IMGS-616-20141 SS #2

1.

(a). $A_a \cdot x = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$ $A_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b). $A_b \cdot x = \begin{bmatrix} c \\ a \\ d \\ b \end{bmatrix}$ $A_b = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(c) $A_c \cdot x = \begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix}$ $A_c = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(d). $A_d \cdot x = \begin{bmatrix} a+c \\ b+d \\ a+b \\ c+d \end{bmatrix}$ $A_d = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(e) $A_a^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A_b^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$A_c^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ A_d^{-1} does not exist. Because. A_d. blocks non-zero vector.

(f). $A_f = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$A^{\dagger} = (A^T A)^{-1} A^T$

$x = A^{\dagger} \cdot b$

2.

(a). $\hat{a}\cdot x = \dfrac{0+1+0}{\sqrt{1^2+1^2}} = \dfrac{1}{\sqrt{2}}$

(b). $\hat{a}\cdot x = \dfrac{i\cdot 0 + i\cdot i + 0\cdot i}{\sqrt{i\cdot(-i) + i(-i)}} = \dfrac{1}{\sqrt{2}}$

(c). $\hat{a}\cdot x = \dfrac{1\cdot(1+i) + (1-i)\cdot 1 + (1+i)(1-i)}{\sqrt{1^2 + (1+i)(1-i) + (1-i)(1+i)}} = \dfrac{4}{\sqrt{5}}$

3.

(e): for A_1:

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 = 0 \qquad \lambda_1 = \lambda_2 = \lambda_3 = 1$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \alpha_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \alpha_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

for A_2:

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = (1-\lambda)^2\cdot(-\lambda) = 0 \qquad \lambda_1 = \lambda_2 = 1 \quad \lambda_3 = 0$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \alpha_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \alpha_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

for A_3:

$$\begin{vmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & -1 \\ -1 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 - 1 = 0. \qquad \begin{aligned} \lambda_1 &= \tfrac{3}{2} + \tfrac{\sqrt{3}}{2} i \\ \lambda_2 &= 0 \\ \lambda_3 &= \tfrac{3}{2} - \tfrac{\sqrt{3}}{2} i \end{aligned}$$

$$\alpha_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} -\frac{1}{2} - \frac{\sqrt{3}}{2} i \\ -\frac{1}{2} + \frac{\sqrt{3}}{2} i \\ 1 \end{bmatrix} \quad \alpha_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \alpha_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ \frac{1}{2} - \frac{\sqrt{3}}{2} i \\ \frac{1}{2} + \frac{\sqrt{3}}{2} i \end{bmatrix}$$

(f).

$$D_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

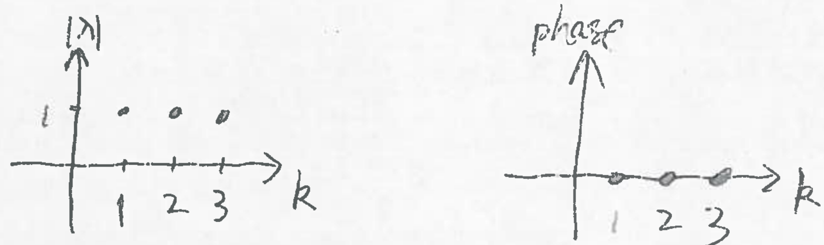
$$D_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} -\frac{1}{2}-\frac{\sqrt{3}}{2}i & 1 & -1 \\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i & 1 & \frac{1}{2}-\frac{\sqrt{3}}{2}i \\ 1 & 1 & \frac{1}{2}+\frac{\sqrt{3}}{2}i \end{bmatrix}$$

(g).

$$\Lambda_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \Lambda_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

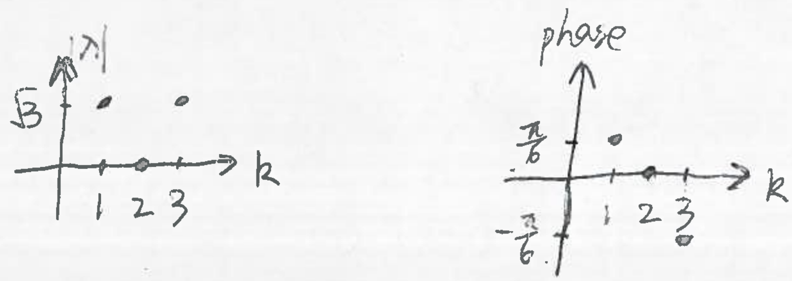
$$\Lambda_3 = \begin{bmatrix} \frac{3}{2}+\frac{\sqrt{3}}{2}i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2}-\frac{\sqrt{3}}{2}i \end{bmatrix}$$

(h). A_1:



A_2:

A_3



(i).

4.

(a). V_1:
$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ & & 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ & & & 1/2 & 1/2 & 0 & 0 & 0 \\ & & & & 1/2 & 1/2 & 0 & 0 \\ & 0 & & & & 1/2 & 1/2 & 0 \\ & & & & & & 1/2 & 1/2 \\ 1/2 & & & & & & & 1/2 \end{bmatrix}$$

V_2:
$$\begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & & & & \\ & 1/3 & 1/3 & 1/3 & & & 0 & \\ & & 1/3 & 1/3 & 1/3 & & & \\ & & & 1/3 & 1/3 & 1/3 & & \\ & 0 & & & 1/3 & 1/3 & 1/3 & \\ & & & & & 1/3 & 1/3 & 1/3 \\ 1/3 & & & & & & 1/3 & 1/3 \\ 1/3 & 1/3 & & & & & & 1/3 \end{bmatrix}$$

V_3
$$\begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 & & & 0 & \\ & 1/4 & 1/4 & 1/4 & 1/4 & & & \\ & & 1/4 & 1/4 & 1/4 & 1/4 & & \\ & & & 1/4 & 1/4 & 1/4 & 1/4 & \\ & 0 & & & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & & & & & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & & & & & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & & & & & 1/4 \end{bmatrix}$$

V_4:
$$\begin{bmatrix} 1/2 & 0 & 1/2 & & & & & \\ & 1/2 & 0 & 1/2 & & & 0 & \\ & & 1/2 & 0 & 1/2 & & & \\ & & & 1/2 & 0 & 1/2 & & \\ & & & & 1/2 & 0 & 1/2 & \\ & 0 & & & & 1/2 & 0 & 1/2 \\ 1/2 & & & & & & 1/2 & 0 \\ 0 & 1/2 & & & & & & 1/2 \end{bmatrix}$$

V_5:
$$\begin{bmatrix} 1 & -1 & & & & & & \\ & 1 & -1 & & & & & \\ & & 1 & -1 & & & 0 & \\ & & & 1 & -1 & & & \\ & 0 & & & 1 & -1 & & \\ & & & & & 1 & -1 & \\ & & & & & & 1 & -1 \\ -1 & & & & & & & 1 \end{bmatrix}$$

V_6:
$$\begin{bmatrix} 1 & -2 & 1 & & & & & \\ & 1 & -2 & 1 & & & 0 & \\ & & 1 & -2 & 1 & & & \\ & & & 1 & -2 & 1 & & \\ & & & & 1 & -2 & 1 & \\ & 0 & & & & 1 & -2 & 1 \\ 1 & & & & & & 1 & -2 \\ -2 & 1 & & & & & & 1 \end{bmatrix}$$

V_7:
$$\begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} & & & & & \\ & -\frac{1}{2} & 1 & -\frac{1}{2} & & & 0 & \\ & & -\frac{1}{2} & 1 & -\frac{1}{2} & & & \\ & & & -\frac{1}{2} & 1 & -\frac{1}{2} & & \\ & & & & -\frac{1}{2} & 1 & -\frac{1}{2} & \\ & 0 & & & & -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & & & & & & -\frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & & & & & & -\frac{1}{2} \end{bmatrix}$$

(b)

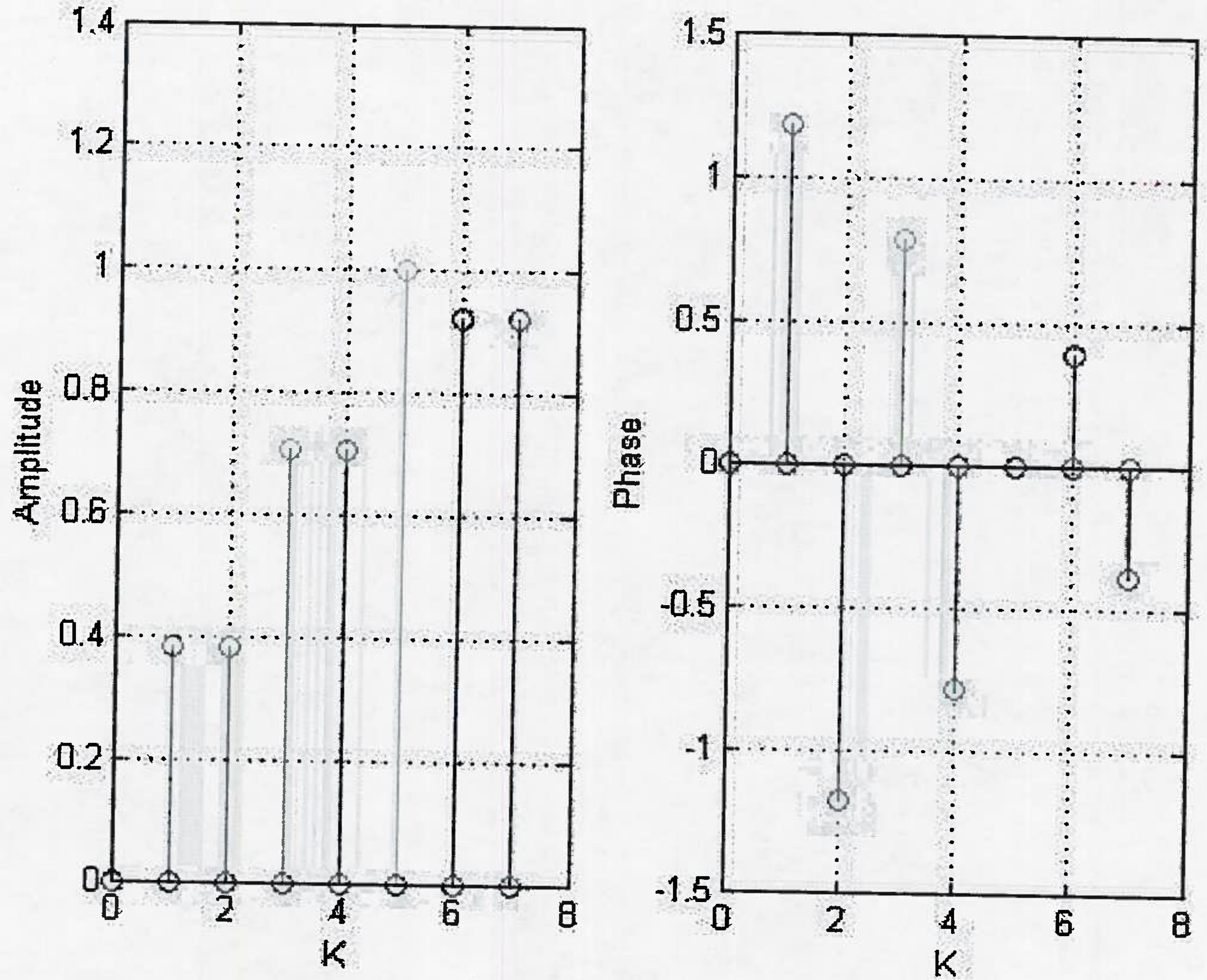
$$\alpha_1 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \alpha_2 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i\frac{\pi}{4}} \\ e^{i\frac{2\pi}{4}} \\ e^{i\frac{3\pi}{4}} \\ e^{i\frac{4\pi}{4}} \\ e^{i\frac{5\pi}{4}} \\ e^{i\frac{6\pi}{4}} \\ e^{i7\pi/4} \end{bmatrix} \quad \alpha_3 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i\pi/2} \\ e^{i2\pi/2} \\ e^{i3\pi/2} \\ e^{i4\pi/2} \\ e^{i5\pi/2} \\ e^{i6\pi/2} \\ e^{i7\pi/2} \end{bmatrix}$$

$$\alpha_4 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i3\pi/4} \\ e^{i6\pi/4} \\ e^{i9\pi/4} \\ e^{i12\pi/4} \\ e^{i15\pi/4} \\ e^{i18\pi/4} \\ e^{i21\pi/4} \end{bmatrix} \quad \alpha_5 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i\pi} \\ e^{i2\pi} \\ e^{i3\pi} \\ e^{i4\pi} \\ e^{i5\pi} \\ e^{i6\pi} \\ e^{i7\pi} \end{bmatrix} \quad \alpha_6 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i5\pi/4} \\ e^{i10\pi/4} \\ e^{i15\pi/4} \\ e^{i20\pi/4} \\ e^{i25\pi/4} \\ e^{i30\pi/4} \\ e^{i35\pi/4} \end{bmatrix}$$

$$\alpha_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i7\pi/4} \\ e^{i14\pi/4} \\ e^{i21\pi/4} \\ e^{i28\pi/4} \\ e^{i35\pi/4} \\ e^{i42\pi/4} \\ e^{i48\pi/4} \end{bmatrix} \quad \alpha_7 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ e^{i3\pi/2} \\ e^{i6\pi/2} \\ e^{i9\pi/2} \\ e^{i12\pi/2} \\ e^{i15\pi/2} \\ e^{i18\pi/2} \\ e^{i21\pi/2} \end{bmatrix}$$

(c) $D = [\alpha_1, \alpha_2, \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6\ \alpha_7\ \alpha_8]$

A1


Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
K
Phase
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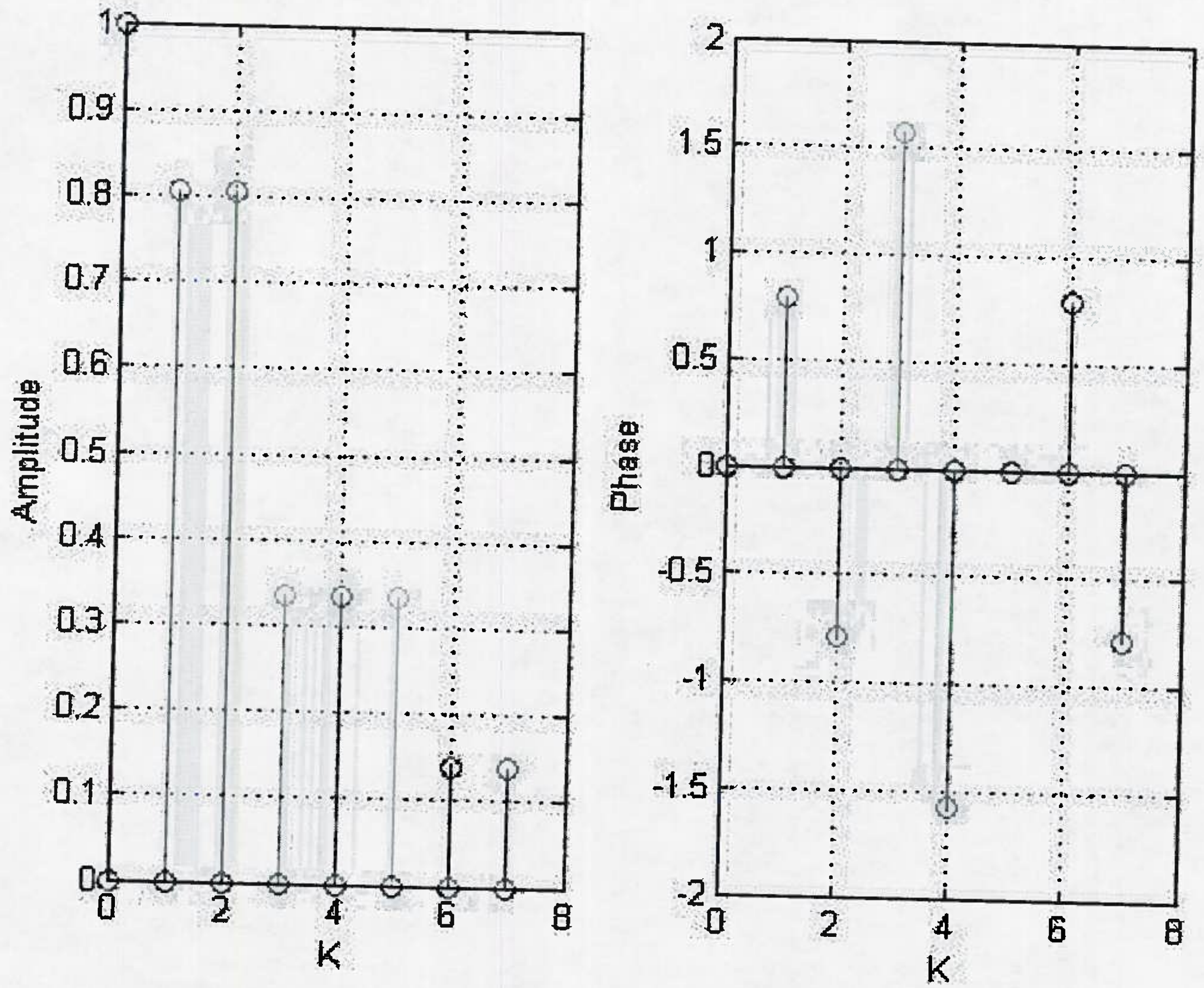
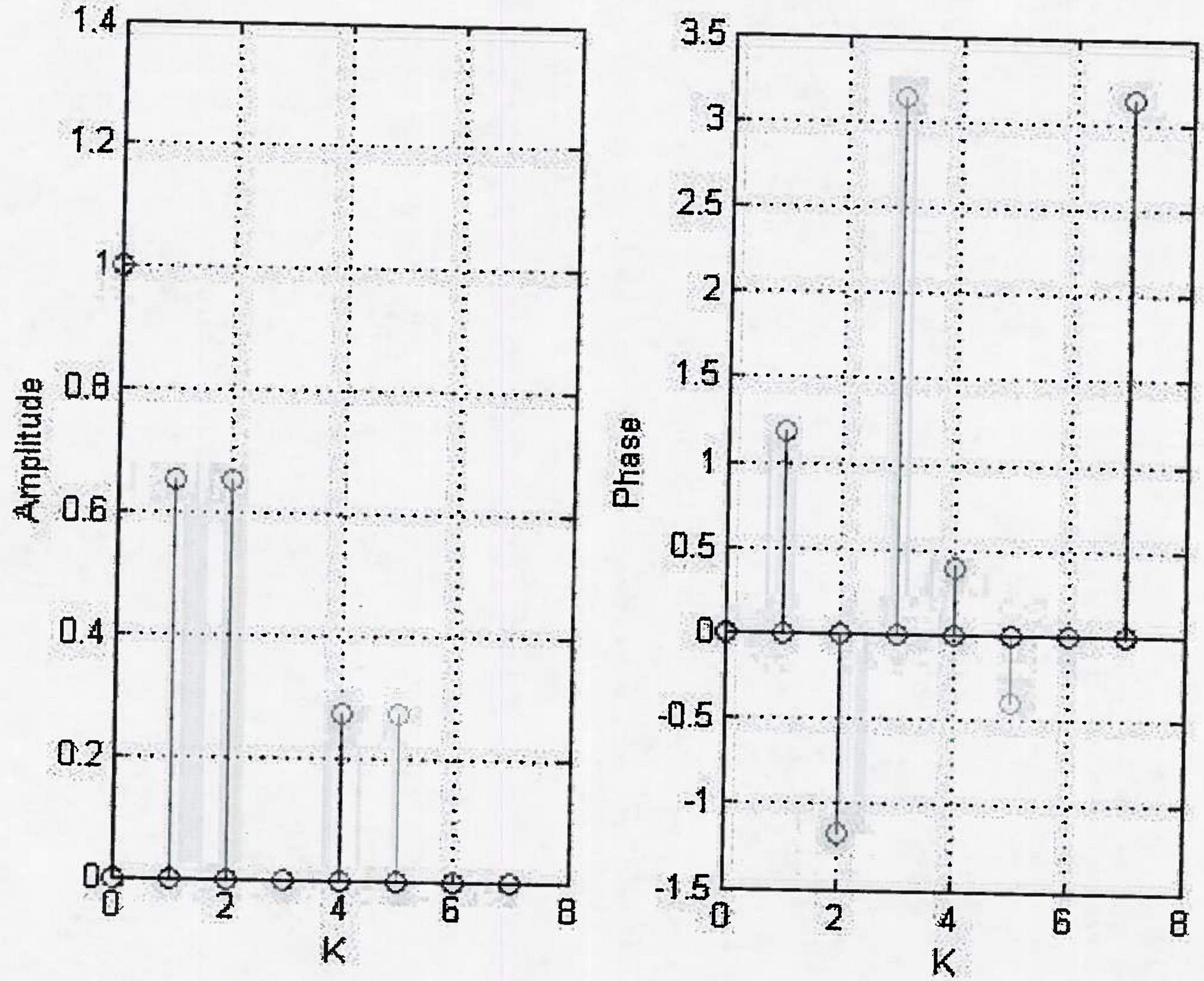
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
K

A 2



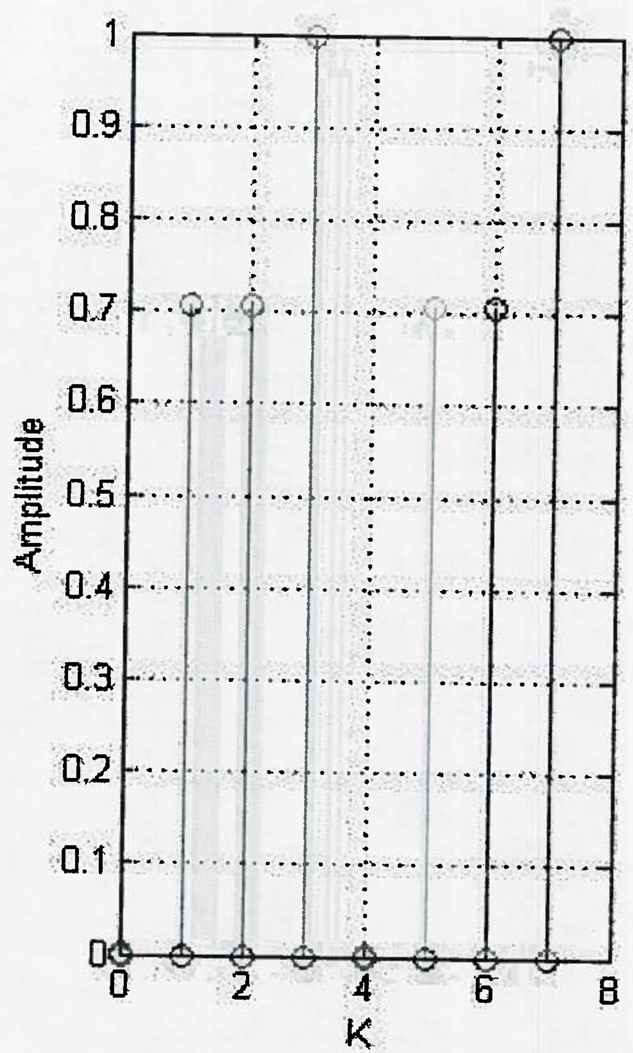
Amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2
4
6
8
K
Phase
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
0
2
4
6
8
K

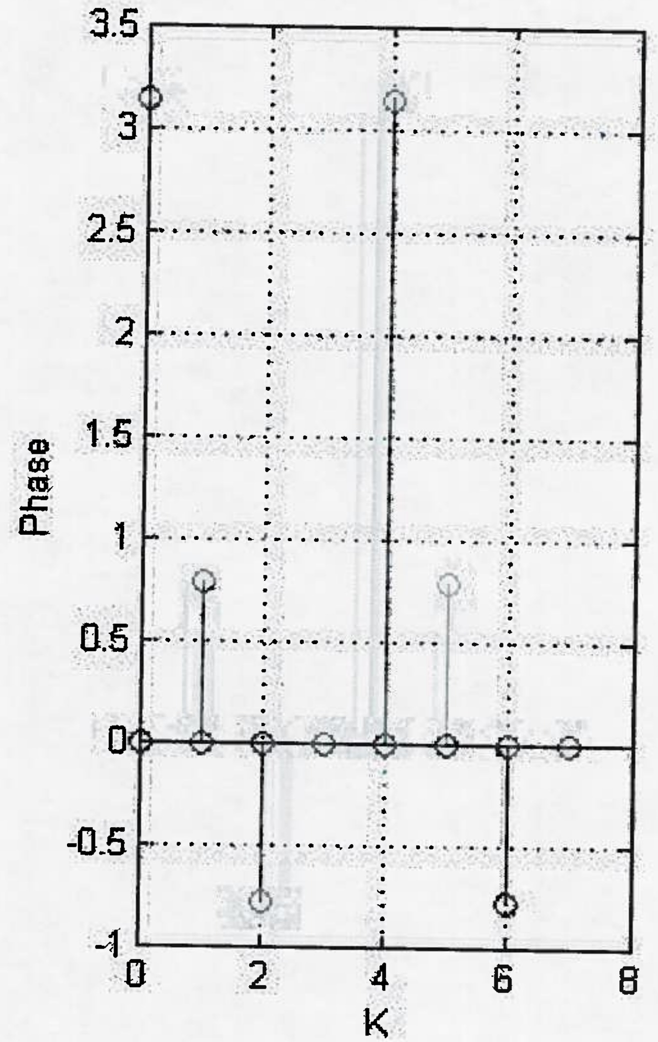
A3



Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
K
Phase
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
K

A4


Amplitude
K
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
4
6
8


Phase
K
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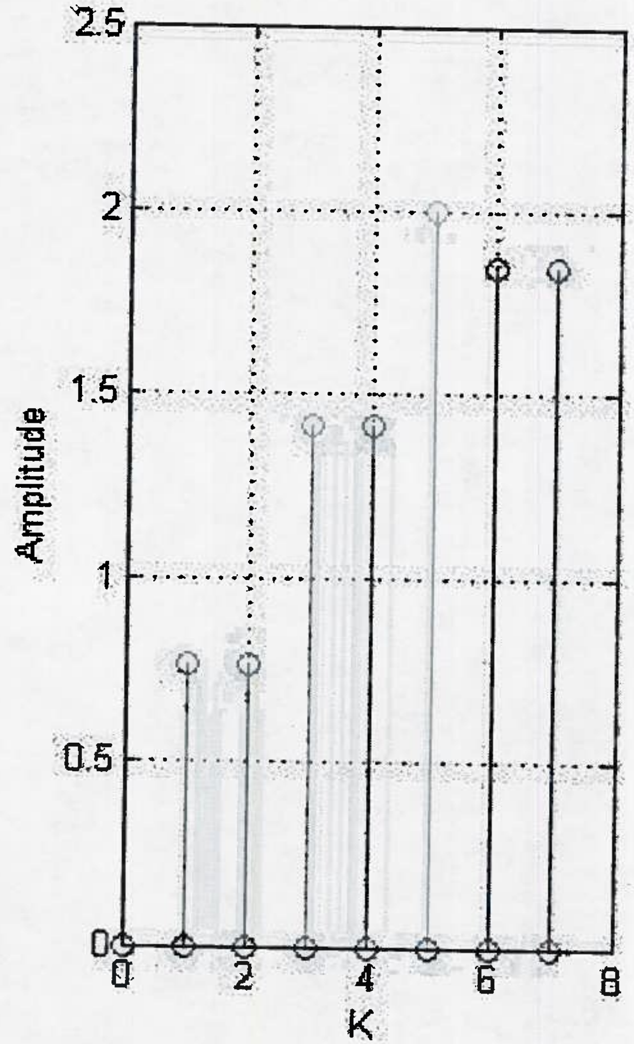
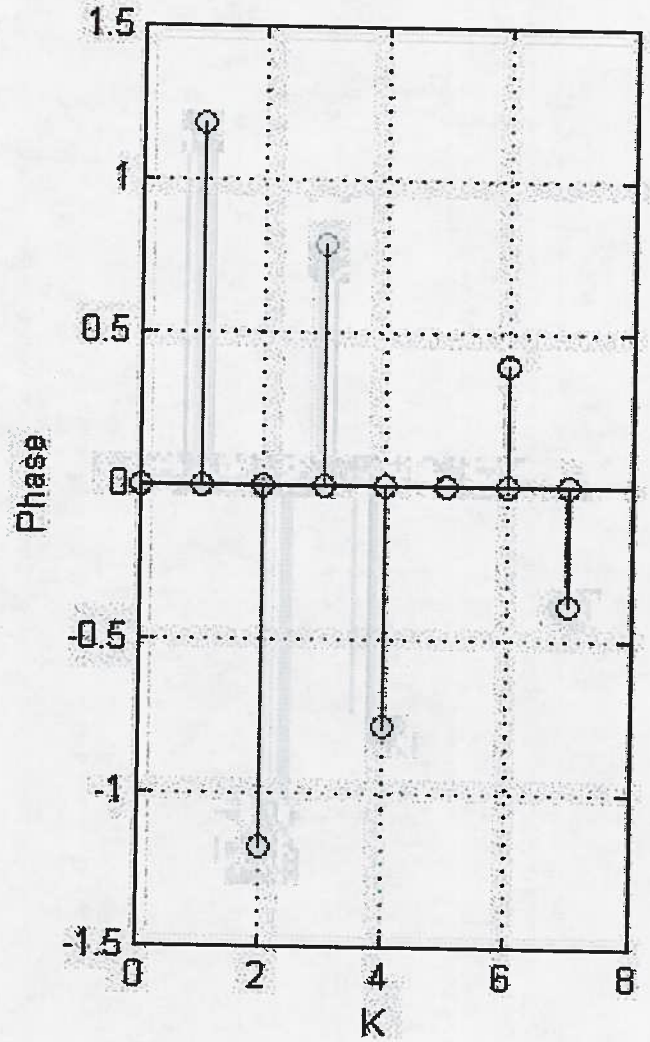
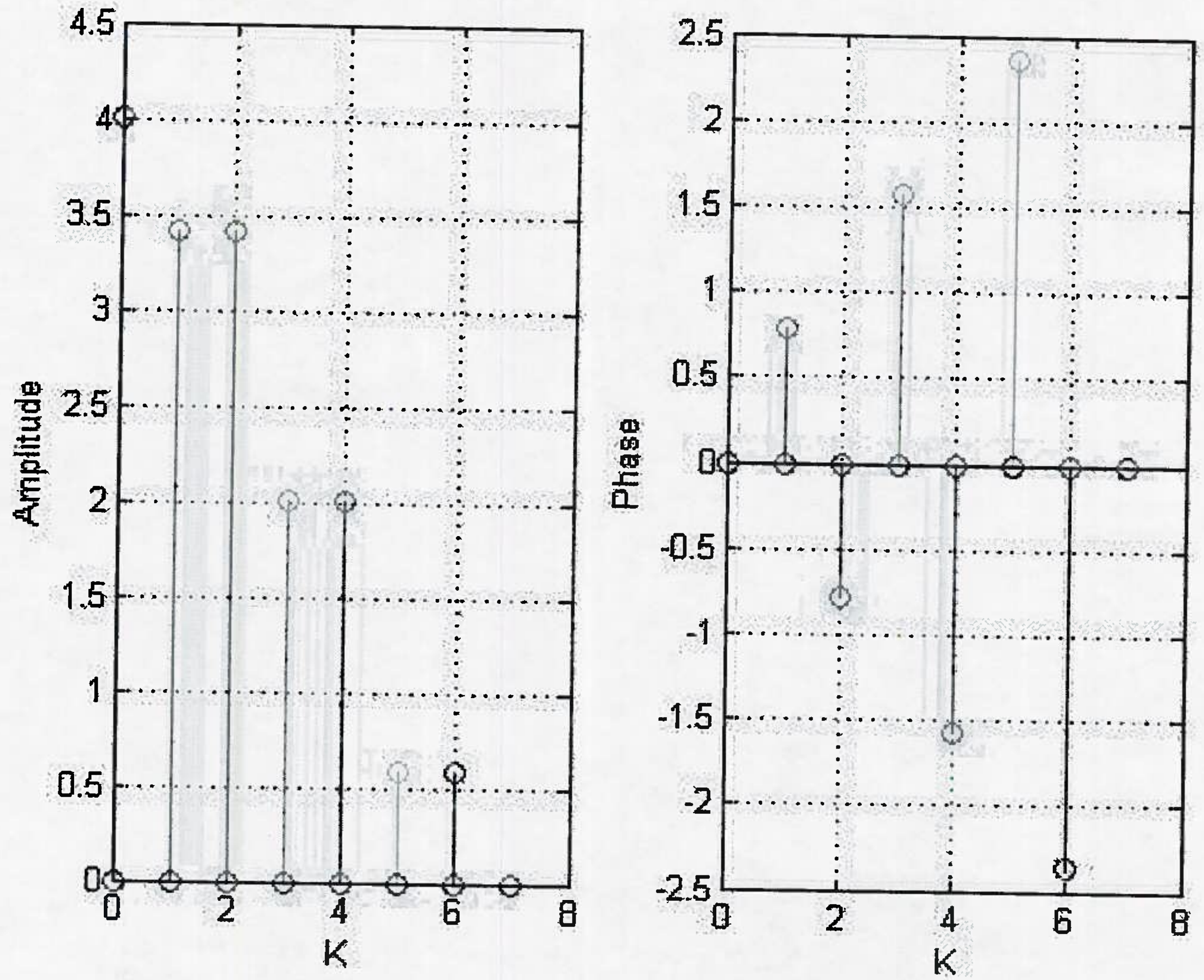
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
2
4
6
8

A5





A6



Amplitude
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
K
Phase
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
2
4
6
8
K

A7