DFT $\quad f(n) \; ; \quad 0 \le n \le N-1$

$N = 2^m$, e.g. 1024

$$F(k) = c_1 \sum_{n=0}^{N-1} f(n)\, e^{-i 2\pi (n) \cdot \left(\frac{k}{N}\right)}$$

FREQ AS
\# OF CYCLES PER PIXEL

$$k = 0 \Rightarrow c_1 \sum_{n=0}^{N-1} f(n)$$

$$c_1 = \frac{1}{N}$$

$$= 1$$

$$= \frac{1}{\sqrt{N}}$$

$$k = 1 \; ; \; \xi = \frac{1}{N} \frac{\text{cycles}}{\text{pixel}}$$

$$f(n) = c_2 \sum_{k=0}^{N-1} F(k)\, e^{+i 2\pi \frac{k}{N} \cdot n}$$

$$c_1 \cdot c_2 = \frac{1}{N}$$

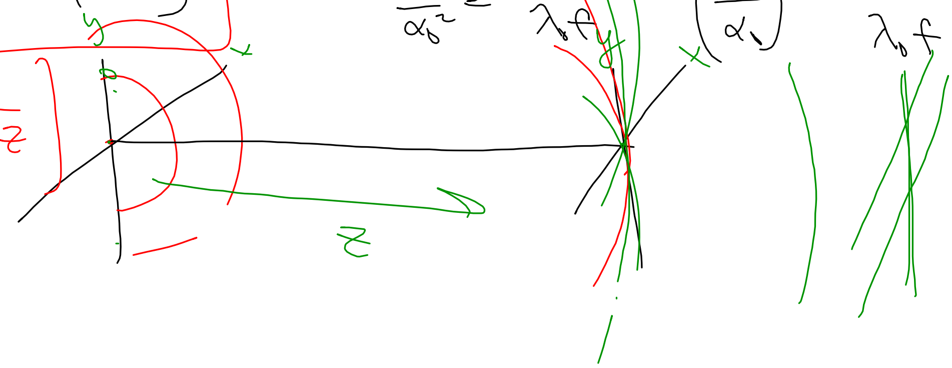
$$F\left[\xi = \frac{x}{\alpha_0^2}\right] = \left[\left(f(x)\cdot e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}\right) * e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}\right] e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}$$

$$\boxed{F[\xi] \to F\left[\frac{x}{\alpha_0^2}\right]}$$

$$\boxed{\alpha_0 = \sqrt{\lambda_0 z}} \to \sqrt{\lambda_0 f}$$

$$\frac{x}{\alpha_0^2} = \frac{x}{\lambda_0 f} \qquad \left(\frac{x}{\alpha_0}\right)^2 = \frac{x^2}{\lambda_0 f}$$

$$F\left[\frac{x}{\lambda_0 z}\right]$$



$$f(x) * 1(x) = \int f(\alpha)\, \underbrace{1(x-\alpha)}_{1}\, d\alpha$$

$$\int f(\alpha)\, d\alpha \quad \text{FOR EACH } x$$

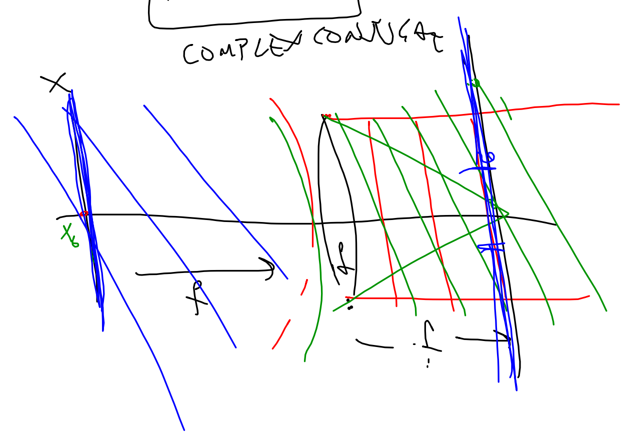
CHIRP FOURIER TRANSFORM

$$F(\xi) = \left[\left[f(x)\cdot e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}\right] * e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}\right]\cdot e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}\Bigg|_{\frac{x}{\alpha_0^2}\to\xi}$$

$$F\left(\frac{x}{\alpha_0^2}\right)$$

COMPLEX CONJUGATE

C-M-C



FILTERING

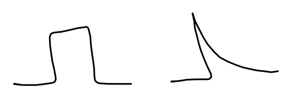
IMAGING TASKS

$$\mathcal{O}\{f(x)\} = g(x)$$

(1) DIRECT
(2) INVERSE
(3) ANALYSIS
(4) "LOCATION" MATCHING

INVERSE $f(x) * \underbrace{h(x)}_{\mathcal{O}} = g(x)$

(WHERE'S WALDO)



$$f(x) * h(x) = g(x) \rightarrow \underbrace{\left(f(x) * h(x)\right)}_{g(x)} * w(x) = f(x)$$

$$f(x) * \underbrace{\left(h(x) * w(x)\right)}_{\delta(x)} = f(x)$$

$$F(\xi) \cdot H(\xi) = G(\xi)$$

$$F(\xi) = G(\xi) \cdot W(\xi)$$

$$= F(\xi) \cdot \left(H(\xi) \cdot W(\xi)\right)$$

$1/\xi$

H

$$W(\xi) = \frac{1}{H(\xi)} =$$

$$W(\xi) = \frac{1}{H(\xi)} = \frac{H^*(\xi)}{H^*(\xi)\cdot H(\xi)} = \frac{H^*(\xi)}{|H(\xi)|^2} = W(\xi) = H^*(\xi)\cdot\frac{1}{|H(\xi)|^2}$$

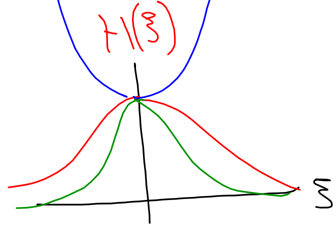
$$w(x) = \mathcal{F}^{-1}\{W(\xi)\} = \mathcal{F}^{-1}\{H^*(\xi)\} * \mathcal{F}^{-1}\left\{\frac{1}{|H(\xi)|^2}\right\}$$

$$= h^*(-x) * \mathcal{F}^{-1}\left\{\frac{1}{|H|^2}\right\}$$

$$g(x) = f(x) * h(x)$$

$$\hat{f}(x) = g(x) * w(x) = f(x) * \underbrace{h(x) * h^*(-x)}_{h(x) \star h(x)} * \mathcal{F}^{-1}\left\{\frac{1}{|H|^2}\right\}$$

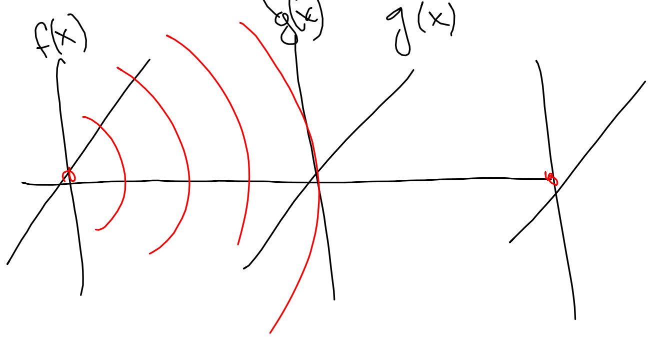
ESTIMATE OF $f(x)$



$$g(x) = f(x) \times e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}$$

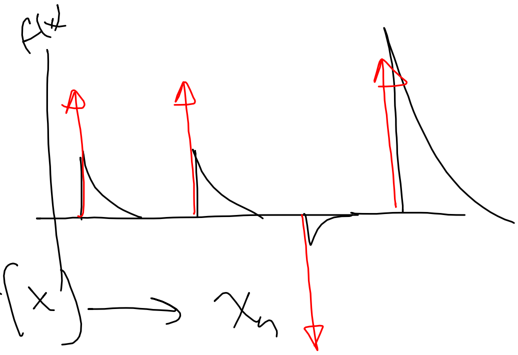
$$g(x) * w(x) = \left(f(x) \times e^{+i\pi\left(\frac{x}{\alpha_0}\right)^2}\right) \times \left(e^{+i\pi\left(\frac{-x}{\alpha_0}\right)^2}\right)^*$$

$$\underbrace{h^*(-x)}_{e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}}$$



WHERE'S WALDO → MATCHED FILTER

$$f(x) = \sum_n \alpha_n r(x - x_n)$$



$$r(x) = \text{"WALDO"}$$

OUTPUT → LOCATE $r(x) \longrightarrow x_n$

$$F(\xi) = \sum_n \alpha_n R(\xi)\, e^{-i2\pi\xi x_n}$$

$$U(\xi) = \sum_n \alpha_n e^{-i2\pi\xi x_n}$$

$$M(\xi) = \frac{1}{R(\xi)}$$

$$f(x) * m(x) = \underbrace{u(x)}_{\sum \alpha_n \delta(x-x_n)} \to \text{ESTIMAT}\ x_n$$

$$F(\xi)\cdot M(\xi) = \left(\sum_n \alpha_n \cancel{R(\xi)}\, e^{-i2\pi\xi x_n}\right)\cdot \frac{1}{\cancel{R(\xi)}}$$

$$\longrightarrow \sum_n \alpha_n\, \delta(x-x_n)$$

$$M(\xi) = \frac{1}{R(\xi)} = \frac{R^*(\xi)}{|R(\xi)|^2}\,;\quad m(x) = \mathcal{F}^{-1}\{R^*(\xi)\} * \mathcal{F}^{-1}\left\{\frac{1}{|R(\xi)|^2}\right\}$$

$$W(\xi) = \frac{1}{H(\xi)}$$

$$m(x) = \frac{r^*(-x)}{|R(\xi)|e^{-i\Phi_R(\xi)}} * \mathcal{F}^{-1}\left\{\frac{1}{|R(\xi)|^2}\right\}$$

WHAT IF $|R(\xi)|^2 = 1(\xi)$?

$$r(x) \Rightarrow R(\xi) = |R(\xi)|\, e^{i\Phi_R(\xi)}$$

$$\frac{1}{R(\xi)} = \frac{1}{|R(\xi)|\, e^{i\phi_R(\xi)}} = \frac{1}{|R(\xi)|}\, e^{-i\phi_R(\xi)}$$

REALISTIC M.F. PROBLEM

$$g(x) = f(x) + \underbrace{n(x)}_{\text{NOISE}} = \left(\sum_n \alpha_n r(x-x_n)\right) + n(x)$$

$$g(x) \star m(x) = \left(\sum_n \alpha_n \underline{r(x-x_n)}\right) * \underline{r^*(-x)} \star \mathcal{F}^{-1}\left\{\frac{1}{|R|^2}\right\}$$

$$+ \underline{n(x)} \star \underline{r^*(-x)} \star \mathcal{F}^{-1}\left\{\frac{1}{|R|^2}\right\}$$

$r(x) \longrightarrow m(x) = r^{*}(-x)$ IN "CLASSICAL" MATCHED FIL

$$f(x) * h(x) + \underline{\underline{n(x)}} = g(x)$$

(1) $f(x) + n(x) = g(x)$; $h(x) = \delta(x)$

TASK: ESTIMATE $f(x)$ IN NOISE

(2) KNOW $g(x)$, $h(x) \rightarrow \hat{f}(x)$

$$f(x) + n(x) = g(x) \quad ; \quad N$$

$G(\xi)$, IF KNOW $N(\xi)$

$$\left(F(\xi) + N(\xi)\right) - N(\xi)$$

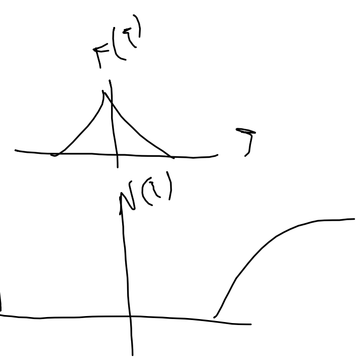
$$g(x) * w(x) = f(x)$$

$$\left(f(x) + n(x)\right) * w(x) = f(x)$$

$$\left(F(\xi) + N(\xi)\right) \cdot W(\xi) = F(\xi)$$

$$W(\xi) = \frac{F(\xi)}{F(\xi) + N(\xi)}$$

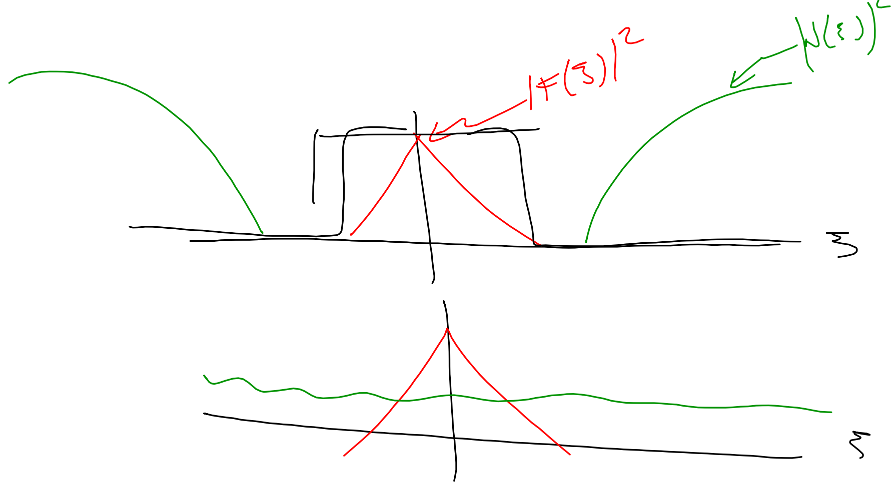
$$w(x) = \mathcal{F}^{-1}\left\{ \frac{F(\xi)}{F(\xi) + N(\xi)} \right\}$$



WIENER FILTER

KNOW $|F(\xi)|^2, \ |N(\xi)|^2$

$$W(\xi) = \frac{|F(\xi)|^2}{|F(\xi)|^2 + |N(\xi)|^2} = \frac{1}{1 + \frac{|N(\xi)|^2}{|F(\xi)|^2}}$$



$$f(x) * h(x) + n(x) = g(x)$$

$$w(x) \text{ S.T. } g(x) * w(x) = \hat{f}(x)$$

WIENER

$$W(\xi) = \frac{|F(\xi)|^2}{|F(\xi)|^2 + |N(\xi)|^2}$$

WIENER-HELSTROM

$$W(\xi) = \frac{|F(\xi)|^2 \, \underline{H^*(\xi)}}{|F(\xi)|^2 \, \underline{|H(\xi)|^2} + |N(\xi)|^2}$$

$$|N(\xi)|^2 = 0 \Rightarrow W(\xi) = \frac{H^*(\xi)}{|H(\xi)|^2} = \frac{1}{H(\xi)}$$

IF $|N(\xi)|^2 >> |F(\xi)|^2 \, |H(\xi)|^2$

$$W(\xi) = H^*(\xi) \left(\frac{|F(\xi)|^2}{|N(\xi)|^2} \right)$$

k_0

$$w(x) = h^*(-x) \cdot k_0$$

$$h(x) = e^{+i\kappa\left(\frac{x}{\alpha_0}\right)^2} \cdot e^{-\pi x^2}$$