IMGS-616-20141 Homework Assignment #6 Due 10/30/2014 (Th)
(NEXT EXAM will be held Thursday, 11/6)

0. In Chapter 7 (2-D special functions), read §7.2, §7.3.1-§7.3.3, §7.3.5, §7.4 (you may skip the sections on rotating functions; also read Chapter 10 (2-D transforms), and Chapter 11 (transforms of circular functions)

1. Evaluate the volumes of and graph 1-D axial profiles (i.e., $f[x,0]$ and $f[0,y]$) of the following functions as "top views". Also find expressions for and sketch the even and odd parts.

   (a) $f[x,y] = SINC\left[\frac{x}{2}, y\right]$

   (b) $g[x,y] = RECT\left[\frac{x}{2}, \frac{y}{4}\right] - RECT\left[x, \frac{y}{2}\right]$

   (c) $p[x,y] = CYL\left(\frac{\sqrt{x^2+y^2}}{2}\right) - CYL\left(\sqrt{x^2+y^2}\right)$

   (d) $q[x,y] = \left[CYL\left(\frac{r}{2}\right) - CYL(r)\right] \cdot STEP[y]$

2. Find the Fourier transforms of the following 2-D separable functions and sketch them as profiles or as "images":

   (a) $COR\left[\frac{x}{2}, 2y\right]$, where "$COR$" is the "corral" function defined in §7.3.6

   (b) $RECT[x,y] * (\delta[x] \cdot 1[y])$

   (c) $RECT[x,y] * (\delta[x-1] \cdot 1[y-1])$

   (d) $RECT[x,y] * CROSS[x,y]$, where $CROSS[x,y] \equiv \delta[x] \cdot 1[y] + 1[x] \cdot \delta[y]$

   (e) $COR[x,y] * COR[x,y]$

3. Use the Fourier transforms of $\exp\left[\pm i\pi x^2\right]$ to derive the 2-D transform

$$\mathcal{F}_2\left\{\exp\left[\pm i\pi\left(x^2\right)\right] \cdot \exp\left[\pm i\pi\left(y^2\right)\right]\right\} = \mathcal{F}_2\left\{\exp\left[\pm i\pi\left(x^2+y^2\right)\right]\right\} = \mathcal{F}_2\left\{\exp\left[\pm i\pi r^2\right]\right\}$$

4. Find the results of the convolution and sketch it:

   (a) $CYL(r) * (\delta[x, y+2] + \delta[x, y-2])$

   (b) $GAUS(r) * \delta[x-1, y]$

5. Evaluate the Fourier transforms of the following functions and sketch them:

   (a) $CYL(r) * (\delta[x] \cdot (\delta[y+2] + \delta[y-2]))$

   (b) $GAUS(r) * \delta[x-1, y]$

   (c) $J_0(2\pi r) + J_0(4\pi r)$

   (d) $\exp\left[-i\pi\frac{r^2}{4}\right] + \exp\left[+i\pi\frac{r^2}{4}\right]$

6. Find the transfer function of the imaging systems with the following impulse responses:

   (a) $h_a(r) = J_0(2\pi r) + J_0(\pi r)$

   (b) $h_b(r) = SOMB\left(\frac{r}{10}\right)$

   (c) $h_c(r) = -r^2 GAUS(r)$

7. Evaluate AND SKETCH the results of the following 2-D operations, where the symbols "$*$" and "$\bigstar$" denote 2-D convolution and correlation, respectively:

   (a) $CYL(r) * (\delta[x] \cdot 1[y])$

   (b) $(\cos[\pi\xi] \cdot SINC[\eta] + SINC[\xi] \cdot \cos[\pi\eta]) \bigstar \left(\cos[2\pi\xi] \cdot SINC\left[\frac{\eta}{2}\right] + SINC\left[\frac{\xi}{2}\right] \cdot \cos[2\pi\eta]\right)$

   (c) $J_0(2\pi\rho_0 r) * J_0(2\pi\rho_1 r)$, where $\rho_0 \neq \rho_1$