SIMG-616 Homework #1 Due 9/2/2014 (T)

0. Read Chapters 1-4 in the book and spend some time in the RIT library looking through other books on the subject of Fourier mathematics, linear systems, etc. from the bibliography.

1. Consider two spatial sinusoids with the same spatial frequency ξ_0 (measured in "cycles per unit length"), but arbitrary amplitudes A_1 and A_2 and arbitrary phase angles ϕ_1 and ϕ_2:

   (a) Prove that the sum of these two sinusoids is a sinusoid with that same frequency ξ_0.

   (b) Find the expression that determines relates the amplitude A and phase ϕ of the summation sinusoid from the amplitudes and initial phase angles of the component sinusoids.

2. For a sinusoidal functions whose phase is a power of the coordinate:

$$f[x] = \cos\left[\pi\left(\frac{x}{\alpha_0}\right)^n + \phi_0\right]$$

   (a) If the variable x has units of length, what are the "units" of the parameter α_0?

   (b) Graph the function for $\alpha = 1$, $\phi_0 = 0$, and $n = 1$, 2, and 3.

   (c) Graph the function for $\alpha = 1$, $\phi_0 = -\frac{\pi}{2}$, and $n = 1$, 2, and 3.

   (d) Determine the equation for the spatial frequency of $f[x]$.

3. Find the lengths of the 3-D vectors:

   (a) a. $\begin{pmatrix} +1 \\ +2 \\ -1 \end{pmatrix}$ b. $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$ c. $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$

4. In each case, find the length of the input vector $\underline{\mathbf{x}}$ in the direction of the "reference" vector $\underline{\mathbf{a}}$ (this is the "projection of $\underline{\mathbf{x}}$ onto $\underline{\mathbf{a}}$).

   (a) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   (b) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (c) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

5. Find k so that the two vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ are orthogonal:

$$\underline{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \underline{\mathbf{b}} = \begin{pmatrix} -1 \\ k \\ -3 \end{pmatrix}$$

1.

(a) $f_1 = A_1 \sin(2\pi\xi_0 + \phi_1)$. $\quad f_2 = A_2 \sin(2\pi\xi_0 + \phi_2)$

$$f = f_1 + f_2 = A_1 \sin(2\pi\xi_0)\cdot\cos\phi_1 + A_1 \sin\phi_1 \cdot \cos(2\pi\xi_0) + A_2 \sin(2\pi\xi_0)\cdot\cos\phi_2 + A_2 \sin\phi_2 \cdot \cos(2\pi\xi_0)$$

$$= (A_1\cos\phi_1 + A_2\cos\phi_2)\cdot\sin(2\pi\xi_0) + (A_1\sin\phi_1 + A_2\sin\phi_2)\cdot\cos(2\pi\xi_0)$$

assume such. A, ϕ, expression shown below.

$$f = A\sin(2\pi\xi_0 + \phi)$$

$$A^2 = (A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2$$

$$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

(b).

$$A = \left((A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2\right)^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}\right)$$

2.

(a). argument of a sinusoidal function is unitless.

α_0, x have the same units.

here, α_0 has units of length.

(b). $\alpha = 1$. $\phi_0 = 0$.

$n=1$. $f(x) = \cos(\pi x)$

$n=2$ $f(x) = \cos(\pi x^2)$

$n=3$ $f(x) = \cos(\pi x^3)$

(c). $\alpha = 1$ $\phi_0 = -\frac{\pi}{2}$

$n=1$ $f(x) = \sin(\pi x)$

$n=2$ $f(x) = \sin(\pi x^2)$

$n=3$ $f(x) = \sin(\pi x^3)$.

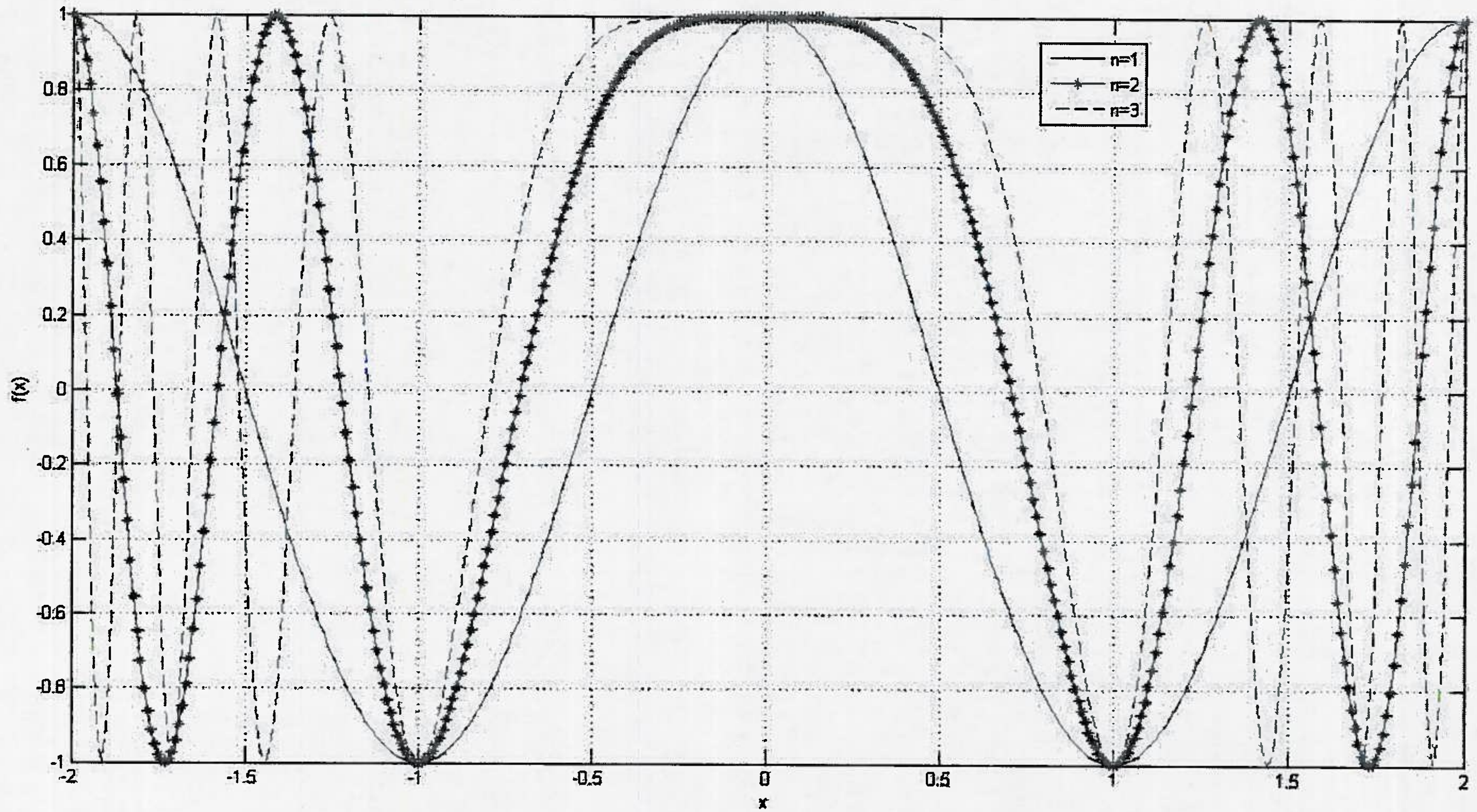
(d).

$$\frac{1}{2\pi} \cdot \frac{\partial \phi}{\partial x} = \frac{1}{2\pi} \frac{\partial}{\partial x}\left(\pi \cdot \left(\frac{x}{\alpha_0}\right)^n + \phi_0\right). = \frac{1}{2\pi} \cdot \pi \cdot n \cdot \left(\frac{x}{\alpha_0}\right)^{n-1} \cdot \frac{1}{\alpha_0}.$$

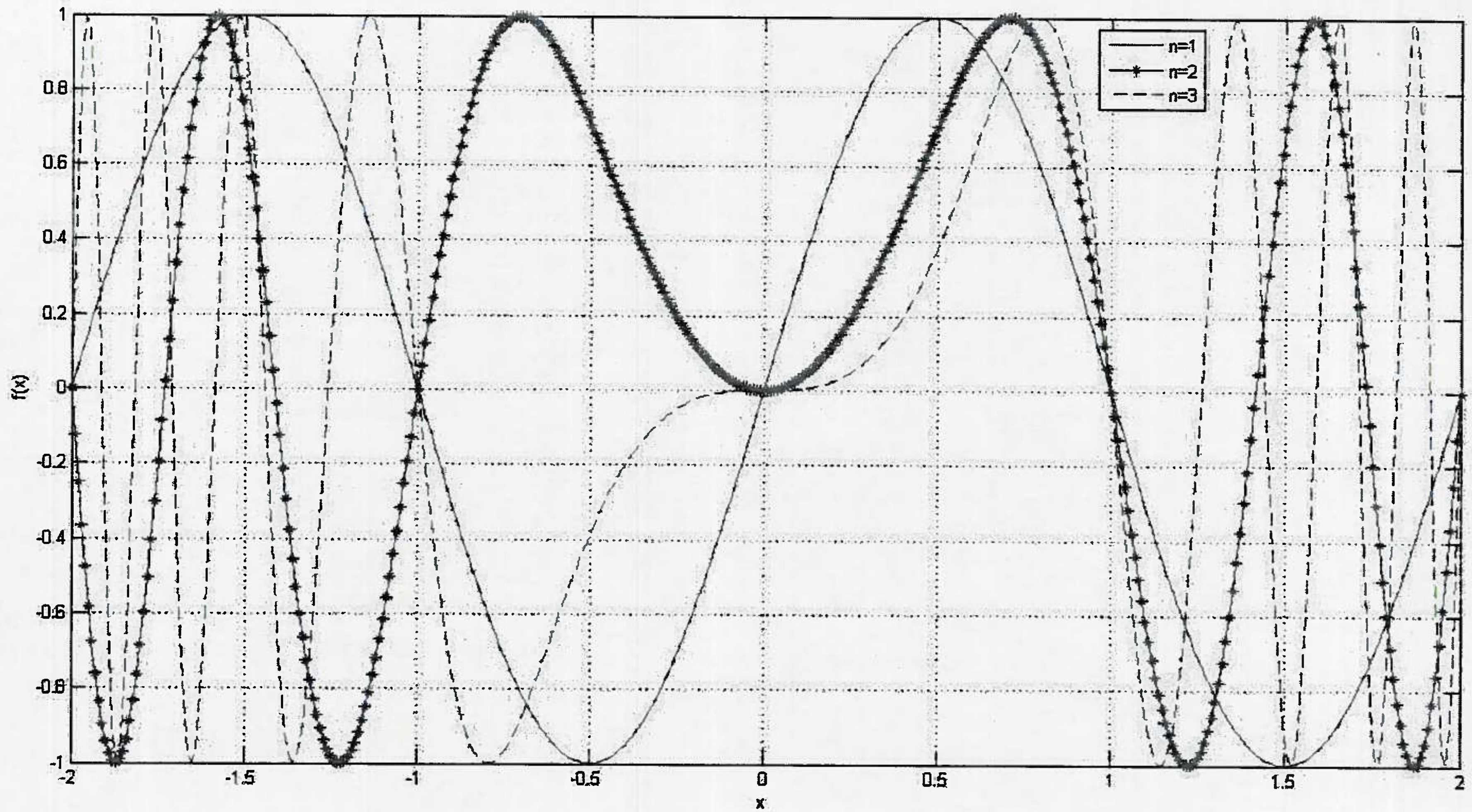
$$= \frac{n}{2\alpha_0} \cdot \left(\frac{x}{\alpha_0}\right)^{n-1}$$

$$= \frac{n \cdot x^{n-1}}{2\alpha_0^n}.$$

2. (b).



2. (c).



3.

(a) $\left((+1)^2+(+2)^2+(-1)^2\right)^{1/2}=\sqrt{6}$

(b) $\left(\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2\right)^{1/2}=1$

(c) $\left(\left(\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2\right)^{1/2}=1$

4.

(a). $\frac{x\cdot a}{\|a\|}=\frac{1\cdot 1+1\cdot 0}{1}=1$

(b) $\frac{x\cdot a}{\|a\|}=\frac{1\cdot 1+0\cdot 1}{(1^2+1^2)^{1/2}}=\frac{1}{\sqrt{2}}$

(c) $\frac{x\cdot a}{\|a\|}=\frac{1\cdot 1+1\cdot 0+1\cdot 1}{(1^2+0^2+1^2)^{1/2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$

5.

$a\cdot b=0.$

$1\cdot(-1)+2\cdot k+3\cdot(-3)=0$

$2k-10=0$

$k=5.$