

Syllabus for IMGS-616 *Fourier Methods in Imaging* (RIT #11857)

3 July 2014

(TENTATIVE and subject to change)

Note that I expect to be in Europe twice during the term: in Paris the week of 9/15 and in Malmö the week of 9/29. The missing times will be made up by some means, possibly by prerecorded lectures, by guest lecturers, or by makeup classes on other days.

Week 1: 8/26, 8/28**I. Signals, Operators, and Imaging Systems**

- A. Imaging “chain”
- B. Three imaging “tasks”
 1. Direct (find output from input and system)
 2. Inverse: (find input from output and system)
 3. System Analysis (find system from input and output)
- C. Examples of the imaging chain and tasks
- D. Constraints of linearity and shift invariance applied to enable analysis:

II. Review of (or “Introduction to”) Linear Algebra

- A. Vectors with real-valued components
 1. Scalar product (“dot product”), “length” or of vector
 2. Scalar projection of “input vector” onto “reference vector,” orthogonal vectors
 3. Simultaneous projections of one “input vector” onto several “reference vectors”
 - a. matrix-vector product as representation of action of imaging system on input
 - b. matrix-matrix multiplication
 4. Orthogonal matrices (scalar product of distinct rows or columns is null)
 5. Matrix transpose, inverse, identity matrix
 6. Vector spaces and subspaces
 - a. criteria for vector space/subspace
 - b. basis vectors for vector space
 - c. Four vector subspaces associated with any imaging system matrix
 1. Input vectors (row subspace, null subspace)
 2. Output vectors (column subspace, left-null subspace)

Week 2: 9/2, 9/4

- B. Shift-invariant systems
 1. Representations as circulant matrices
 2. Examples of matrices for translation operator, averager, and differencer
- C. Complex numbers and their geometric interpretation
 1. complex numbers as real-valued vectors
 2. representations: real+imaginary, magnitude+phase, phasor/Argand diagram
 3. complex arithmetic
 4. Euler relation, deMoivre's theorem, roots of complex numbers
- D. Vectors with complex-valued components
 1. Inner product (analogue of scalar product)
 2. Products of complex-valued matrices and vectors
 3. Eigenvectors and eigenvalues
 4. Diagonal form of circulant matrix
 5. discrete Fourier transform
- E. Matrix formulation of imaging task
 1. direct problem: matrix-vector product

2. inverse problem: inverse and pseudoinverse matrices
3. system-analysis task: find matrix from input and output

Week 3: 9/9, 9/11

III. Representations of systems, inputs, and outputs by functions

- A. Symmetry properties of functions: even and odd
- B. Projections of functions onto “reference” functions, scalar products, orthogonality
- C. special functions
 1. Support
 2. Area (1-D) or volume (2-D)
 3. Shifting and scaling
 4. Definitions of deterministic real-valued functions
 - a. unit function, null function
 - b. rectangle
 - c. triangle
 - d. signum and step
 - e. Gaussian
 - f. linear-phase sinusoid
 - g. quadratic-phase sinusoid
 - h. Dirac delta function
 - i. Stochastic functions

Week 4: 9/16, 9/18 (Paris)

5. Complex-valued 1-D special functions
 - a. representations as real and imaginary parts and as magnitude and phase
 - b. linear- and quadratic-phase sinusoids
6. 2-D functions that are separable in Cartesian coordinates
7. 2-D functions that are separable in polar coordinates
8. 2-D Dirac delta function and its relatives (line delta and cross)
9. Rotation of 2-D functions

Midterm Exam 1 (linear algebra and discrete case)

Week 5: 9/23, 9/25

IV. Classes of Imaging Operators

- A. Linearity
- B. Shift invariance
- C. Crosscorrelation
- D. Convolution
 1. theorems
 2. examples

V. Alternative representations of functions: Fourier analysis and synthesis

- A. Projection of functions onto sinusoidal “reference” functions
- B. Projections onto combinations of “reference” functions
 1. projection of $f[x]$ onto cosine + sine (Hartley transform)
 2. projection of $f[x]$ onto cosine + i -sine (Fourier transform)
- C. Fourier synthesis, inverse Fourier transform
- D. Fourier transforms of the 1-D special functions

Week 6: 9/30, 10/2 (Malmö meeting)

VI. Theorems of the Fourier transform

- A. Transform of transform
- B. Scaling theorem
- C. Shift theorem
- D. Filter theorem
- E. Modulation theorem
- F. Derivative theorem
- G. Fourier transform of autocorrelation, Wiener spectrum
- H. Rayleigh's and Parseval's theorems
- I. Fourier transform of periodic function
- J. Fourier transform of sampled function
- K. Fourier transform of discrete periodic function
- L. Effect of nonlinear operations on spectra
- M. Central-limit theorem
- N. Uncertainty relation(s)

Week 7: 10/7, 10/9

VII. Fourier transforms of multidimensional functions

- A. Separable 2-D functions
- B. Circularly symmetric 2-D functions, Hankel transform

VIII. Filtering of continuous functions

- A. Magnitude filters
 - 1. Lowpass (averagers, integrators)
 - 2. Highpass (differencers, differentiators)
 - 3. Bandpass (difference of averages) filters and relatives (bandboost, bandstop)

Week 8: No class 10/14 (Monday class schedule), 10/16

- B. Phase filters ("allpass")
 - 1. constant phase
 - 2. linear phase
 - 3. quadratic phase
 - 4. chirp Fourier transform by multiplication (M) and convolution (C) with quadratic-phase factors
 - a. multiplication + convolution + multiplication (*M-C-M*)
 - b. convolution + multiplication + convolution (*C-M-C*)

Week 9, 10/21 and 10/23

- C. Magnitude-and-phase filters
 - 1. causality

IX. Applications of linear filters

- A. Inverse imaging task, "deconvolution"
 - 1. Inverse filter
 - 2. Wiener filter
 - 3. Wiener-Helstrom filter
- B. Detection of known signal: "matched filter"
- C. Analogies between inverse and matched filters

Week 10: 10/28 and 10/30
Midterm Exam 2

Week 11: 11/4 and 11/6

- X. Fourier transforms and optical imaging systems
- A. propagation of light as spherical waves, Huygens' principle, convolution with spherical waves
 - B. approximation to propagation of light as paraboloidal waves, Fresnel diffraction, convolution with constant-magnitude quadratic-phase factor
 - C. approximation to propagation of light as plane waves, Fraunhofer diffraction, shift-variant Fourier transform
 - D. Imaging system formed from Fraunhofer propagation + aperture + Fraunhofer propagation
 - E. Approximation of lens action as multiplication by apodized quadratic-phase factor
 - F. optical Fourier transform via C-M-C chirp transform
 - G. optical filtering by cascade of two C-M-C chirp transforms
 - H. Schlieren imaging

Week 12: 11/11 and 11/13

- XI. Sampling
- A. Ideal sampling of special functions
 - B. Ideal sampling of $\delta[x]$ and $COMB[x]$
 - C. Ideal interpolation of sampled functions
 - D. Nyquist limit
 - E. Whittaker-Shannon sampling theorem
 - F. Aliasing
 - G. Realistic Sampling (finite-sized detector elements)
 - H. Realistic Interpolation
 - I. Quantization, quantization error

Week 13: 11/18 and 11/20

- XII. Discrete Fourier Transform (DFT)
- A. Infinite-Support DFT
 - B. Finite-Support DFT; "blurring" of spectrum
 - C. Efficient evaluation of DFT via fast Fourier transform

Week 14: 11/25

- D. Practical considerations
 1. 2D DFT
 2. location of constant (DC) component
 - a. Centered and "uncentered" arrays
 - b. conversion between renderings
 3. Units of measure in the two domains
 - a. uncertainty relation for N , Δx , $\Delta \xi$
 4. Leakage, sidebands
 5. Data windows
 - a. Hanning

- b. Hamming
- c. Bartlett (triangle)

Week 15: 12/2 and 12/4

Week 16: 12/9

XIII. Cleanup and Review