IMGS-616-20141 OPTIONAL HW #9

Due 12/11/2014 (Th): hand directly to me or slide under my office door 76-2112
(You may submit some or all of problems)

1. The impulse response of an imaging system is:

$$h[x] = \exp\left[+i\pi\left(\frac{x}{\alpha_0}\right)^2\right]$$

   Evaluate the expression for the inverse filter $w[x]$

2. The transfer function of a filter is:

$$H_1[\xi] = 1[\xi] \cdot \exp[i \cdot 2\pi \cdot (1 - 2 \cdot RECT[\xi])]$$

   (a) Characterize this filter (highpass, ...)

   (b) Evaluate and sketch the impulse response $h[x]$
   Find and sketch the outputs $g[x]$ for the following inputs:

   (c)
$$f_1[x] = \cos\left[2\pi\frac{x}{8}\right]$$

   (d)
$$f_2[x] = \cos[4\pi x]$$

   (e) Repeat (c) and (d) for the modified transfer function:

$$H_2[\xi] = 1[\xi] \cdot \exp\left[i \cdot 2\pi \cdot \left(\frac{1}{2} - RECT[\xi]\right)\right]$$

3. The transfer function of a 1-D system is:

$$H[\xi] = \left(1 - 2 \cdot RECT\left[\frac{\xi}{0.5}\right] * COMB[\xi]\right) \cdot \exp[+i\pi(1 - COMB[\xi] * RECT[2\xi])]$$

   (a) Sketch this transfer function as (real, imaginary) parts and as (magnitude, phase).

   (b) Classify this filter (e.g., highpass, ...).

   (c) Derive and sketch the impulse response $h[x]$.

   (d) Derive and sketch the inverse filter $w[x]$ for this impulse response.

4. Determine the areas of:

   (a) $\cos[2\pi x] \cdot \cos\left[\pi x^2\right]$

   (b) $SINC^4[x]$

5. An imaging system acting on the 1-D input function $f[x]$ includes the following steps:

   (a) evaluate $\mathcal{F}\{f[x]\}|_{\xi \to \frac{x}{(\alpha_0)^2}}$ where α_0 is a constant parameter with units of "length";

   (b) multiply by the real-valued pupil function $p[x]$ (which has finite support) to form $g_1[x]$

   (c) evaluate $\mathcal{F}\{g_1[x]\}|_{\xi \to \frac{x}{(\alpha_0)^2}}$
   For an input function of your choice, evaluate and sketch the output function if the pupil $p[x] = RECT\left[\frac{x}{b_0}\right]$. Note that this is a real imaging system where the first and last steps propagate light a long distance to the Fraunhofer diffraction region.

6. Modify the imaging system in the previous problem so that the explicit steps in the process are now:

   (a) evaluate $\mathcal{F}\{f[x]\}|_{\xi \to \frac{x}{(\alpha_0)^2}}$ where α_0 is a constant parameter with units of "length";

   (b) multiply by $p[x] = RECT\left[\frac{x}{b_0}\right] \cdot \exp\left[-i\pi\left(\frac{x}{\alpha_1}\right)^2\right]$, where both b_0 and α_1 are system parameters with units of length

   (c) convolve the product in step (b) with impulse response $h[x] = \exp\left[+i\pi\left(\frac{x}{\alpha_1}\right)^2\right]$

   In this system, steps (b) and (c) are the first two steps in an *M-C-M* chirp Fourier transformer. This is also a realistic imaging imaging system step (a) represents propagation from the object to a location in the Fraunhofer diffraction region, step (b) is the action of a lens, and step (c) is propagation to the focal plane of the lens. In this example, the constants $\alpha_1 = \sqrt{\lambda_0 z_1}$ and $\alpha_1 = \sqrt{\lambda_0 \mathbf{f}}$, where z_1 is the propagation distance from the object to the lens and $\mathbf{f}$ is the focal length of the lens.

7. A measured signal $g[x]$ is the sum of a deterministic part $f[x]$ and a "stochastic" random part $n[x]$

   (a) Describe the constraints on $f[x]$ AND on $n[x]$ that would allow a filter $w[x]$ to be constructed such that the output of the filter is *exactly* equal to $f[x]$, i.e.,

   $$(f[x] + n[x]) * w[x] = f[x]$$

   (b) Specify AND SKETCH the transfer function AND impulse response of the filter that will "extract" $f[x]$ from $g[x]$ if the signal and noise spectra are the functions::

   $$\begin{aligned} F[\xi] &= STEP[\xi] \cdot \exp\left[+i\pi\xi^2\right] \\ N[\xi] &= STEP[-\xi] \cdot \exp\left[+i\pi \cdot R[\xi]\right] \end{aligned}$$

   where $R[\xi]$ is a random number selected from the interval $-\pi \leq R[\xi] < +\pi$

8. We already know that the output of a "perfect" imaging system presented with the input $f[x]$ is $f[x]$:

   $$f[x] * \delta[x] = f[x]$$

   The output of a second imaging system consists of the sum of two identical but translated replicas of the input, where the two replicas are displaced by the distance b_0.

   (a) Write down the expression for the impulse response of the second system.

   (b) Evaluate and sketch the outputs for the following inputs:

   $$\begin{aligned} f_1[x] &= \cos\left[2\pi\left(\frac{x}{b_0} + \frac{1}{2}\right)\right] \\ f_2[x] &= \cos\left[2\pi\left(\frac{x}{2b_0}\right)\right] \end{aligned}$$

   (c) Evaluate and sketch the transfer function of the system

   (d) Is it possible to extract the input $f[x]$ from the output $g[x]$ and the impulse response $h[x]$? Explain why or why not.

9. Modify the previous problem so that the second exposure of the system is attenuated relative to the first by a factor of 2..

10. Modify the problem so that the image is not created from the two exposures but rather from a single exposure made over the displacement by the distance b_0.