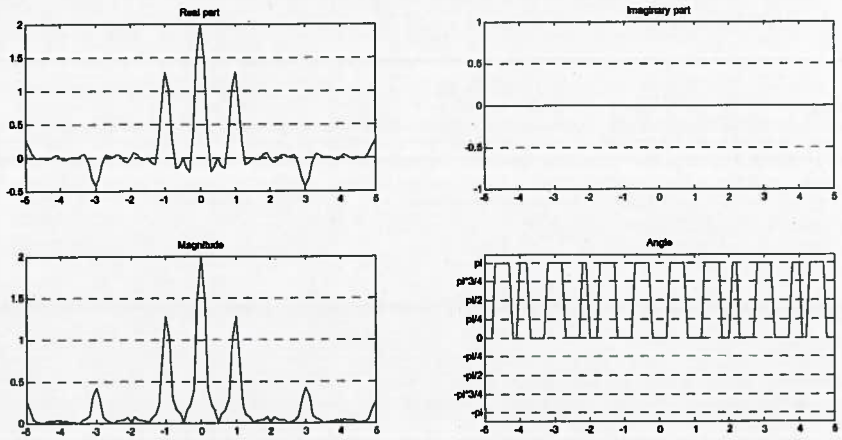

Real part
Imaginary part
Magnitude
Angle

(b)

g[x]


Real part
Imaginary part
Magnitude
Angle

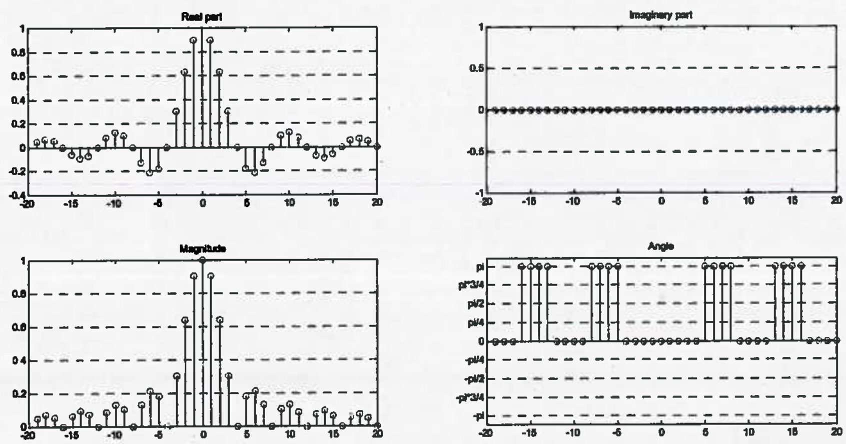
G[ξ]



Real part
Imaginary part
Magnitude
Angle

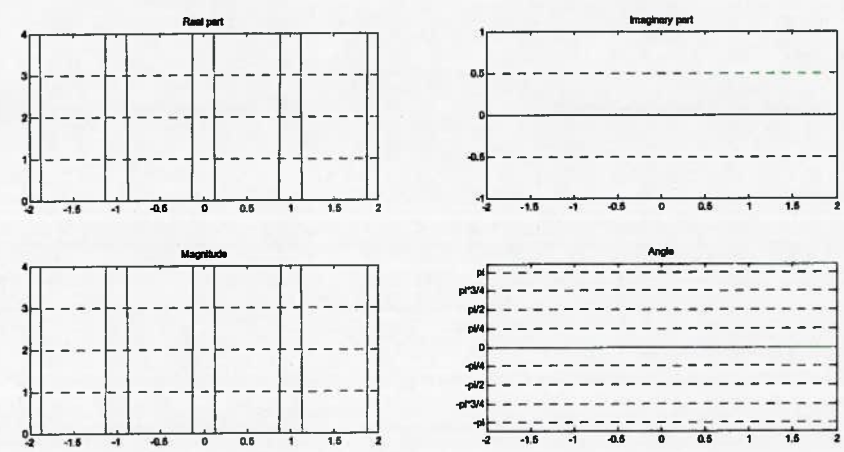
(c)

r[x]



Real part
Imaginary part
Magnitude
Angle

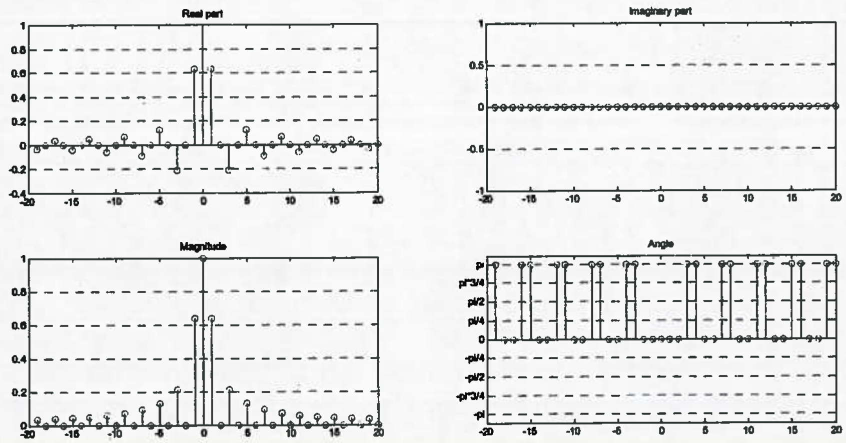
R[ξ]



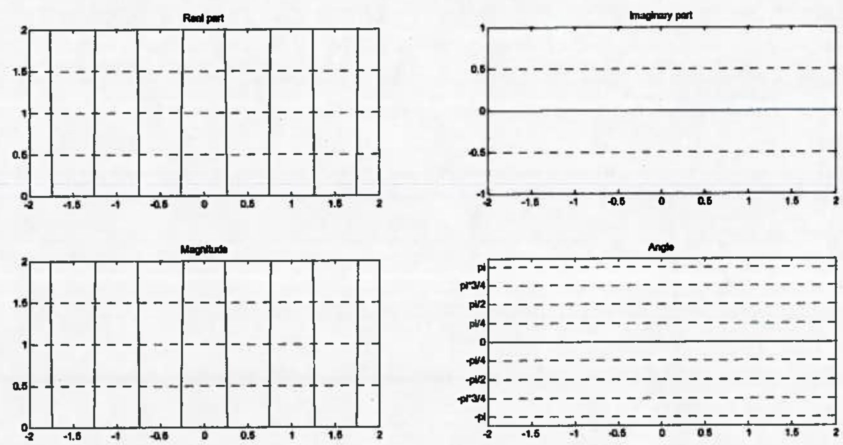
Real part
Imaginary part
Magnitude
Angle

(d)

s[x]

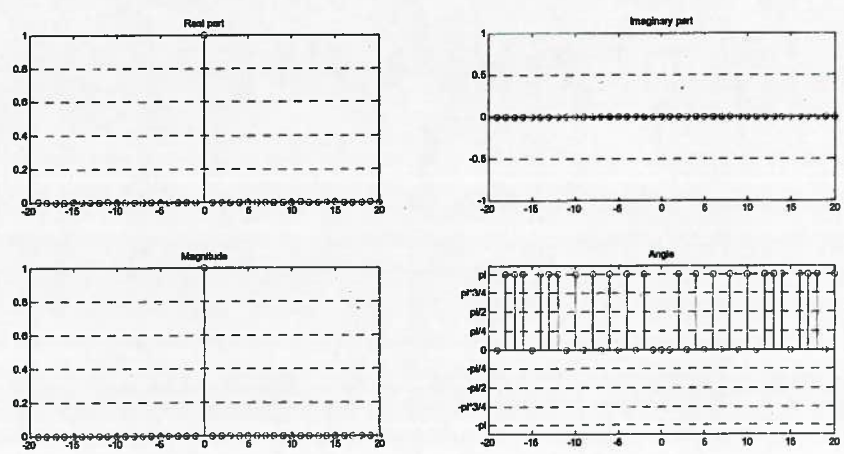

Real part
Imaginary part
Magnitude
Angle

S[ξ]

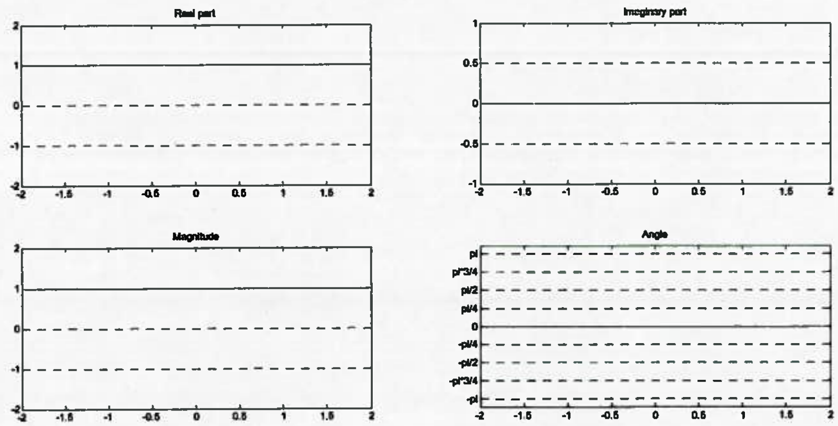

Real part
Imaginary part
Magnitude
Angle

(e)

t[x]

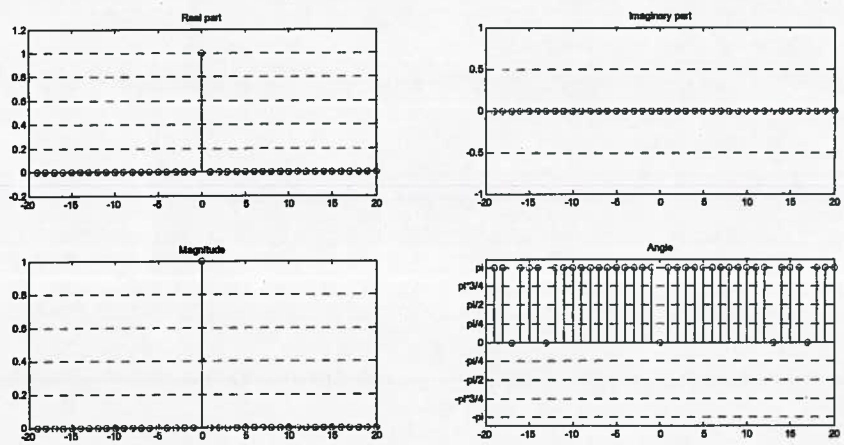

Real part
Imaginary part
Magnitude
Angle

T[ξ]

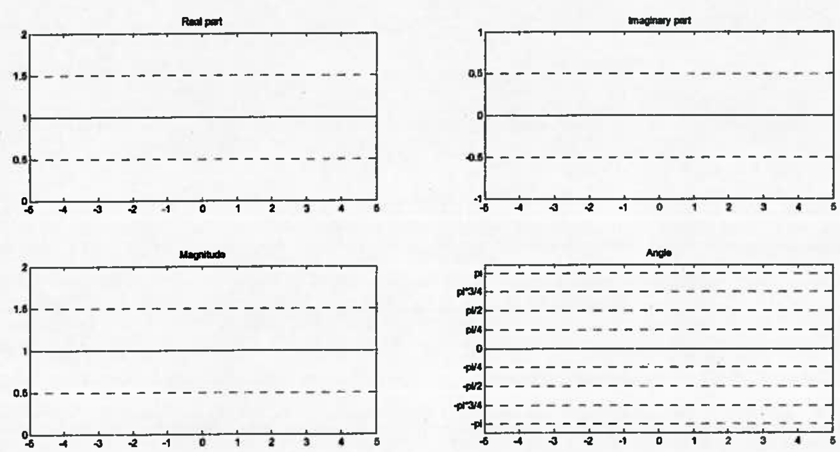

Real part
Imaginary part
Magnitude
Angle

(f)

u[x]


Real part
Imaginary part
Magnitude
Angle

U[ξ]


Real part
Imaginary part
Magnitude
Angle

5.

a). $\mathrm{GAUS}(\frac{x}{3}) * \mathrm{GAUS}(\frac{x}{4}) = \mathcal{F}^{-1}[\mathcal{F}(\mathrm{GAUS}(\frac{x}{3}) * \mathrm{GAUS}(\frac{x}{4}))]$

$= \mathcal{F}^{-1}[3\,\mathrm{GAUS}(3\xi) \cdot 4 \cdot \mathrm{GAUS}(4\xi)]$

$= \mathcal{F}^{-1}[12 \cdot \mathrm{GAUS}(5\xi)]$

$= \frac{12}{5}\mathrm{GAUS}(\frac{x}{5})$.

b).

$\mathrm{SINC}(3x) * \mathrm{SINC}(2x) = \mathcal{F}^{-1}[\mathcal{F}(\mathrm{SINC}(3x) * \mathrm{SINC}(2x))]$

$= \mathcal{F}^{-1}[\frac{1}{3}\mathrm{RECT}(\frac{\xi}{3}) \cdot \frac{1}{2} \cdot \mathrm{RECT}(\frac{\xi}{2})]$

$= \mathcal{F}^{-1}[\frac{1}{6} \cdot \mathrm{RECT}(\frac{\xi}{2})]$

$= \frac{1}{6} \cdot 2 \cdot \mathrm{SINC}(2x)$

$= \frac{1}{3}\mathrm{SINC}(2x)$.