$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

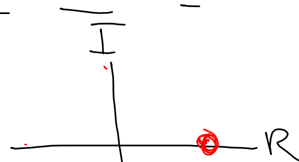
$$\underset{\sim}{A} \leftarrow * h(x), \quad \underset{\sim}{x} \leftarrow f(x), \quad \underset{\sim}{b} \leftarrow g(x)$$

IF $\underset{\sim}{A}$ IS CIRCULANT

$$\begin{bmatrix}\alpha_0 & \beta_0 & \gamma_0 & \delta_0\\ \delta_0 & \alpha_0 & \beta_0 & \gamma_0\\ \gamma & \delta & \alpha & \beta\\ \beta & \gamma & \delta & \alpha\end{bmatrix}\underbrace{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}_{\delta(x)} = \underbrace{\begin{bmatrix}\alpha_0\\ \delta_0\\ \gamma_0\\ \beta_0\end{bmatrix}}_{h(x)}$$

$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}\underbrace{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}}_{\delta(x-1)} = \underbrace{\begin{bmatrix}\beta_0\\ \alpha_0\\ \delta_0\\ \gamma_0\end{bmatrix}}_{h(x-1)}$$

$$\hat{A}\,\hat{x} = \underset{\sim}{b}$$



$$f(x) * h(x) = g(x)$$

$$\hat{A}\,\hat{x} = \underset{\sim}{b}$$

$$\underset{\sim}{\Lambda}\,x' = \underset{\sim}{b'}$$

$$H(\xi)\cdot F(\xi) \quad \begin{pmatrix}\lambda_0 & & & 0\\ & \lambda_1 & & \\ & & \lambda_2 & \\ 0 & & & \lambda_3\end{pmatrix}$$

$$\underset{\sim}{\hat{x}_0} = \frac{1}{\sqrt{4}}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \quad \xi = 0$$

$$\underset{\sim}{\hat{x}_1} = \frac{1}{2}\begin{bmatrix}1\\i\\-1\\-i\end{bmatrix}; \quad \xi = \frac{1}{4}\,\frac{\text{cycle}}{\text{pixel}}$$

$$\underset{\sim}{D} = \begin{bmatrix}\underset{\sim}{\hat{x}_0} & \underset{\sim}{\hat{x}_1} & \underset{\sim}{\hat{x}_2} & \underset{\sim}{\hat{x}_3}\end{bmatrix}; \quad \underset{\sim}{D}^{-1} = \underset{\sim}{D}^{*}$$

$$\underset{\sim}{x}' = \underset{\sim}{D}^{-1}\underset{\sim}{x} \longrightarrow \mathcal{F}\{f(x)\} = F(\xi)$$

LSI

$$f(x) * h(x) = g(x)$$

$$F(\xi) \cdot H(\xi) = G(\xi)$$

$$\int f(x) \left[e^{+i2\pi\xi x} \right]^* dx = F(\xi)$$

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

$$\underset{\sim}{\Lambda}\,\underset{\sim}{x}' = \underset{\sim}{b}'$$

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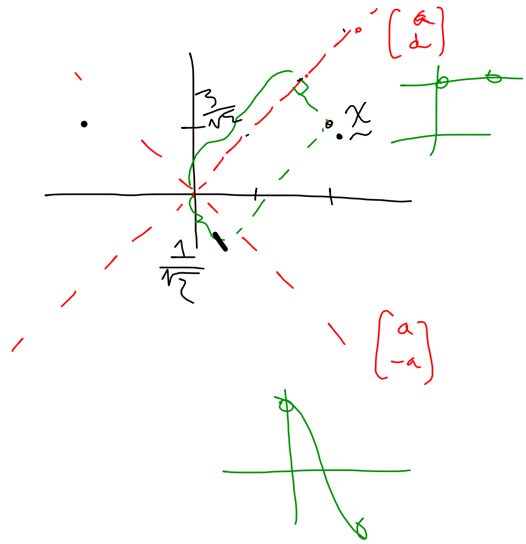
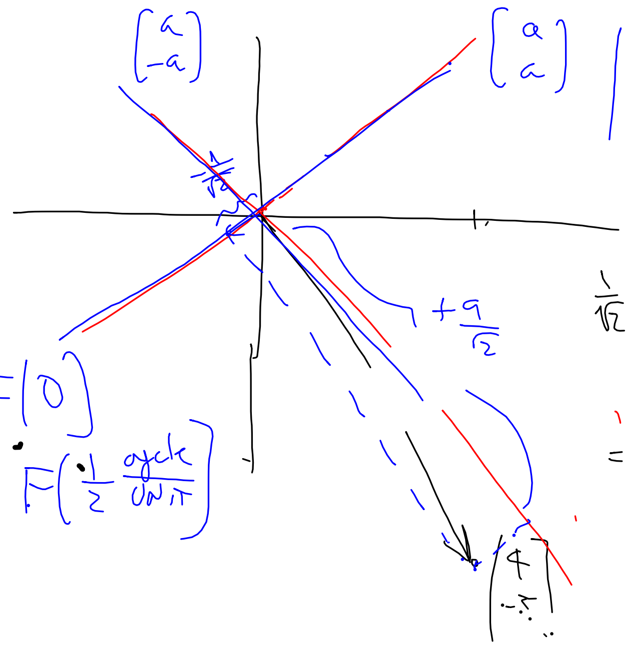
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}3\\ 1\end{pmatrix}.$$





$$\left\| \begin{bmatrix} 4 \\ -5 \end{bmatrix} \right\| = \sqrt{16+25} = \sqrt{41}$$

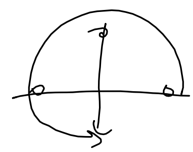
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 9 \end{bmatrix} = \frac{\sqrt{1^2+9^2}}{\sqrt{2}}$$

$$\sqrt{\frac{82}{2}} = \sqrt{41}$$

$$f(x) \longrightarrow F(0)$$

$$f(x) \longrightarrow F\left(\frac{1}{2}\,\frac{\text{cycle}}{\text{unit}}\right)$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & +i \\ 1 & -1 & 1 & -1 \\ 1 & +i & -1 & -i \end{pmatrix} \underset{f(x)}{\overset{\underset{\sim}{x}}{\begin{pmatrix} 1 \\ 4 \\ 2 \\ -1 \end{pmatrix}}} = \underset{F(\xi)}{\overset{\underset{\sim}{x}'}{\begin{pmatrix} 6/2 \\ \frac{1}{2}(-1-5i) \\ 0 \\ \frac{1}{2}(-1+5i) \end{pmatrix}}}$$



$$\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} = 0$$

$$\int_{-\infty}^{\infty} f(x)\, e^{-i2\pi \xi x}\, dx = F(\xi) \qquad \alpha_0 = \sqrt{\lambda_0 z_1} \leftarrow f$$

$$\left(\left(f(x)\, e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}\right) * e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}\right) \cdot e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}\Bigg|_{x \to \alpha_0^2 \xi} = F(\xi)$$

M C

$$F\left(\frac{x}{\alpha_0^2}\right)$$

$$f(x) = \delta(x - x_0) \longrightarrow \mathcal{F}[\xi]\; e^{-i2\pi \xi x_0} = F(\xi)$$

(1) $\delta(x-x_0)\, e^{-i\pi\frac{x^2}{\alpha_0^2}} = \delta(x-x_0)\, e^{-i\pi\frac{x_0^2}{\alpha_0^2}}$

$$f(x)\,\delta(x-x_0) = f(x_0)\,\delta(x-x_0)$$

(2) $e^{-i\pi\frac{x_0^2}{\alpha_1^2}}\left[\delta(x-x_0) * e^{+i\pi\frac{x^2}{\alpha_0^2}}\right] = e^{-i\pi\frac{x_0^2}{\alpha_1^2}}\left(e^{+i\pi\frac{(x-x_0)^2}{\alpha_0^2}}\right)$

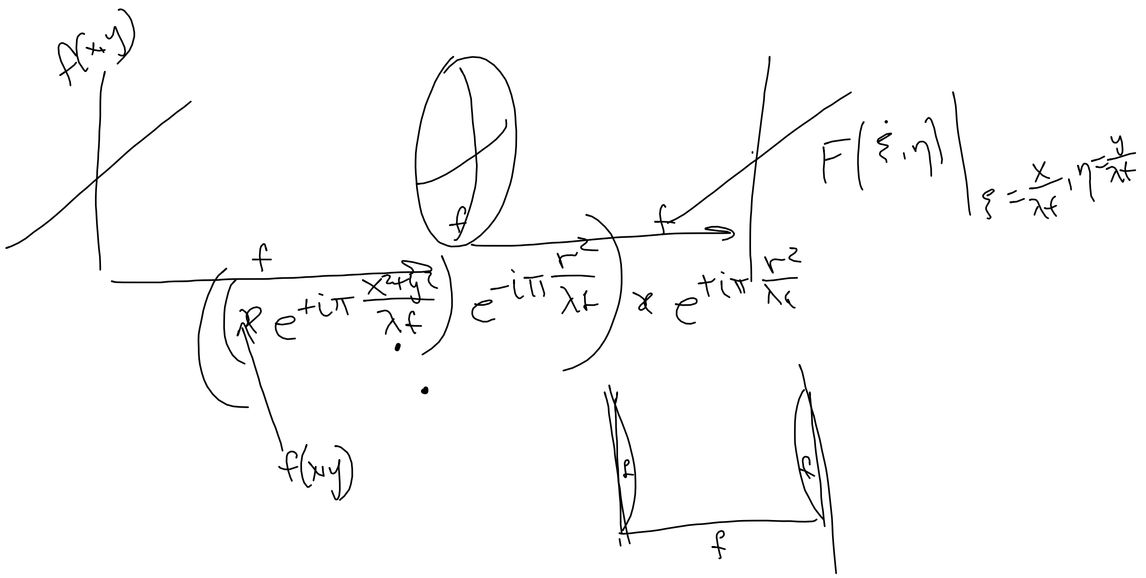
(3) $\left(e^{-i\pi\frac{x_0^2}{\alpha_1^2}}\; e^{+i\pi\frac{(x-x_0)^2}{\alpha_0^2}}\right) \cdot e^{-i\pi\frac{x^2}{\alpha_0^2}}$

$$e^{-i\pi\frac{x_0^2}{\alpha_1^2}}\; e^{+i\pi\frac{x^2+x_0^2-2xx_0}{\alpha_1^2}}\; e^{-i\pi\frac{x^2}{\alpha_1^2}}$$

$$\cancel{e^{-i\pi\frac{x_0^2}{\alpha_1^2}}}\; \cancel{e^{+i\pi\frac{x^2}{\alpha_1^2}}}\; \cancel{e^{+i\pi\frac{x_0^2}{\alpha_1^2}}}\; e^{-i2\pi\frac{xx_0}{\alpha_1^2}}\; \cancel{e^{-i\pi\frac{x^2}{\alpha_0^2}}}$$

$$e^{-i2\pi\frac{x \cdot x_0}{\alpha_0^2}} = F\left(\xi = \frac{x}{\alpha_1^2}\right)$$

$$e^{-i2\pi \xi x_0}$$



$$\left(\left(f\, e^{+i\pi \frac{x^2+y^2}{\lambda f}}\right) e^{-i\pi \frac{r^2}{\lambda f}}\right) \times e^{+i\pi \frac{r^2}{\lambda f}}$$

$$F(\xi,\eta)\Big|_{\xi=\frac{x}{\lambda f},\ \eta=\frac{y}{\lambda f}}$$

$$w(\xi) = \frac{1}{H(\xi)} = \frac{H^*(\xi)}{|H(\xi)|^2}$$

$$w(x) = \mathcal{F}^{-1}\left\{\frac{1}{H(\xi)}\right\} = \mathcal{F}^{-1}\left\{H^*(\xi)\right\} * \mathcal{F}^{-1}\left\{\frac{1}{|H(\xi)|^2}\right\}$$

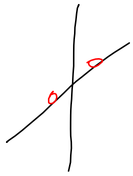
$$= h^*(-x) * \mathcal{F}^{-1}\left\{\frac{1}{|H(\xi)|^2}\right\}$$

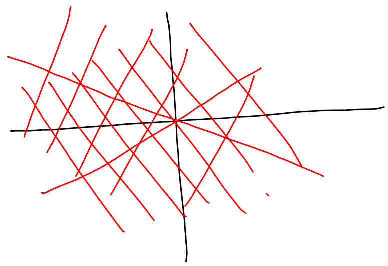
$$f(x) * h(x) + n(x) = g(x)$$

$$W(\xi) = \frac{|F(\xi)|^2 H^*(\xi)}{|F(\xi)|^2 |H(\xi)|^2 + |N(\xi)|^2}$$

$$|F(\xi)|^2 \to |F(\xi)|$$

$$\Phi_F(\xi)$$





$$\left(\delta(x+x_0)+\delta(x-x_0)\right) * e^{+i\pi \frac{x^2}{\alpha_0^2}}$$

(λz)

$$e^{+i\pi\frac{(x+x_0)^2}{\alpha_0^2}} + e^{+i\pi\frac{(x-x_0)^2}{\alpha_0^2}}$$

$$e^{+i\pi\frac{x^2+x_0^2+2xx_0}{\alpha_0^2}} + e^{+i\pi\frac{x^2+x_0^2-2xx_0}{\alpha_0^2}}$$

$$e^{+i\pi\frac{x^2}{\alpha_0^2}}\, e^{+i\pi\frac{x_0^2}{\alpha_0^2}}\, e^{+i2\pi\frac{xx_0}{\alpha_0^2}} + e^{+i\pi\frac{x^2}{\alpha_0^2}}\, e^{+i\pi\frac{x_0^2}{\alpha_0^2}}\, e^{-i2\pi\frac{xx_0}{\alpha_0}}$$

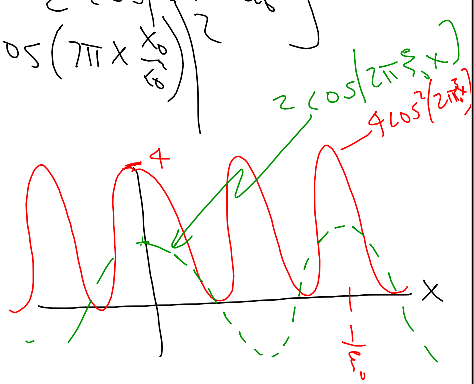
$$e^{+i\pi\frac{x^2}{\alpha_0}}\, e^{+i\pi\frac{x_0^2}{\alpha_1^2}} \left[\underbrace{e^{+i2\pi\frac{xx_0}{\alpha_0^2}} + e^{-i2\pi\frac{xx_0}{\alpha_0^2}}}_{2\cos\left[2\pi x\frac{x_0}{\alpha_0^2}\right]}\right]$$

$$\left| e^{+i\pi\frac{x^2}{\alpha_0^2}}\, e^{+i\pi\frac{x_0^2}{\alpha_0^2}} \cdot 2\cos\left(2\pi x\frac{x_0}{\alpha_0}\right)\right|^2$$

$$\left|2\cos\left[2\pi x\left(\frac{x_0}{\alpha_0^2}\right)\right]\right|^2$$

$$= 4\cos^2\left[2\pi x\left(\frac{x_0}{\alpha_0^2}\right)\right]$$

ξ_0



$\cos A \cos B$

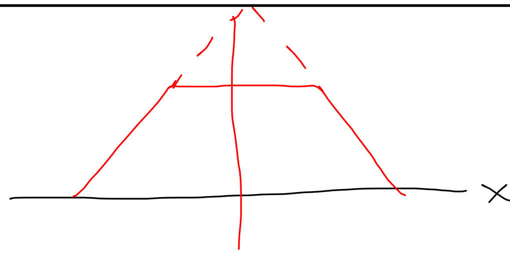
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

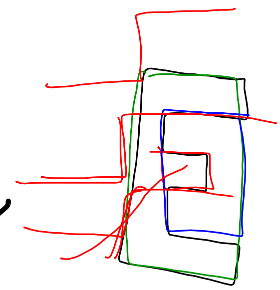
$$\cos(A-B) + \cos(A+B) = 2\cos A \cos B$$

$$\cos(2\pi x) + \cos(6\pi x) = 2\cos(4\pi x)\cos(2\pi x)$$

$A - B = 2\pi x \quad A = 4\pi x$

$A + B = 6\pi x \quad B = 2\pi x$



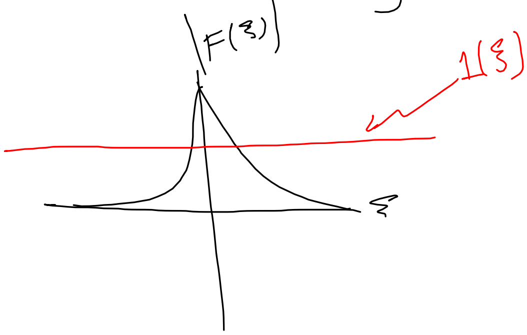


$$|F(\xi)| = O(\xi)$$

$$f(x) = O(x)$$

$$|F(\xi)| \to 1(\xi)$$

$$\phi_F(\xi)$$



$$1(\xi)\, e^{i\phi_F(\xi)} \xrightarrow{\mathcal{F}^{-1}}$$

$$|F(\xi)|\, e^{i\cdot 0} \xrightarrow{\mathcal{F}^{-1}}$$