

# IMGS-616-20141    Homework #5    Due 10/23/2014 (Th)

0. Read (or finish) Chapter 9

1. Find the Fourier transforms of the following functions and sketch them as BOTH real and imaginary parts AND as magnitude and phase:

(a)  $f[x] = \text{RECT}\left[\frac{x}{2}\right] + \text{RECT}\left[\frac{x}{4}\right]$

(b)  $h[x] = \frac{1}{2}\text{RECT}\left[\frac{x-1}{2}\right] + i \cdot \text{RECT}\left[\frac{x+1}{2}\right]$

(c)  $p[x] = \cos\left[\pi\frac{x^2}{4}\right]$

(d)  $r[x] = \sin\left[\pi\frac{x^2}{4}\right]$

(e)  $u[x] = p[x] + i \cdot r[x]$

2. Find expressions for (AND SKETCH) the inverse Fourier transforms of the following functions:

(a)  $R[\xi] = \left(\frac{1}{10}\right) \cdot \left(\delta\left[\frac{\xi}{10} + 1\right] + \delta\left[\frac{\xi}{10} - 1\right]\right)$

(b)  $G[\xi] = \text{SINC}^2\left[\frac{\xi-1}{2}\right]$

(c)  $S[\xi] = \text{TRI}[\xi+1] + \text{TRI}[\xi-1]$ , sketch  $S[\xi]$  and  $s[x]$

3. Evaluate the Fourier transforms of the outputs of the following operations and sketch them as real-and-imaginary parts and as magnitude-phase:

(a)  $\text{RECT}[x] * \text{RECT}[x]$

(b)  $\text{RECT}[x-1] * \text{RECT}[x]$

(c)  $\text{RECT}[x-1] * \text{RECT}[x+1]$

(d)  $\text{RECT}[x-1] \star \text{RECT}[x+1]$

4. Find the Fourier transforms of the following functions and sketch both representations:

(a)  $f[x] = \text{COMB}[x] \cdot \text{RECT}\left[\frac{x}{4}\right]$

(b)  $g[x] = \left(\text{COMB}[x] \cdot \text{RECT}\left[\frac{x}{4}\right]\right) * \text{RECT}[2x]$

(c)  $r[x] = \text{COMB}[x] \cdot \text{SINC}\left[\frac{x}{4}\right]$

(d)  $s[x] = \text{COMB}[x] \cdot \text{SINC}\left[\frac{x}{2}\right]$

(e)  $t[x] = \text{COMB}[x] \cdot \text{SINC}[x]$

(f)  $u[x] = \text{COMB}[x] \cdot \text{SINC}[2x]$

5. In HW#4-4, you solved (or tried to) the convolutions of two scaled Gaussian functions and of two SINC functions:

(a)  $\text{GAUS}\left[\frac{x}{3}\right] * \text{GAUS}\left[\frac{x}{4}\right] = \int_{-\infty}^{+\infty} \exp\left[-\pi\left(\frac{\alpha}{3}\right)^2\right] \exp\left[-\pi\left(\frac{x-\alpha}{4}\right)^2\right] d\alpha$

(b)  $\text{SINC}[3x] * \text{SINC}[2x] = \int_{-\infty}^{+\infty} \left(\frac{\sin[3\pi\alpha]}{3\pi\alpha}\right) \left(\frac{\sin[2\pi(x-\alpha)]}{2\pi(x-\alpha)}\right) d\alpha$

Use the filter theorem and known transforms to evaluate these convolutions, which is MUCH easier than direct integration.