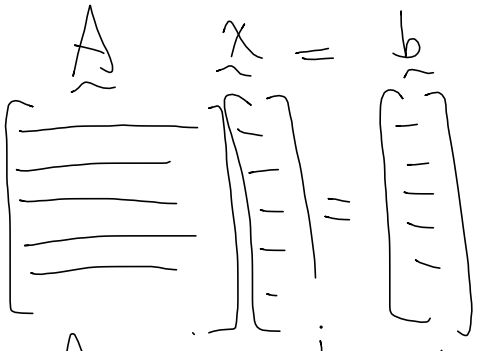
IMGS-616 FOURIER METHODS

15 MINUTES FOR ASSESSMENT "QUIZ"

(LECTURE STARTS 15 MINUTES INTO VIDEO)

$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$

SYSTEM, INPUT, OUTPUT

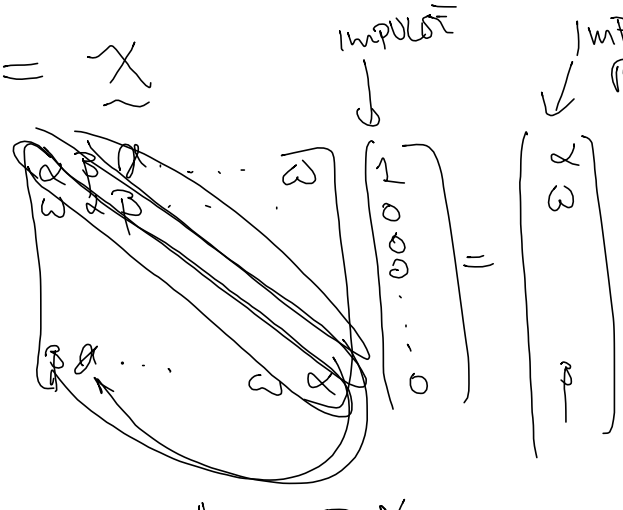


(1) DIRECT "TASK", KNOW $\underset{\sim}{A}, \underset{\sim}{x}$; $\Rightarrow$ FIND $\underset{\sim}{b}$

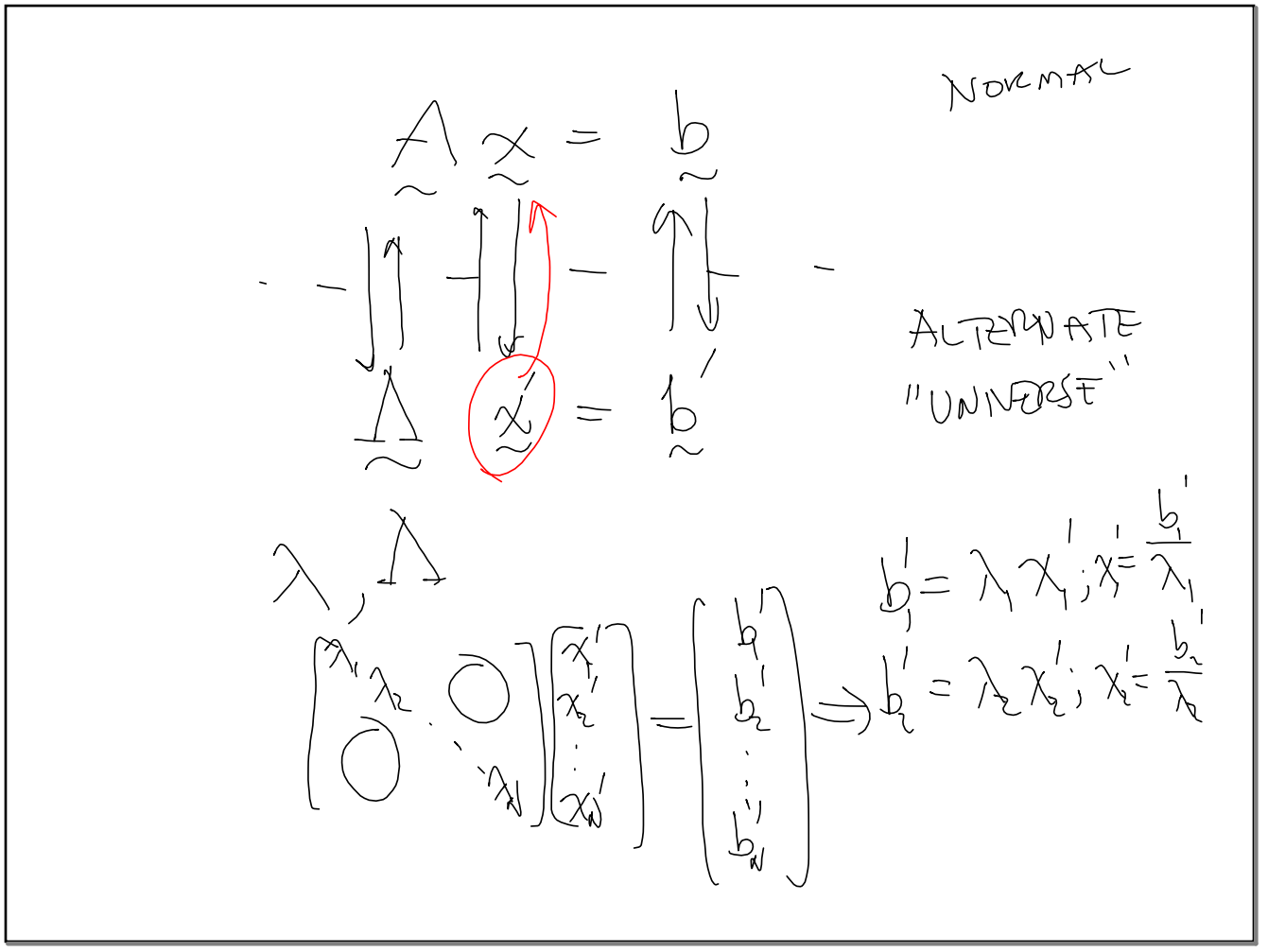
(2) INVERSE TASK, KNOW $\underset{\sim}{A}, \underset{\sim}{b}$; $\Rightarrow$ FIND $\underset{\sim}{x}$

$\underset{\sim}{A}^{-1}\,\underset{\sim}{b} = \underset{\sim}{x}$

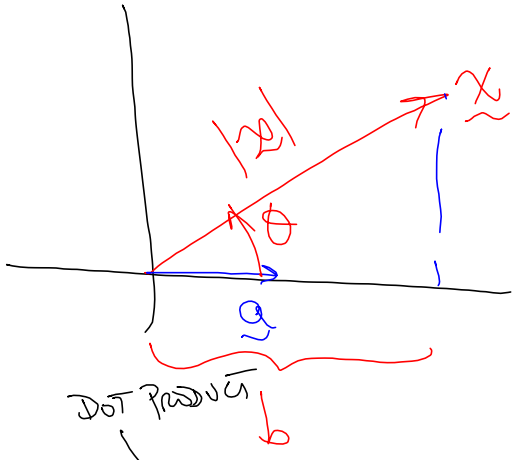
$\underset{\sim}{A} \rightarrow$



"CIRCULANT" MATRIX


NORMAL
ALTERNATE
"UNIVERSE"

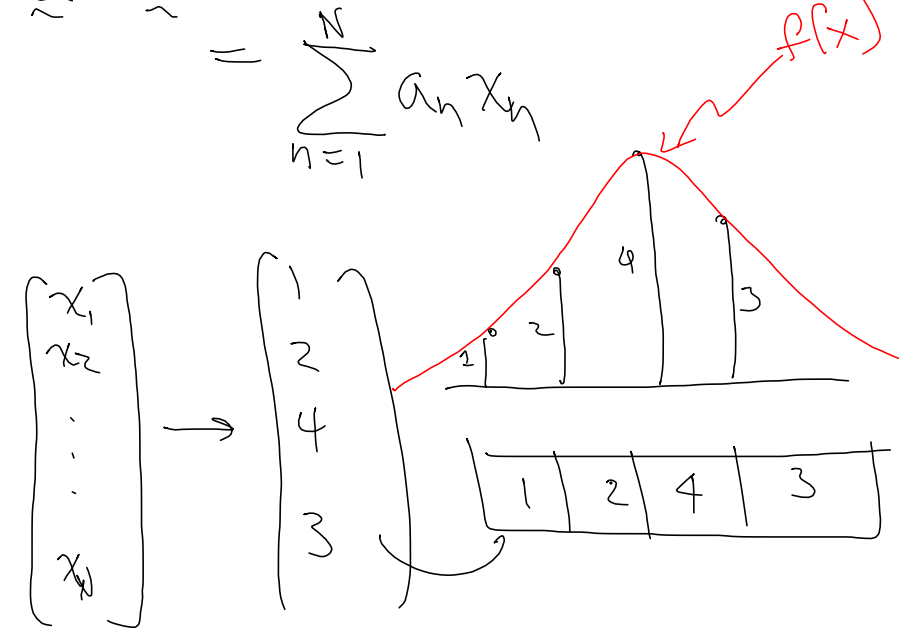
$\underset{\sim}{a}$ $\underset{\sim}{x}$



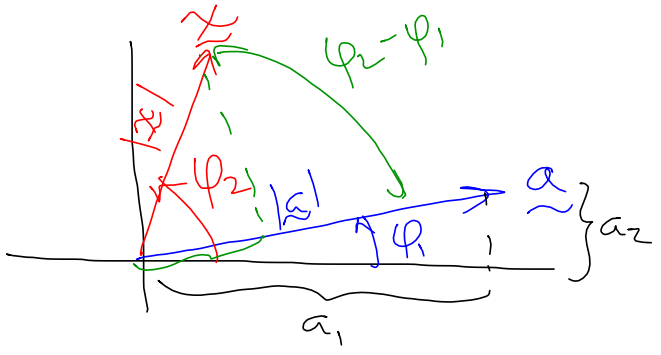
$b = |\underset{\sim}{x}| \cos\theta$

DOT PRODUCT

$$\underset{\sim}{a} \circ \underset{\sim}{x} = a_1 x_1 + a_2 x_2 + \cdots + a_N x_N = \sum_{n=1}^{N} a_n x_n$$



$$\int_{-\infty}^{+\infty} a(x)\, f(x)\, dx = b$$



ψ PSI

φ PHI

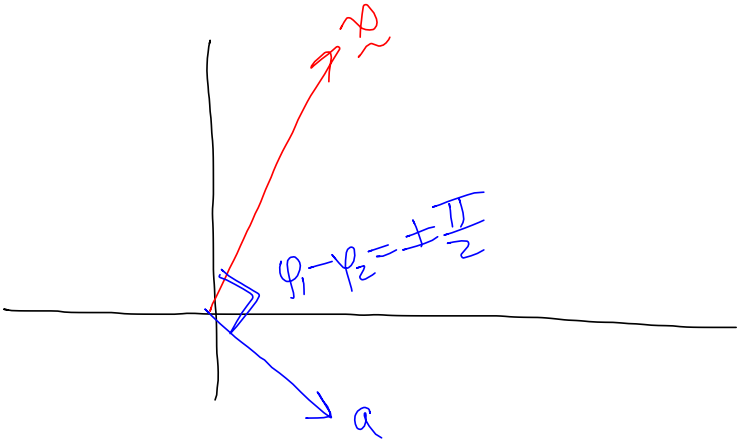
$$\underset{\sim}{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = a_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left.\begin{aligned} a_1 &= |\underset{\sim}{a}| \cos\varphi_1 \\ a_2 &= |\underset{\sim}{a}| \sin\varphi_1 \end{aligned}\right\} \quad \left.\begin{aligned} \chi_1 &= |\underset{\sim}{\chi}| \cos\varphi_2 \\ \chi_2 &= |\underset{\sim}{\chi}| \sin\varphi_2 \end{aligned}\right.$$

$$\underset{\sim}{a} \circ \underset{\sim}{\chi} = |\underset{\sim}{a}|\,|\underset{\sim}{\chi}| \underbrace{\left(\cos\varphi_1 \cos\varphi_2 + \sin\varphi_1 \sin\varphi_2\right)}_{\cos(\varphi_1 - \varphi_2)}$$

$$= |\underset{\sim}{a}|\,|\underset{\sim}{\chi}| \cos(\varphi_1 - \varphi_2)$$

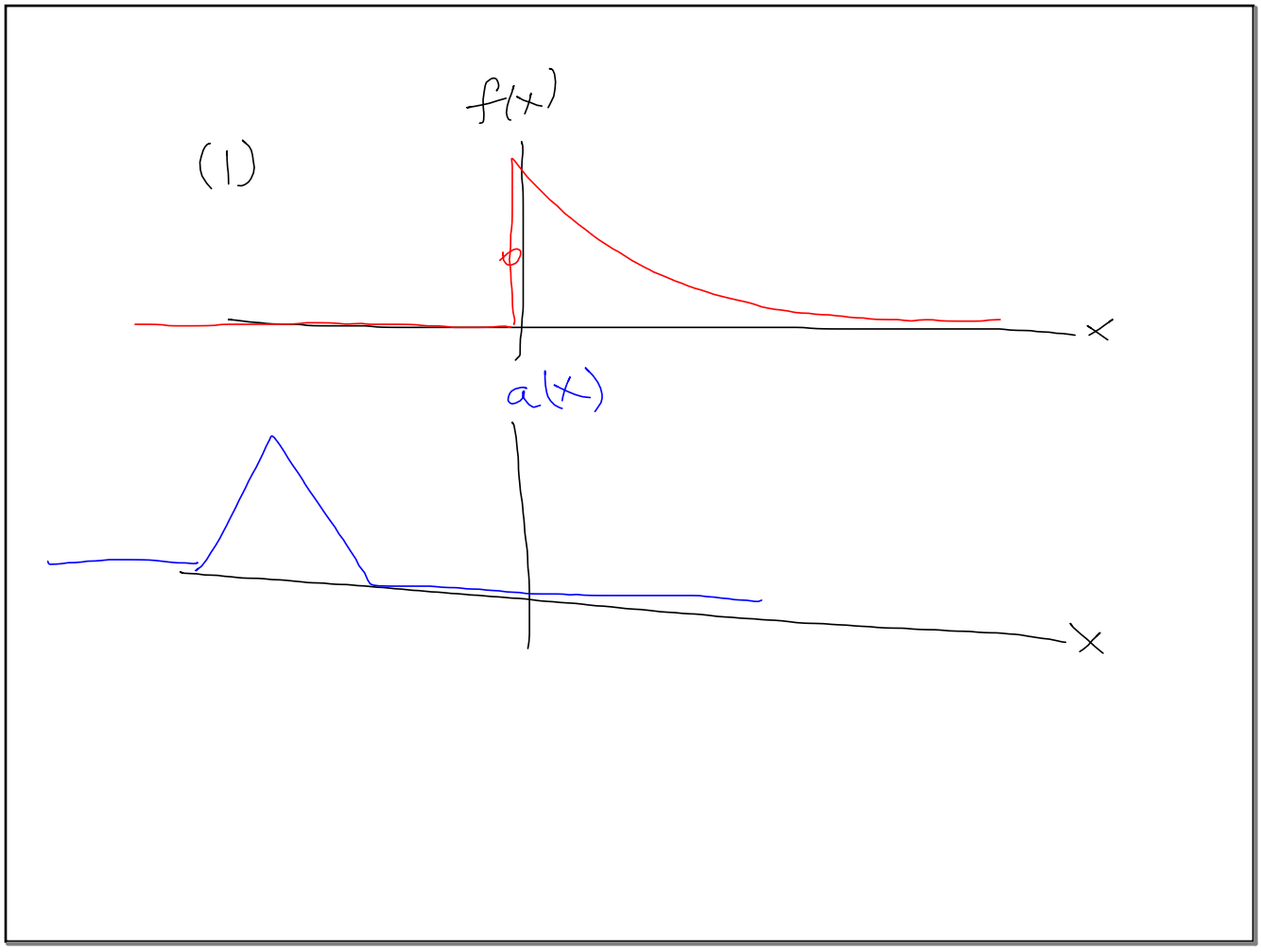
PROJECTION OF $\underset{\sim}{\chi}$ ONTO $\underset{\sim}{a}$ IS $\dfrac{\underset{\sim}{a} \circ \underset{\sim}{\chi}}{|\underset{\sim}{a}|} = \dfrac{|\underset{\sim}{a}|\,|\underset{\sim}{\chi}| \cos(\varphi_1 - \varphi_2)}{|\underset{\sim}{a}|}$

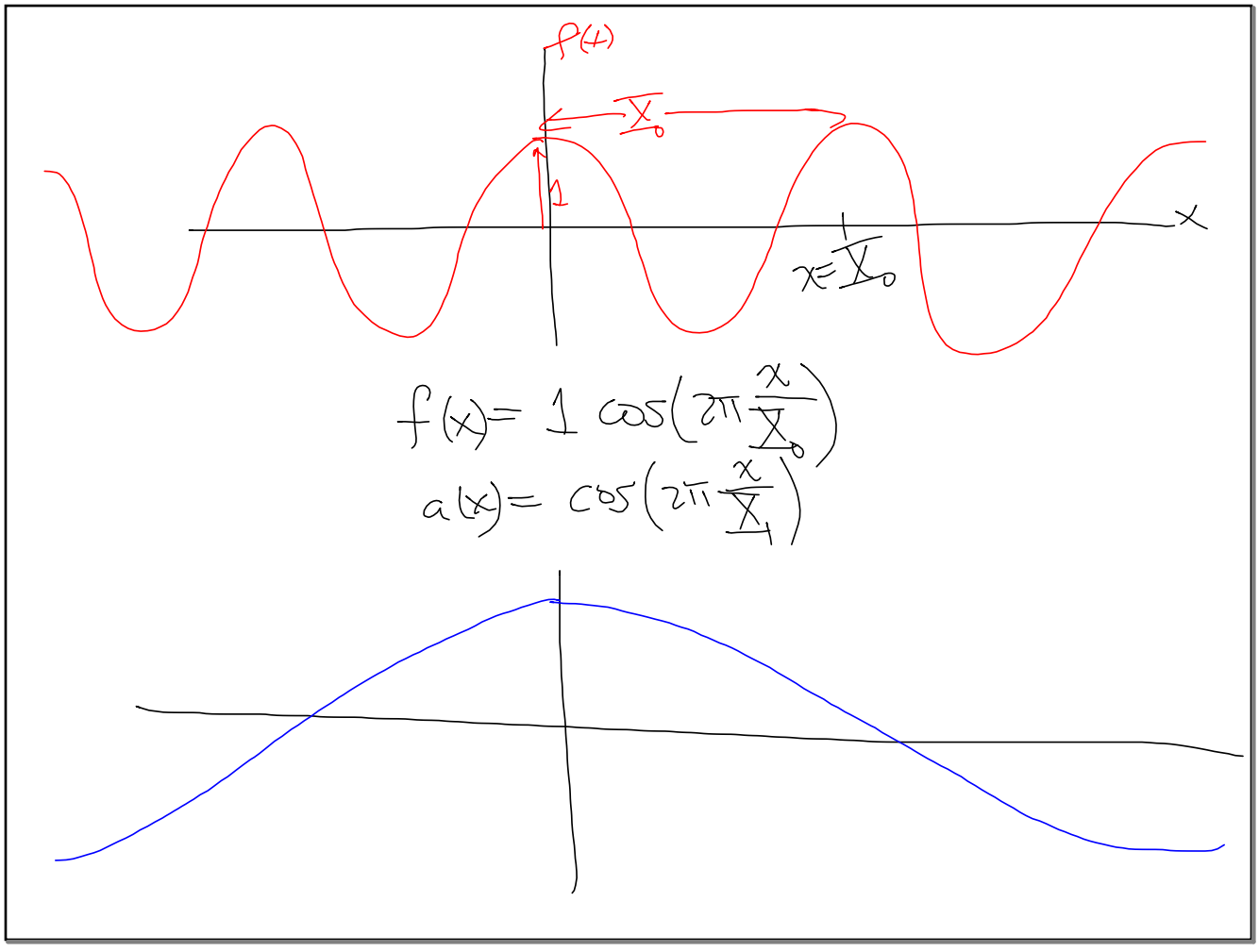


$$\underset{\sim}{a} \cdot \underset{\sim}{x} = 0 \Rightarrow \underset{\sim}{x} \perp \underset{\sim}{a}$$

$$\int_{-\infty}^{+\infty} f(x)\, a(x)\, dx \longrightarrow \text{"LENGTH" OF } f(x) \text{ IN "DIRECTION" OF } a(x)$$

$$\longrightarrow 0 \Rightarrow f(x) \perp a(x)$$


f(x)
(1)
x
a(x)
x


p(x)
X_0
1
x
$x = X_0$
$f(x) = 1 \cos\left(2\pi \frac{x}{X_0}\right)$
$a(x) = \cos\left(2\pi \frac{x}{X_1}\right)$

$$f(x) = \cos\left(2\pi \frac{x}{4}\right)$$

$$a(x) = \sin\left(2\pi \frac{x}{4}\right) = \cos\left(2\pi \frac{x}{4} - \frac{\pi}{2}\right)$$

$$\int_{-\infty}^{+\infty} \cos\left(2\pi \frac{x}{4}\right) \sin\left(2\pi \frac{x}{4}\right) dx$$

$$\cos A \sin B = \frac{1}{2}\sin(B-A) + \frac{1}{2}\sin(B+A)$$

$$\int \frac{1}{2}\sin(0)\,dx + \frac{1}{2}\int \sin(\pi x)\,dx$$