SIMG-616 Homework #4 Due 10/16/2014 (Th)

0. Read Chapters 8 and 9 in the book.

1. Use the definition of the Dirac delta function to evaluate and sketch the following:

   (a) $\delta\left[x^2-1\right]+\delta\left[x^2-4\right]+\delta\left[x^2-9\right]+\cdots=\sum_{n=1}^{\infty}\delta\left[x^2-n^2\right]$

   (b) $\delta\left[\sin\left[2\pi\xi_0 x\right]\right]$

2. Given a system $\mathcal{S}$ that is characterized by the operator

$$\mathcal{S}\{f[x]\}=\left[a\left(\frac{d^2}{dx^2}\right)+b\left(\frac{d}{dx}\right)+c\right]f[x]$$

   where a, b, and c are arbitrary constants.

   (a) Determine if the system us linear

   (b) Determine if the system is shift invariant.

   (c) Sketch the output if the input $f[x]=\cos\left[2\pi\frac{x}{2}\right]$

3. Assume the system to be characterized by the operator:

$$\mathcal{S}\{f[x]\}=a_0\cdot(f[x])^2+b_0\cdot f[x]$$

   (a) Is the system linear? Shift invariant?

   (b) Calculate and sketch the output of the system for $f[x]=GAUS[x]=\exp\left[-\pi x^2\right]$.

4. The convolutions of two scaled Gaussian functions and of two *SINC* functions may be written as:

   (a) $GAUS\left[\frac{x}{3}\right]*GAUS\left[\frac{x}{4}\right]=\int_{-\infty}^{+\infty}\exp\left[-\pi\left(\frac{\alpha}{3}\right)^2\right]\exp\left[-\pi\left(\frac{x-\alpha}{4}\right)^2\right]d\alpha$

   (b) $SINC[3x]*SINC[2x]=\int_{-\infty}^{+\infty}\left(\frac{\sin[3\pi\alpha]}{3\pi\alpha}\right)\left(\frac{\sin[2\pi(x-\alpha)]}{2\pi(x-\alpha)}\right)d\alpha$

   Evaluate these integrals either rigorously (by direct integration) or approximately by graphical means. Comment on the difficulty of the evaluation. We shall revisit these problems after proving the so-called "filter theorem" of the 1-D Fourier transform, where the solution becomes "trivial".

5. Use “direct integration” to evaluate the convolution of a triangle function with unit witdth parameter with a rectangle function with unit width:

$$\begin{aligned} TRI[x] * RECT[x] &= \int_{-\infty}^{+\infty} TRI[\alpha] \cdot RECT[x-\alpha] \; d\alpha \\ &= \int_{-\infty}^{+\infty} (1-|\alpha|) \cdot RECT\left[\frac{\alpha}{2}\right] \cdot RECT[x-\alpha] \; d\alpha \end{aligned}$$

6. Given $f[x] = (\delta[x] + \delta[x-1]) + i \cdot (\delta[x-1] + \delta[x-4])$, evaluate the autocorrelation and “autoconvolution” of $f[x]$, i.e., evaluate:

   (a) $f[x] \bigstar f[x] \equiv \int_{-\infty}^{+\infty} f[\alpha] \cdot f^*[\alpha - x] \; d\alpha$

   (b) $f[x] * f[x] \equiv \int_{-\infty}^{+\infty} f[\alpha] \cdot f[x-\alpha] \; d\alpha$

7. Find the Fourier transforms of the following functions and sketch them as BOTH real and imaginary parts AND as magnitude and phase:

   (a) $f[x] = RECT\left[\frac{x}{2}\right]$

   (b) $g[x] = RECT[x-1]$

   (c) $h[x] = \frac{1}{2} RECT\left[\frac{x-1}{2}\right]$

   (d) $p[x] = RECT[x-2] \cdot \exp[+2\pi i x]$

   (e) $r[x] = \frac{1}{2} RECT\left[\frac{x}{2}\right] + TRI[x]$

8. Evaluate the Fourier transforms of the outputs of the following operations and sketch them are real-and-imaginary parts and as magnitude-phase:

   (a) $RECT[x] * RECT[x]$

   (b) $RECT[x-1] * RECT[x]$

   (c) $RECT[x-1] * RECT[x+1]$

   (d) $RECT[x-1] \bigstar RECT[x+1]$