PHASE FILTERS

$$H[\xi] = 1[\xi]\, e^{i\pi W[\xi]}$$

IF WELL "BEHAVED"

$$W[\xi] = a_0 \xi^0 + a_1 \xi^1 + a_2 \xi^2 + \cdots$$

$$= \alpha_0 \xi^0 + (\alpha_1 \xi)^1 + (\alpha_2 \xi)^2 + \cdots$$

$$\alpha_n^n = a_n$$

$$H[\xi] = e^{+i\pi(\alpha_0 + \alpha_1 \xi + \alpha_2^2 \xi^2 + \cdots)}$$

$$= e^{+i\pi\alpha_0} \cdot e^{i\pi\alpha_1 \xi} \cdot e^{i\pi \alpha_2^2 \xi^2} \cdots$$

$$h[x] = \mathcal{F}^{-1}\{e^{i\pi\alpha_0}\} * \mathcal{F}^{-1}\{e^{i\pi\alpha_1\xi}\} * \mathcal{F}^{-1}\{e^{i\pi\alpha_2^2\xi^2}\} * \cdots$$

ZERO-ORDER ⇒ CONSTANT PHASE

$$H_0(\xi) = e^{i\pi\alpha_0} \cdot 1(\xi) \rightarrow h_0(x) = e^{i\pi\alpha_0}\,\delta(x)$$

$$f(x) * h_0(x) = f(x) * \boxed{e^{i\pi\alpha_0}\,\delta(x)}$$

INFINITESIMAL SUPPORT

$$= e^{i\pi\alpha_0} f(x) = g(x)$$

$$g(x) \star g(x) = g(x) * g^*(-x)$$

$$= f(x) \star f(x)$$

AUTOCORRELATION IS UNCHANGED AFTER PHASE FILTER

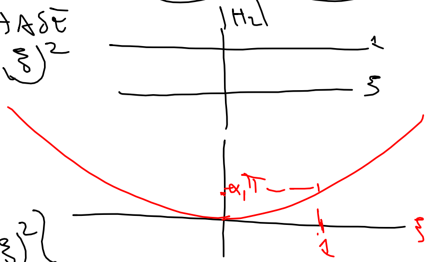
FIRST-ORDER, LINEAR PHASE

$$H_1[\xi] = 1[\xi]\, e^{+i\pi\alpha_1\xi} = e^{+i2\pi\left(\frac{\alpha_1}{2}\right)\xi}$$

$$h_1[x] = \delta\left[x + \frac{\alpha_1}{2}\right]$$ INFINITESIMAL SUPPORT

SECOND ORDER, QUADRATIC PHASE

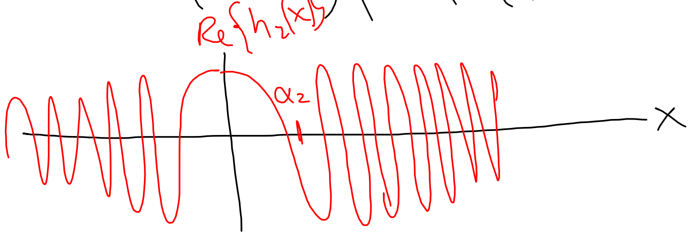
$$H_2[\xi] = 1[\xi]\, e^{+i\pi(\alpha_2\xi)^2}$$



$$h_2[x] = \mathcal{F}^{-1}\left\{e^{+i\pi(\alpha_2\xi)^2}\right\}$$

$$= \left(\frac{1}{|\alpha_2|}e^{+i\frac{\pi}{4}}\right)e^{-i\pi\left(\frac{x}{\alpha_2}\right)^2}$$ CHIRP

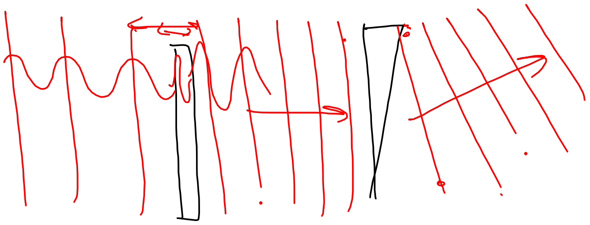
$$= \left(\quad\right)\left(\cos\left(\pi\left(\frac{x}{\alpha_2}\right)^2\right) - i\sin\left(\pi\left(\frac{x}{\alpha_2}\right)^2\right)\right)$$

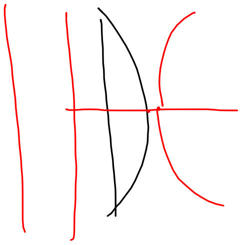


INFINITE SUPPORT

CONSTANT PHASE

$$h_0(x) = e^{i\pi\alpha_0}\,\delta(x)$$





$$H(\xi) = 1(\xi)\, e^{i\pi W(\xi)}$$

$$|H(\xi)|^2 = 1(\xi)$$

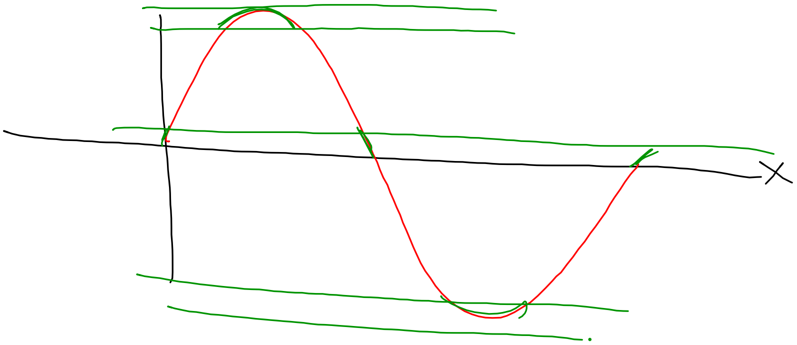
$$\mathcal{F}^{-1}\left\{ |H(\xi)|^2 \right\} = h(x) \star h(x) = \mathcal{F}^{-1}\{1(\xi)\} = \delta(x)$$

$$h(x) \ast h^*(-x)$$

$$g(x) = f(x) \ast h(x)$$

$$g(x) \star g(x) = \left(f(x) \ast h(x)\right) \star \left(f(x) \ast h(x)\right) = \left(f(x) \star f(x)\right) \ast \cancelto{\delta(x)}{\left(h(x) \star h(x)\right)}$$

$$H(\xi) = 1(\xi)\, e^{\underbrace{i\pi N(\xi)}_{\text{RANDOM}}}$$



STELLAR SPECKLE INTERFEROMETRY
A. LABEYRIE

$$F(\xi) = \underline{|F(\xi)|}\, e^{\underline{i\phi_F(\xi)}}$$

PHASE MORE IMPORTANT

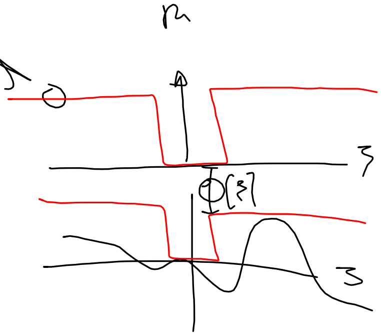
$$f(x) = 1(x)\, e^{\boxed{i\phi_f(x)}}$$ PHASE OBJECT

$$|f(x)|^2 = 1(x)$$

SCHLIEREN IMAGING $e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!}$

$$f(x) = e^{i\phi_f(x)} = \sum_n \frac{(i\phi)^n}{\underset{4}{n!}} = \underline{1} + i\phi + \frac{(i\phi)^2}{2} + \cdots$$

$$\simeq 1 + i\phi(x) - \cancel{\frac{\phi^2(x)}{2}} \to 0$$

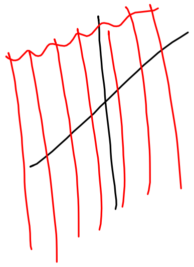


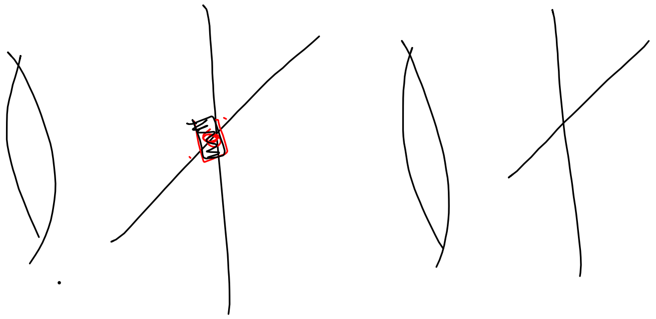
$$F(\xi) \simeq \delta(\xi) + i\mathcal{F}\{\phi(x)\}$$

$$H(\xi) = \underline{1} - \mathrm{RECT}\left(\frac{\xi}{b_0}\right)$$

$$G(\xi) \simeq i\mathcal{F}\{\phi(x)\}$$

$$g(x) \simeq i\phi(x)$$





CHIRP FOURIER TRANSFORM

$$\int_{-\infty}^{+\infty} f(x)\, e^{-i2\pi\xi x}\, dx \qquad e^{-i2\pi\xi x} = e^{+i\pi(\underbrace{-2\xi x})}$$

$$(\xi - x)^2 = \xi^2 + x^2 - 2\xi x$$

$$(\xi - x)^2 - \xi^2 - x^2 = -2\xi x$$

$$\left(\alpha_0\xi - \frac{x}{\alpha_0}\right)^2 = (\alpha_0\xi)^2 + \left(\frac{x}{\alpha_0}\right)^2 - 2\cancel{\alpha_0}\xi\,\frac{x}{\cancel{\alpha_0}}$$

$$-2\xi x = \left(\alpha_0\xi - \frac{x}{\alpha_0}\right)^2 - (\alpha_0\xi)^2 - \left(\frac{x}{\alpha_0}\right)^2$$

$$e^{+i\pi(-2\xi x)} = e^{+i\pi\left(\alpha_0\xi - \frac{x}{\alpha_0}\right)^2}\, e^{-i\pi(\alpha_0\xi)^2}\, e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}$$

$$\underset{M}{e^{-i\pi(\alpha_0\xi)^2}} \int_{-\infty}^{+\infty} \left(\underbrace{f(x)\, e^{-i\pi\left(\frac{x}{\alpha_0}\right)^2}}_{M}\right) \underset{C}{e^{+i\pi\left(\alpha_0\xi - \frac{x}{\alpha_0}\right)^2}}\, dx = F(\xi)$$

$$M - C - M$$