

# IMGS-616-20141 Homework Assignment #7 Due 11/18/2014 (T)

0. Read §13.1.1-§13.1.4 on the moment theorem, §13.2 on stationary phase, §13.3 on the central-limit theorem, and §14 on Sampling; you may ignore any items in §13 on the so-called “superchirp function”  $\exp [+i\pi x^n]$  (though a problem involving the sampling of this function that is given below).

Note that there are two OPTIONAL problems (#7 and #8); the second of these is very useful in describing light amplitude in optical systems

1. For the function  $f[x] = \delta[x - x_0]$ 
  - (a) Find an expression for the moments of the function
  - (b) Show that the moments produce the correct spectrum for some different choices of  $x_0$ .
2. Determine the centroid of  $RECT[x - x_0]$  in two ways: (a) via direct integration and (b) via the moment theorem.
3. Evaluate expressions for the following and be aware of any similarities in the results ;-)

(a)  $COMB\left[\frac{x}{\Delta x}\right] * RECT\left[\frac{x}{\Delta x}\right]$

(b)  $COMB\left[\frac{x}{\Delta x}\right] * TRI\left[\frac{x}{\Delta x}\right]$

(c)  $COMB\left[\frac{x}{\Delta x}\right] * SINC\left[\frac{x}{\Delta x}\right]$

(d)  $COMB\left[\frac{x}{\Delta x}\right] * SINC^2\left[\frac{x}{\Delta x}\right]$

4. What are the amplitudes of the samples of  $SINC[x]$  and  $SINC^2[x]$  when sampled exactly at the Nyquist frequency?
5. Describe how to sample a function most efficiently without aliasing (i.e., fewest number of samples) if the spectrum is “asymmetric,” (in other words, if the largest positive and negative frequencies are different).
6. Consider sampling of a real-valued cosine function whose phase is proportional to  $x^n$  with an ideal comb function with separation  $\Delta x$

$$f[x] = \cos\left[\pi\left(\frac{x}{b_0}\right)^n\right]$$

$$s[x; \Delta x] = \frac{1}{\Delta x} COMB\left[\frac{x}{\Delta x}\right]$$

determine the relationship between  $\Delta x$  and  $b_0$  for the sample index where the chirp function is “just” aliased.

7. (Problem 13.7) (OPTIONAL BONUS) Consider the 1-D function:

$$f[x] = RECT\left[\frac{x - 2.5}{5}\right] \cdot \exp [+i\pi x^2]$$

- (a) Try to evaluate  $g[x] = f[x] * f[x]$  (HINT: I dare you to get this right!)
  - (b) Evaluate the stationary-phase approximation  $\hat{F}[\xi]$  of the Fourier transform of  $f[x]$ .
  - (c) Use the result of part (b) to find an approximation for the convolution of  $\hat{g}[x] \cong f[x] * f[x]$
  - (d) Use the result of part (b) to approximately evaluate  $f[x] \star f[x] = f[x] * f^*[-x]$
  - (e) Use the result of part (b) to approximately the Fourier transform of  $\Re\{f[x]\}$ .
8. (OPTIONAL BONUS) Use the stationary phase approximation to derive expressions for and sketch the approximate Fourier spectra of the following:

(a)  $\cos\left[\pi\left(\frac{x}{2}\right)^2\right] \cdot RECT\left[\frac{x-4}{2}\right]$

(b)  $\cos\left[\pi\left(\frac{x}{2}\right)^2\right] \cdot SINC\left[\frac{x-4}{2}\right]$

(c)  $\sin\left[\pi\left(\frac{x}{4}\right)^2\right] \cdot SINC\left[\frac{x-4}{4}\right]$