SIMG-616-20142 EXAM #1 2 October 2014

75 minutes
Six problems (2 @ 10%, 2 @ 20%, 2 @ 40%). Select problems totaling 100% (e.g., 2 @ 10%, 2@20%, 1@40% or 1 @ 20% and 2 @ 40%)
Submit ONLY the selected problems IN NUMERICAL ORDER
Closed Book, No Notes, UNPROGRAMMED Calculators ONLY, TURN OFF ALL PHONES, PAGERS, Ipods, etc.; PUT THEM AWAY
Show your work. If you solve problems "by inspection", then write down the thought process that led to your conclusion. You may leave square roots in expressions without evaluating numerically.

1. (10%) On an Argand diagram for an arbitrary complex number $z_0 = a_0 + i \cdot b_0$, where $a_0 < 0$ and $b_0 < 0$, show the locations of the five complex numbers z, z^*, z^2, $z^{\frac{1}{2}}$,and $z^{-1} \equiv \frac{1}{z}$; the superscript "$*$" denotes the "complex conjugate" of the associated complex number.

2. (10%) For an arbitrary complex number z, find an expression for the imaginary part of $(z^* \cdot z)^*$; the superscript "$*$" again denotes the "complex conjugate" of the associated complex number.

3. (20%) Evaluate the discrete Fourier transform of each of these 4-element vectors with elements indexed from $n = 0$ to $n = N - 1 = 3$.

   (a) $\underline{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\underline{\mathbf{x}}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

   (c) Compare the inputs and outputs of (a) and (b) to comment on the similarities

4. (20%) The eigenvalues of a particular 6×6 circulant matrix:

$$\underline{\mathbf{A}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   satisfy the equation:

$$\lambda^6 - 1 = 0$$

   where the λ_n are generally complex valued. Find expressions for the eigenvalues and associated normalized eigenvectors of $\underline{\mathbf{A}}$ as BOTH real and imaginary parts AND as magnitude and phase.

5. (40%) For the 4×4 circulant matrix listed:

$$\underline{\mathbf{A}} = \begin{bmatrix} +1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \\ -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & +1 \end{bmatrix}$$

   (a) Find expressions for the vectors in the null subspace that are "blocked" by the system AND for the vectors that are "passed" by the system so that the output is nonzero.

   (b) Evaluate the projection of any vector in the null subspace onto any vector "passed" by the system

   (c) Determine if the matrix is invertible and give reasons.

6. (40%) A shift-invariant operation acts on 4 samples of a function $f[n]$ that may be represented as a 4-element vector $\underline{\mathbf{x}}$. For the "first" output pixel , the operator evaluates a weighted sum of the four "input" pixels:

$$g[0] = 0.1 \cdot f[0] + 0.2 \cdot f[1] + 0.3 \cdot f[2] + 0.4 \cdot f[3]$$

   (a) Write down the 4×4 matrix $\underline{\mathbf{A}}$ that implements this operation.

   (b) Evaluate the corresponding 4×4 diagonal matrix $\underline{\mathbf{\Lambda}}$

   (c) Use the result of part (b) to determine if the action of the system is invertible

   (d) If the answer to (c) is "yes," then find the inverse matrix $\underline{\mathbf{A}}^{-1}$; if the answer to (c) is "no," then find the pseudoinverse matrix $\underline{\mathbf{A}}^{\dagger}$