

0. Read Chapters 1-4 in the book and spend some time in the RIT library looking through other books on the subject of Fourier mathematics, linear systems, etc. from the bibliography.
1. Consider two spatial sinusoids with the same spatial frequency  $\xi_0$  (measured in “cycles per unit length”), but arbitrary amplitudes  $A_1$  and  $A_2$  and arbitrary phase angles  $\phi_1$  and  $\phi_2$ :

- (a) Prove that the sum of these two sinusoids is a sinusoid with that same frequency  $\xi_0$ .
- (b) Find the expression that determines relates the amplitude  $A$  and phase  $\phi$  of the summation sinusoid from the amplitudes and initial phase angles of the component sinusoids.

2. For a sinusoidal functions whose phase is a power of the coordinate:

$$f[x] = \cos \left[ \pi \left( \frac{x}{\alpha_0} \right)^n + \phi_0 \right]$$

- (a) If the variable  $x$  has units of length, what are the “units” of the parameter  $\alpha_0$ ?
- (b) Graph the function for  $\alpha = 1$ ,  $\phi_0 = 0$ , and  $n = 1, 2$ , and  $3$ .
- (c) Graph the function for  $\alpha = 1$ ,  $\phi_0 = -\frac{\pi}{2}$ , and  $n = 1, 2$ , and  $3$ .
- (d) Determine the equation for the spatial frequency of  $f[x]$ .

3. Find the lengths of the 3-D vectors:

(a) a.  $\begin{pmatrix} +1 \\ +2 \\ -1 \end{pmatrix}$     b.  $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$     c.  $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$

4. In each case, find the length of the input vector  $\underline{\mathbf{x}}$  in the direction of the “reference” vector  $\underline{\mathbf{a}}$  (this is the “projection of  $\underline{\mathbf{x}}$  onto  $\underline{\mathbf{a}}$ ”).

(a)  $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ,  $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(b)  $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c)  $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

5. Find  $k$  so that the two vectors  $\underline{\mathbf{a}}$  and  $\underline{\mathbf{b}}$  are orthogonal:

$$\underline{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \underline{\mathbf{b}} = \begin{pmatrix} -1 \\ k \\ -3 \end{pmatrix}$$