

Answer the following questions as well as you can **WITHOUT** assistance from books or people; this is to assess your background and preparation

1. Determine the length of the listed input vector $\underline{\mathbf{x}}$ in the direction of the listed “reference” vector $\underline{\mathbf{a}}$.

(a) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

2. Find all of the values of z such that $z^4 = -1$. Express in two ways: as real and imaginary parts and as magnitude and phase angle.

3. Write down the matrix that evaluates the difference of adjacent components of the arbitrary 4-element vector $\underline{\mathbf{x}}$.

4. For the matrix \mathbf{A} :

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- (a) Determine if $\underline{\mathbf{A}}$ is invertible, i.e., if $\underline{\mathbf{A}}^{-1}$ exists
- (b) Find the eigenvectors and associated eigenvalues of $\underline{\mathbf{A}}$.

5. A function of one variable is defined by the following expression:

$$f[x] \equiv \begin{cases} 0 & \text{if } |x| > 1 \\ \frac{1}{2} & \text{if } |x| = 1 \\ 1 & \text{if } |x| < 1 \end{cases}$$

(a) Sketch $f[x]$

(b) For this function $f[x]$, evaluate the integral and sketch as a function of u

$$\int_{x=-\infty}^{x=+\infty} f[x] \cdot \exp[+i \cdot 2\pi \cdot x \cdot u] dx, \text{ where } i \equiv \sqrt{-1}$$