IMGS-616-20141 Homework Assignment #8 Due 12/02/2014 (T)

0. Read §13.3 (Central Limit Theorem) and §16 (Magnitude Filters), §17.1-§17.4 and §17.6-§17.9.2 (All-pass Filters), §18.1, and §19

1. The function $f[x] = SINC^2[100x]$ is sampled by the ideal COMB function with period Δx and therefore sampling "rate" $\xi_s = (\Delta x)^{-1}$. The sampled signal is passed through an ideal lowpass filter that "cuts off" at the sampling frequency.

   (a) Find the minimum sampling rate ξ_0 that will permit exact recovery of $f[x]$ from $f_s[x]$, so that $g[x] \propto f[x]$ (hint, you've done this already :-)

   (b) Find $g[x]$ if $\xi_s = 0.75 \cdot \xi_0$ and sketch.

   (c) Find $g[x]$ if $\xi_s = 0.50 \cdot \xi_0$ and sketch.

2. A 1-D "50%" square wave grating (half "on" and half "off") may be written:

$$f[x] = COMB[x] * RECT[2x]$$

   (a) Evaluate and sketch the spectrum $F[\xi]$; as a side comment, this is a scaled replica of the diffraction pattern from a 50% grating if viewed in the Fraunhofer diffraction region.

   For each of the system functions $H[\xi]$ or $h[x]$ listed below, evaluate and sketch the representation, the "other representation" (i.e., $h[x]$ if $H[\xi]$ is given, or vice versa), and the output function $g[x] = f[x] * h[x]$. Classify the filters as lowpass, highpass, phase, etc.

   (b) $H[\xi] = RECT[\xi]$

   (c) $h[x] = RECT[x]$

   (d) $H[\xi] = -RECT[4\xi + 2] + RECT[4\xi - 2]$

   (e) $H[\xi] = 1 + \xi^2$

3. Sketch each of the transfer functions listed and evaluate and sketch the corresponding impulse response $h_n[x]$.

   (a) $H_1[\xi] = \exp[+i\pi]$

   (b) $H_2[\xi] = \exp[+i\pi\xi]$

   (c) $H_3[\xi] = \exp[+i\pi \cdot (1 - RECT[2\xi])]$

   (d) $H_4[\xi] = \exp\left[-i\pi \cdot (2\xi)^2\right]$

   (e) (OPTIONAL BONUS) $H_5[\xi] = SINC[2\xi - 4] \cdot \exp\left[-i\pi \cdot (2\xi)^2\right]$ (HINT: stationary phase)

4. The following signals are applied separately to LSI systems with impulse response $h[x]$ to be determined:

$$\begin{aligned} s_1[x] &= e^{-x} \cdot STEP[x] \\ s_2[x] &= RECT[2x] * (\delta[x] + \delta[x-4] + \delta[x-7] + \delta[x-9]) \end{aligned}$$

   For each case:

   (a) Describe the impulse response $h[x]$ and transfer function $H[\xi]$ of the matched filter that will maximize the output at $x = 2$. (assume that $H[0] = 1$). Sketch the output.

   (b) Is it possible to construct a transfer function $H[\xi]$ that, when applied to $s[x]$, will produce $g[x] = \delta[x-2]$? Explain your reasoning.

MORE→→→

5. Given an LSI system and a "phase-function" input:

$$\begin{aligned} f[x] &= 1[x] \cdot \exp[+i\Phi[x]] \\ \Phi[x] &= -\pi + \pi \left(COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right) * RECT[4x] \end{aligned}$$

(a) Sketch the phase function $\Phi[x]$ and the input function $f[x]$.

(b) Show that the input may be written as:

$$f[x] = 2\left[COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right] * RECT[4x] - 1$$

(c) Sketch the transfer function:

$$H[\xi] = RECT\left[\frac{\xi}{51}\right] \cdot \exp[+i\pi \cdot RECT[8\xi]]$$

(d) Show that the output $g[x] = f[x] * h[x]$ is approximately equal to:

$$g[x] \cong 2\left[COMB[x] \cdot RECT\left[\frac{x}{101}\right]\right] * RECT[4x]$$

(e) Sketch $g[x]$

6. For $f[x] = \delta[x+2]$

(a) evaluate the *M-C-M* chirp Fourier transform, showing the output after each M or C operation

(b) evaluate the *C-M-C* chirp Fourier transform, showing the output after each M or C operation. The expressions for the transforms are given in §19 and below, where α_0 is the *chirp rate*, i.e., the distance from the origin where the phase changes by π radians.

$$F\left[\frac{x}{\alpha_0^2}\right] = \left(\left(f[x] \cdot \exp\left[-i\pi\left(\frac{x}{\alpha_0}\right)^2\right]\right) * \exp\left[+i\pi\left(\frac{x}{\alpha_0}\right)^2\right]\right) \cdot \exp\left[-i\pi\left(\frac{x}{\alpha_0}\right)^2\right] \quad \text{(M-C-M)}$$

$$= \frac{1}{|\alpha_0|}\exp\left[-i\frac{\pi}{4}\right] \cdot \left(\left\{\left(f[x] * \exp\left[+i\pi\left(\frac{x}{\alpha_0}\right)^2\right]\right) \cdot \exp\left[-i\pi\left(\frac{x}{\alpha_0}\right)^2\right]\right\} * \exp\left[+i\pi\left(\frac{x}{\alpha_0}\right)^2\right]\right) \quad \text{(C-M-C)}$$

Note that $\frac{x}{\alpha_0^2}$ has dimensions of "reciprocal length" (e.g., "cycles per millimeter).

(c) (OPTIONAL BONUS) evaluate the *M-C-M* AND *C-M-C* chirp Fourier transforms and sketch after each step for $f[x] = \cos\left[\pi\left(\frac{x}{\alpha_0}\right)^2\right]$

(d) (OPTIONAL BONUS) repeat (c) for $f[x] = \cos\left[\pi\left(\frac{x-1}{\alpha_0}\right)^2\right]$