SIMG-616 Homework #3 Due 9/23/2014 (T)

0. These problems are based on the material in Chapters 5 and 6.1

1. The discrete Fourier transform (DFT) of a sampled one -dimensional function $f[n]$ with $0 \leq n \leq N-1$ is defined:

$$\sum_{n=0}^{N-1} f[n] \cdot \left(\exp\left[+i \cdot 2\pi \cdot n \cdot \frac{k}{N} \right] \right)^* \equiv F[k], \text{ where } k = 0, 1, 2, 3$$

   Evaluate the DFT of these sampled functions with $N = 4$ and plot the real part, imaginary part, magnitude, and phase angle of $F[k]$

   (a) $f_1[n=0] = 1, f_1[n=1] = f_1[n=2] = f_1[n=3] = 0$
   (b) $f_2[0] = 0, f_2[1] = 1,\ f_2[2] = f_2[3] = 0$
   (c) $f_3[0] = f_3[1] = 0,\ f_3[2] = 1,\ f_3[3] = 0$
   (d) $f_4[0] = f_4[1] = f_4[2] = 0,\ f_4[3] = 1$
   (e) $f_5[0] = f_5[1] = f_5[2] = f_5[3] = 1$

2. For $N = 8$, evaluate the DFT's and plot the results for the following sampled functions; the boxes represent the sample locations and are indexed from $n = 0$ to $n = 7$

   (a) $f_1[n] =$

1	0	0	0	0	0	0	0

   (b) $f_2[n] =$

1	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{2}$

   (c) $f_3[n] =$

1	1	0	0	0	0	0	1

3. (Gaskill 3-3) Given positive real-valued and positive constants b_0 and x_0, and the function $f[x] \equiv TRI[x] \cdot STEP[-x]$, sketch the following:

   (a) $f[x]$
   (b) $f\left[\frac{x}{b_0}\right]$
   (c) $f[x + x_0]$
   (d) $f[-x] = f\left[\frac{x}{-1}\right]$
   (e) $f\left[\frac{-x - x_0}{b_0}\right]$.

4. For $f[x] = RECT\left[x + \frac{1}{2}\right] + i \cdot STEP[x] \cdot TRI[x]$, sketch $f\left[\frac{-x+1}{2}\right]$ and $f[-2x+4]$.

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5. For the 1-D function of x

$$f[x] = RECT\left[\frac{x+x_0}{b_0}\right] + RECT\left[\frac{x-x_0}{b_0}\right],$$

sketch $f[x]$ for the following parameters:

(a) $x_0 = 2b_0$

(b) $x_0 = b_0$

(c) $x_0 = \frac{b_0}{2}$ (also write down a different and more concise expression for this function)

6. Sketch the following complex-valued functions over the interval $-4 \leq x \leq +4$ as (1) real part, (2) imaginary part, (3) magnitude, and (4) phase. You may plot them by computer (but you will also need to know how to sketch them!)

(a) $f[x] = RECT[x-1] - RECT[x+1]$

(b) $g[x] = STEP[x] \cdot TRI\left[\frac{x}{2}\right] + i\,STEP[-x] \cdot TRI\left[\frac{x}{2}\right]$

(c) $h[x] = RECT\left[\frac{x-1}{2}\right] + i\,RECT\left[\frac{-x-1}{3}\right]$

7. Express the following product of two translated replicas of the same Gaussian function as a Gaussian function, i.e., where the parameters $x_0, b_0 \geq 0$, evaluate A_0 and d_0 in:

$$GAUS\left[\frac{x+x_0}{b_0}\right] \cdot GAUS\left[\frac{x-x_0}{b_0}\right] = A_0 \cdot GAUS\left[\frac{x}{d_0}\right]$$