25 NOVEMBER (LAST "OFFICIAL" EARLY CLASS)

DFT

$$\left(\left(f[x] * \frac{1}{\Delta x}\,\mathrm{SINC}\left[\frac{x}{\Delta x}\right]\right)\cdot\frac{1}{\Delta x}\,\mathrm{COMB}\left[\frac{x}{\Delta x}\right]\right)\cdot\mathrm{RECT}\left(\frac{x+\epsilon}{N\cdot\Delta x}\right)$$

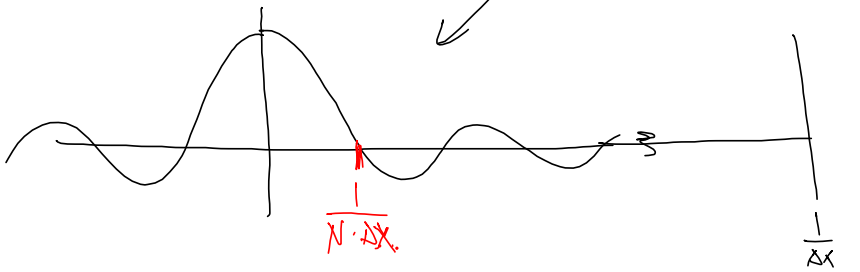
N SAMPLES

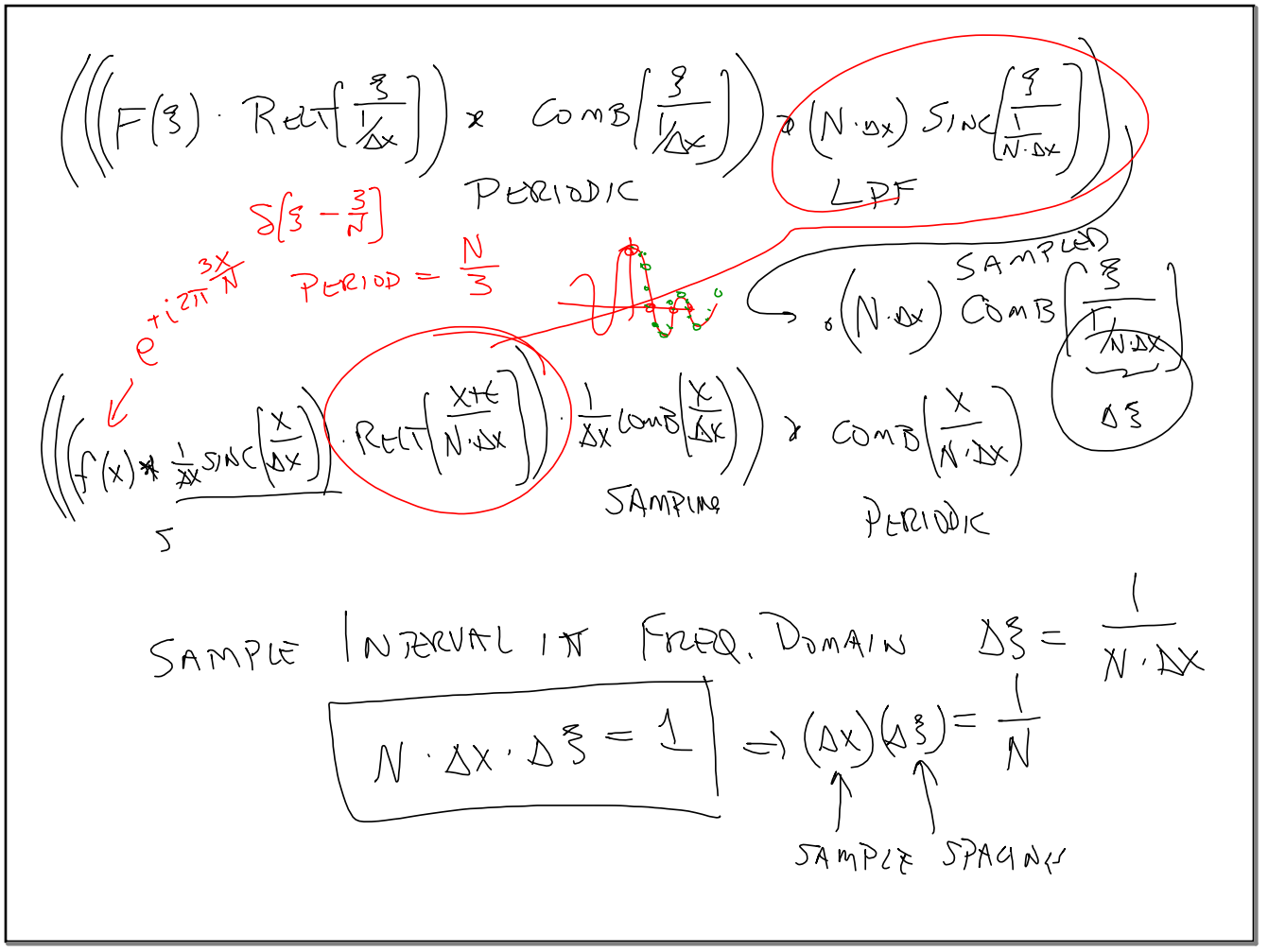
$$\left(F[\xi]\cdot\mathrm{RECT}\left(\frac{\xi}{1/\Delta x}\right)\right) * \mathrm{COMB}\left(\frac{\xi}{1/\Delta x}\right) * (N\cdot\Delta x)\,\mathrm{SINC}\left(N\cdot\Delta x\cdot\xi\right)$$

LPF (ANTIALIAS)

PERIODIC

$$\left(\frac{\xi}{1/(N\cdot\Delta x)}\right)$$



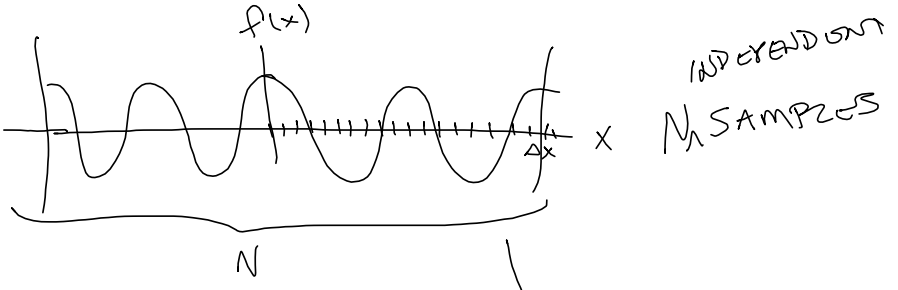

PERIODIC
LPF
SAMPLED
PERIOD = N/3
SAMPLING
PERIODIC
SAMPLE INTERVAL IN FREQ. DOMAIN
SAMPLE SPACING

$$N \cdot \Delta x \cdot \Delta \xi = 1$$

$$\Delta x = 0.1\ mm = 100\ \mu m$$

$$N = 1000$$

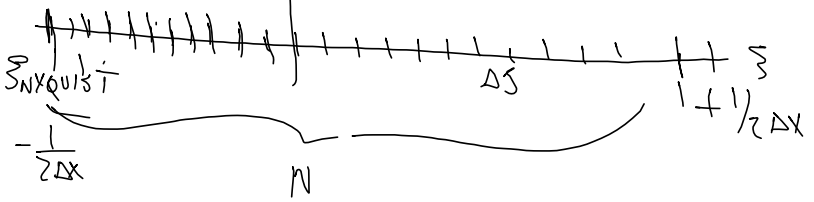
$$\Delta \xi = \frac{1}{1000 \cdot 0.1\ mm} = \frac{1}{100}\ mm^{-1} = \frac{1}{100}\ \frac{cycle}{mm}$$



$$N \cdot \Delta \xi = \frac{1}{\Delta x}$$

$$\xi_{NYQ} = \frac{1}{2\,\Delta x}$$

$$\lambda_{NYQ} = 2 \cdot \Delta x$$



DFT $\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} f(n)\, e^{-i 2\pi \frac{nk}{N}} = F(k)$

$\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} F(k)\, e^{+i 2\pi \frac{nk}{N}} = f(n)$

$N \cdot \Delta x \cdot \Delta s = 1$

$\xi_{MAX} = \frac{1}{2} \frac{\text{cycle}}{\text{SAMPLE}}$

$\lambda_{MIN} = 2 \frac{\text{SAMPLE}}{\text{CYCLE}}$

$$F(\xi) = \sum_{n=0}^{N-1} f(n)\, e^{-i 2\pi n \cdot \Delta x \cdot \xi}$$

ξ: frequency

$N = 256$

$T = 16 \Rightarrow \xi = \frac{1}{T} = \frac{1}{16}$ cycle / SAMPLE

$N \cdot \Delta x \cdot \Delta\xi = 1$

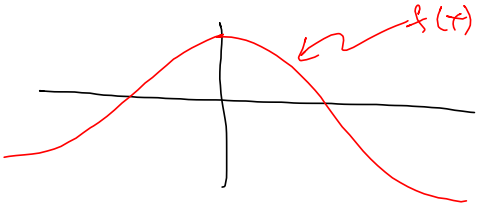
$256 \cdot 1 \cdot \Delta\xi = 1$

$\Delta\xi = \frac{1}{256}$ cycles / SAMPLE

$\xi = \frac{1}{16} = k \cdot \Delta\xi = k \cdot \frac{1}{256} \Rightarrow k = 16$

ξ: frequency

k: INDEX



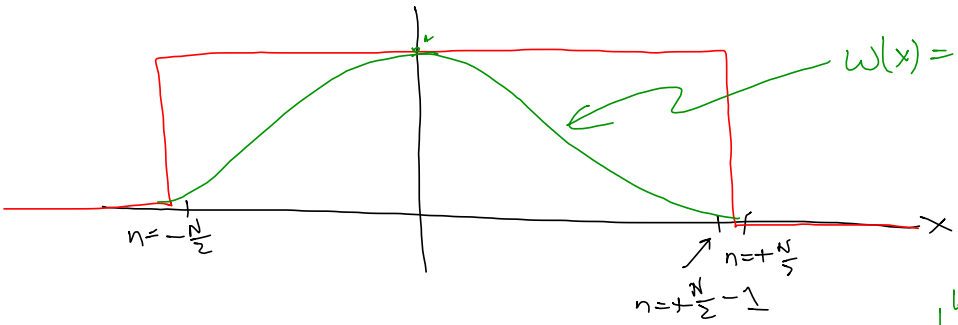
$$\xi_0 = \frac{1}{X} = \frac{1}{256 \cdot \Delta x}$$

$$\sum_{n=0}^{255} \cos\left[2\pi \frac{x}{256}\right] e^{-i 2\pi x \xi}$$

$$\frac{1}{2}\delta(\xi + \xi_0) + \frac{1}{2}\delta(\xi - \xi_0)$$

SINUSOID WITH ℓ CYCLES IN N SAMPLES

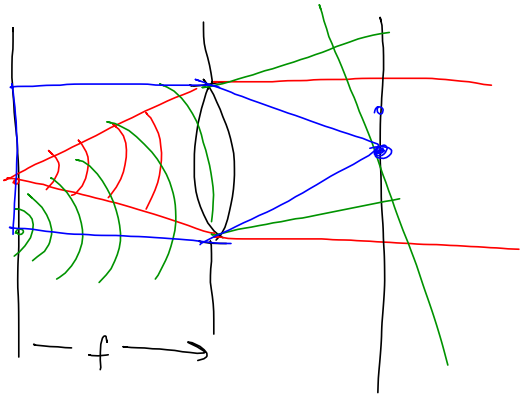
DFT → IMPULSES AT $k = \pm\ell$



$$w(x) = \frac{1}{2} \cdot 1(x) + \frac{1}{2}\cos\left(2\pi x \cdot \frac{1}{N \cdot \Delta x}\right)$$

$$W(\xi) = \frac{1}{2}\delta(\xi) + \frac{1}{4}\delta\left(\xi + \frac{1}{N \cdot \Delta x}\right) + \frac{1}{4}\delta\left(\xi - \frac{1}{N \cdot \Delta x}\right)$$

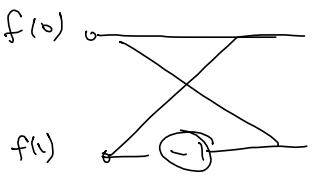
November 25, 2014



DFT — FFT EFFICIENT

N $\quad N = 2^m \quad m=1 \Rightarrow N=2$

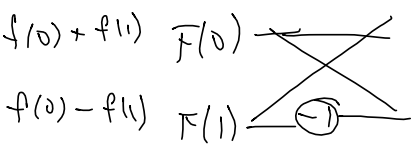
ANALYSIS



$F(0) = f(0) + f(1)$

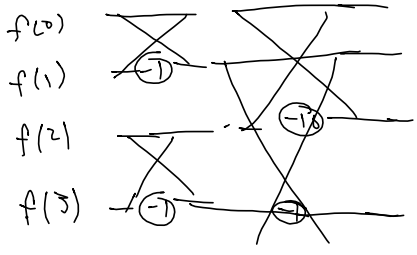
$F(1) = f(0) - f(1)$

SYNTHESIS



$\frac{F(0)+F(1)}{2} = f(0)$

$\frac{F(0)-F(1)}{2} = f(1)$



$(\log_2 N)$ BANKS OF $\frac{N}{2}$ BUTTERFLIES

$N \log_2 N$ OPERATIONS

$$f(k) = \sum f(n)\, e^{-i2\pi \frac{n \cdot k}{N}} \quad ; \quad N^2 \text{ OPERATIONS}$$