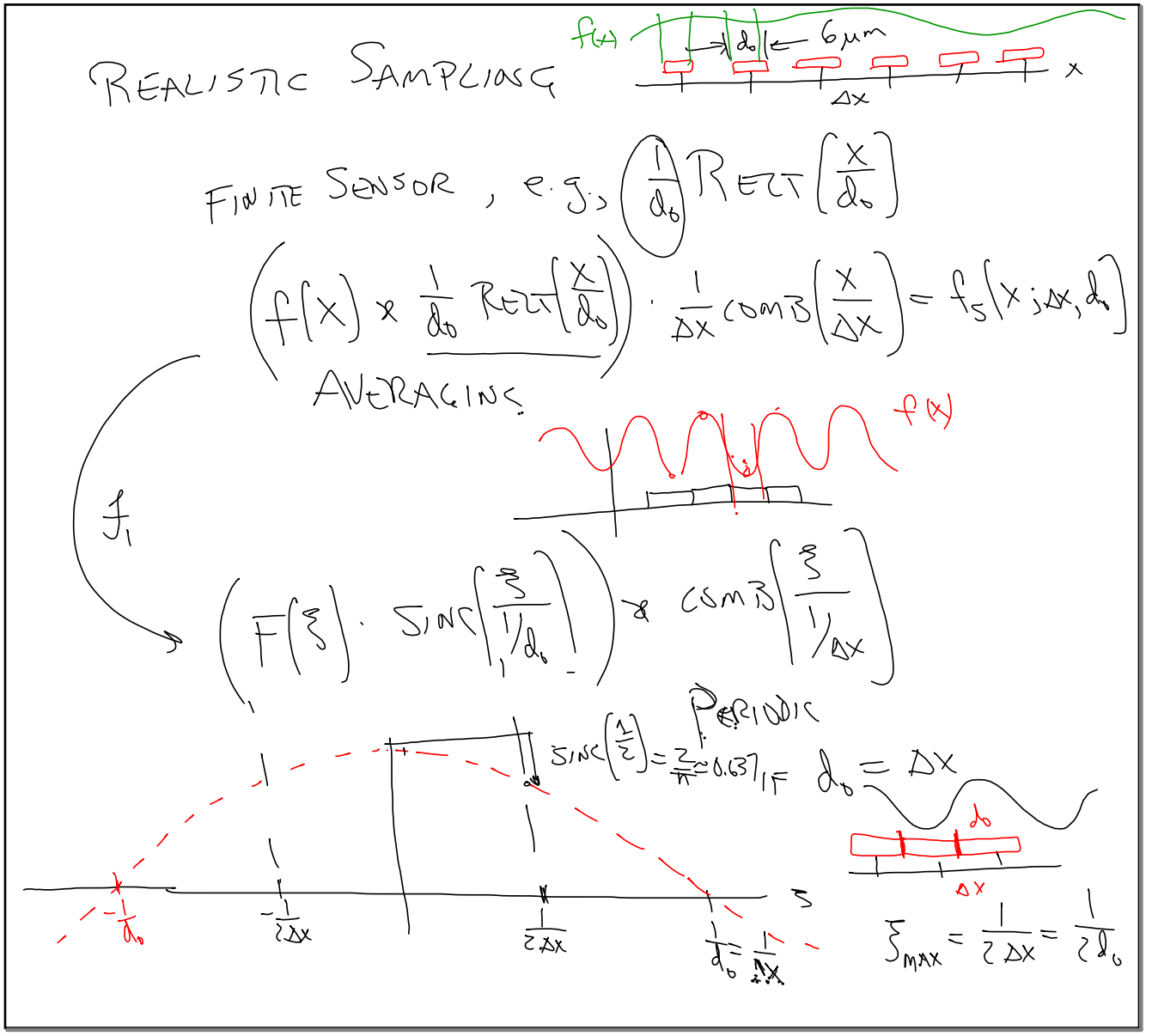
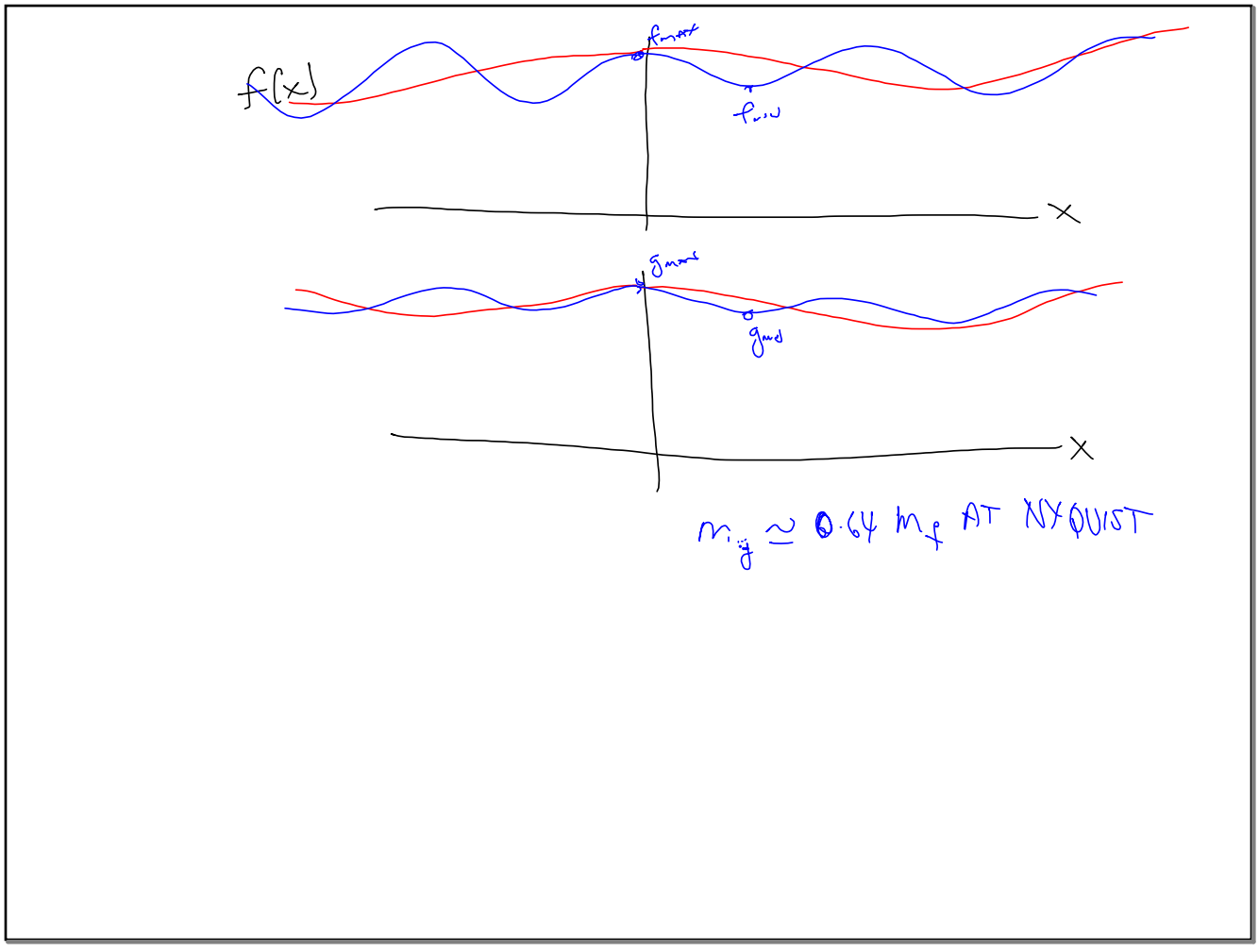
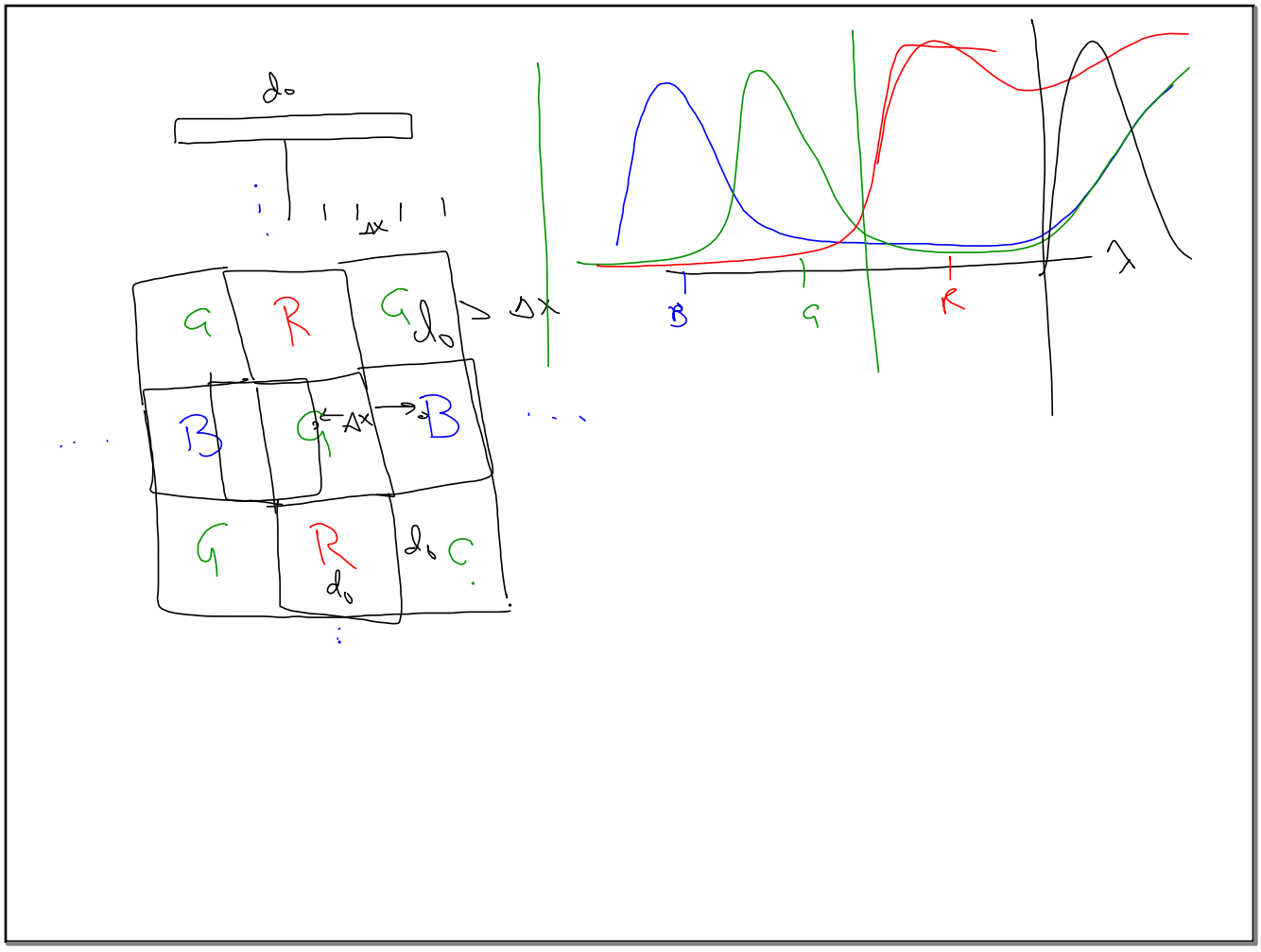
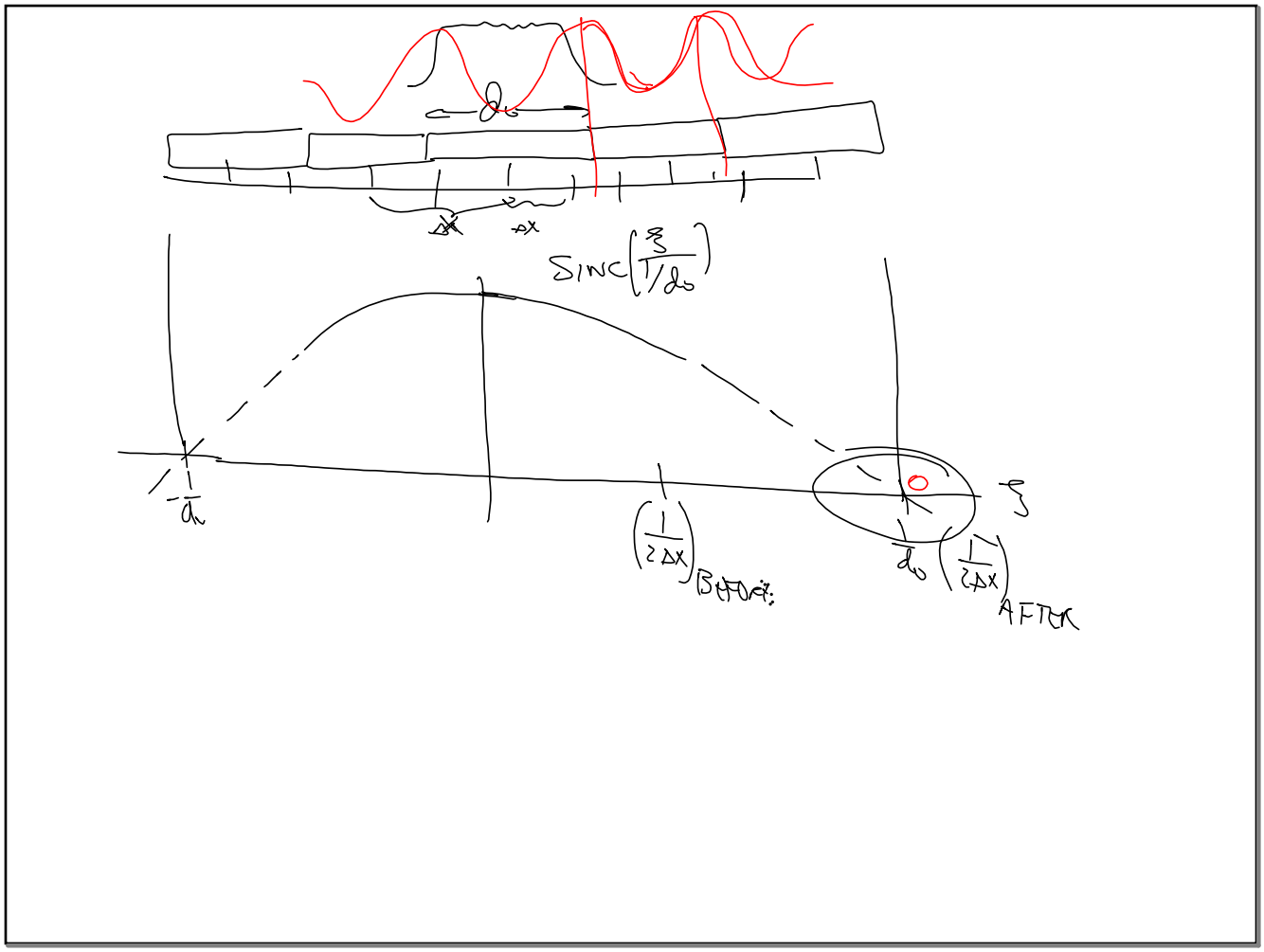
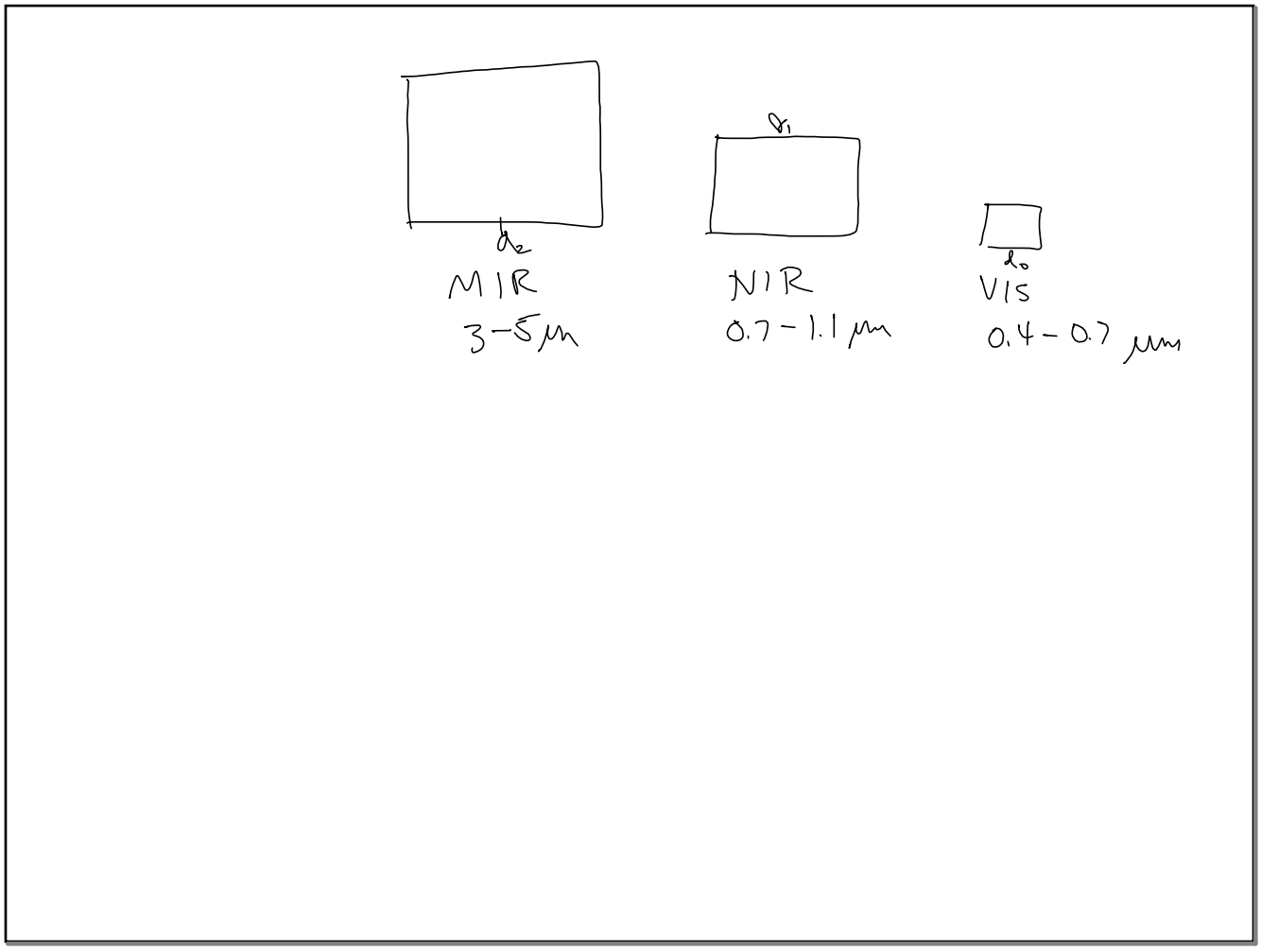

REALISTIC SAMPLING
f(x)
d_0
6μm
Δx
x
FINITE SENSOR, e.g.: $\frac{1}{d_0}\,\mathrm{RECT}\left(\frac{x}{d_0}\right)$
$\left(f(x) \times \frac{1}{d_0}\,\mathrm{RECT}\left(\frac{x}{d_0}\right)\right) \cdot \frac{1}{\Delta x}\,\mathrm{COMB}\left(\frac{x}{\Delta x}\right) = f_S(x;\Delta x, d_0)$
AVERAGING
f_1
$\left(F(\xi) \cdot \mathrm{SINC}\left(\frac{\xi}{1/d_0}\right)\right) * \mathrm{COMB}\left(\frac{\xi}{1/\Delta x}\right)$
PERIODIC
$\mathrm{SINC}\left(\frac{1}{2}\right) = \frac{2}{\pi} \approx 0.637$
IF $d_0 = \Delta x$
$\xi_{MAX} = \frac{1}{2\Delta x} = \frac{1}{2d_0}$



$m_{ij} \simeq 0.64\, m_f$ AT NYQUIST

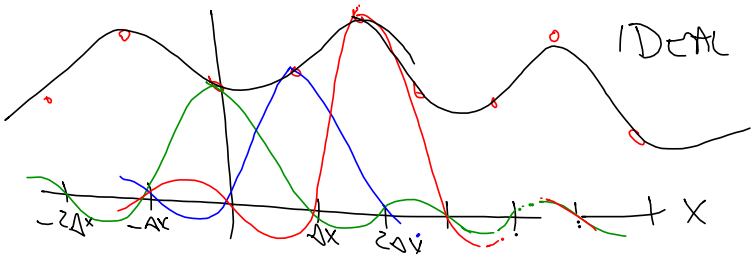

d_0
Δx
G
R
G
d_0
Δx
B
G
Δx
B
G
R
d_0
d_0
B
G
R
λ

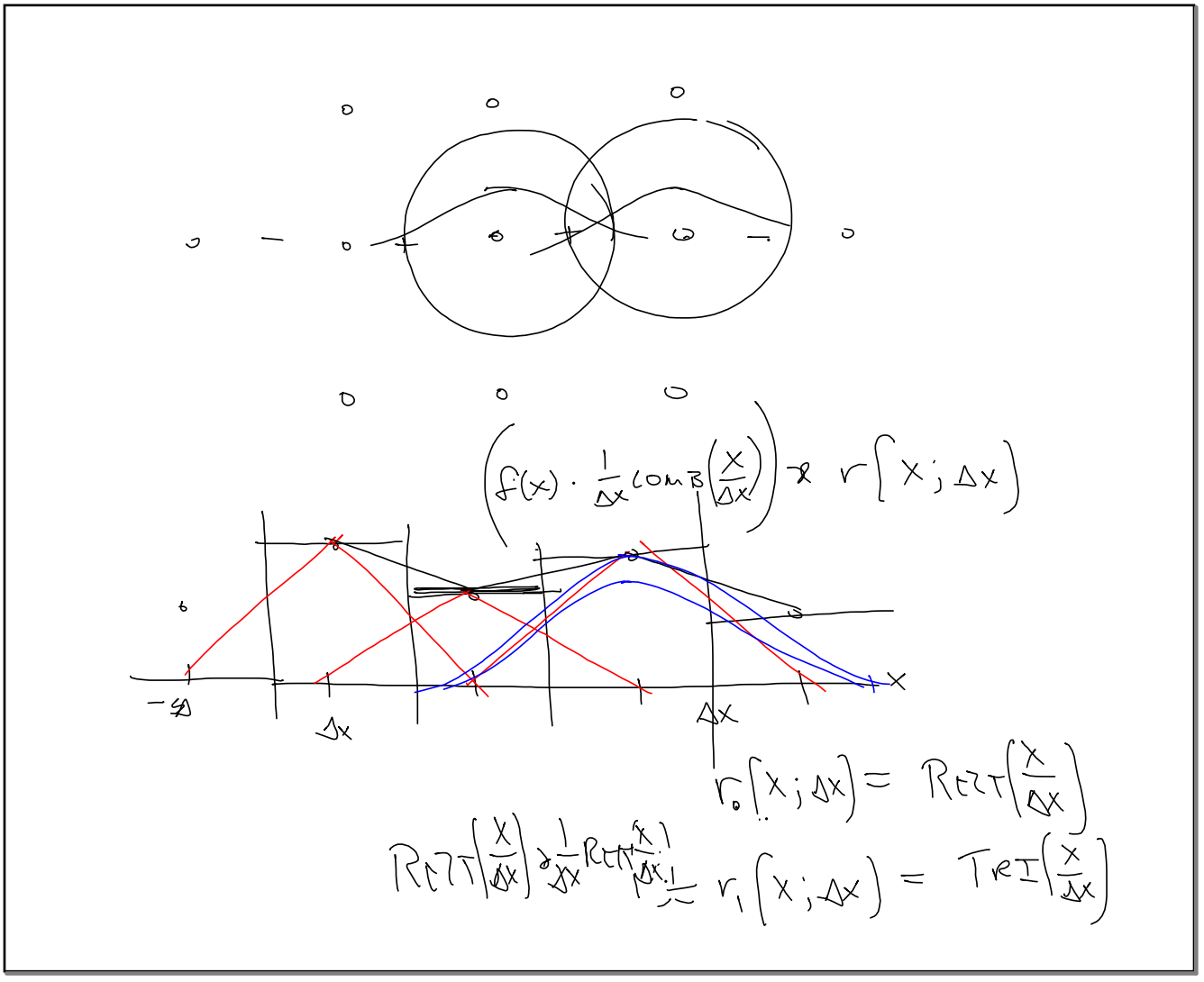

d_0
Δx
Δx
$\mathrm{SINC}\left(\frac{\xi}{1/d_0}\right)$
$-\frac{1}{d_0}$
$\left(\frac{1}{2\Delta x}\right)$
BEFORE
d_0
$\left(\frac{1}{2\Delta x}\right)$
AFTER
ξ

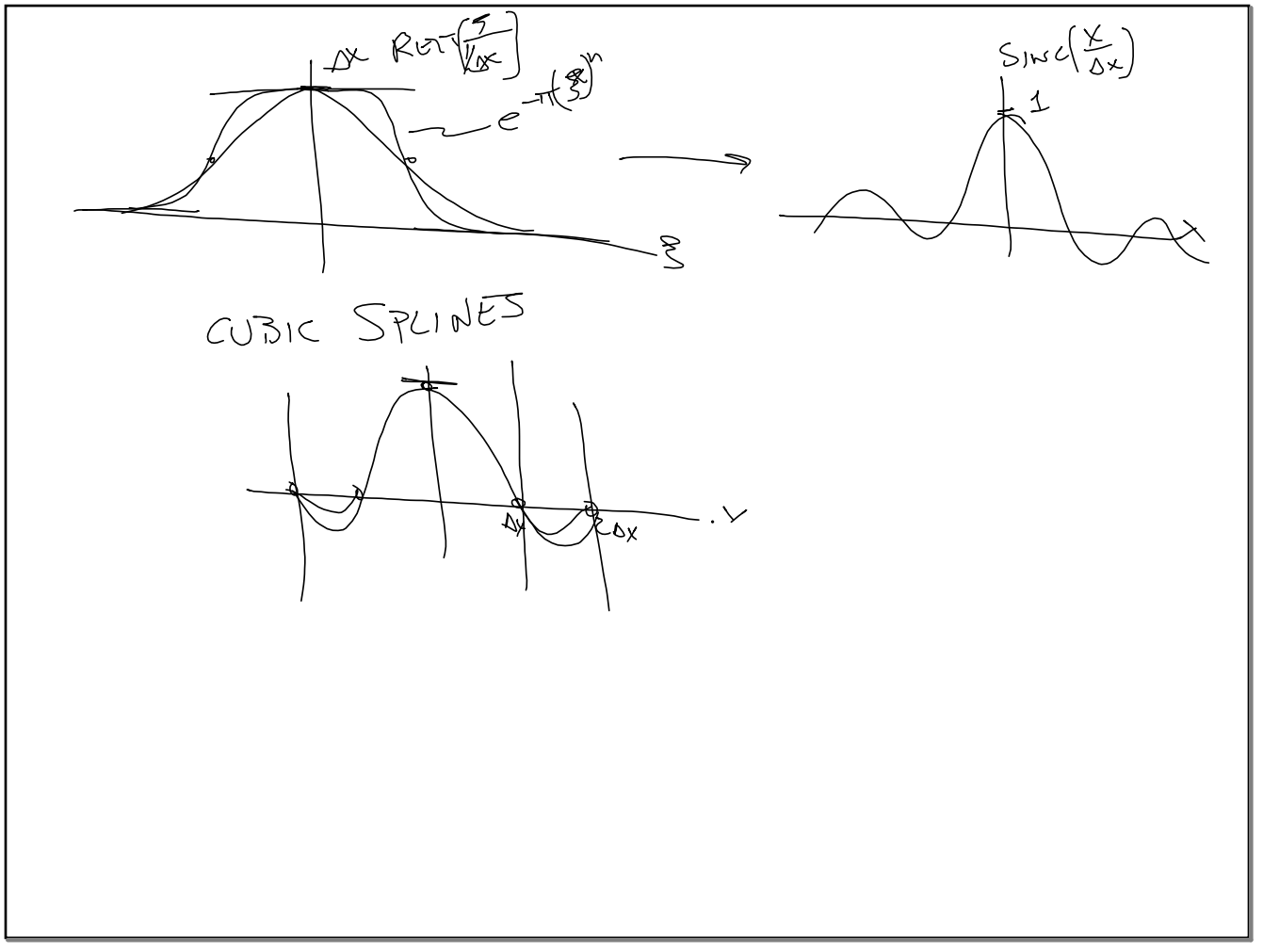

MIR
3-5μm
NIR
0.7-1.1μm
VIS
0.4-0.7 μm

REALISTIC INTERPOLATION

$$\left(f(x) \cdot \frac{1}{\Delta x}\,\mathrm{comb}\left(\frac{x}{\Delta x}\right)\right) * \underbrace{\mathrm{sinc}\left(\frac{x}{\Delta x}\right)}_{\text{IDEAL INTERPOLATION}} = \hat{f}(x)$$




$\left(f(x) \cdot \frac{1}{\Delta x} \mathrm{comb}\left(\frac{x}{\Delta x}\right)\right) * r(x;\Delta x)$
$-\Delta$
Δx
Δx
x
$r_0(x;\Delta x) = \mathrm{Rect}\left(\frac{x}{\Delta x}\right)$
$\mathrm{Rect}\left(\frac{x}{\Delta x}\right) * \frac{1}{\Delta x}\mathrm{Rect}\left(\frac{x}{\Delta x}\right) = r_1(x;\Delta x) = \mathrm{Tri}\left(\frac{x}{\Delta x}\right)$


Δx RECT(x/Δx)
e^{-π(x/Δx)^n}
x
SINC(x/Δx)
1
x
CUBIC SPLINES
Δx
2Δx
x

DISCRETE FOURIER TRANSFORM DFT

$$\mathcal{F}\left\{ f_s(x;\Delta x)\right\} = \mathcal{F}\left(f(x)\cdot \frac{1}{\Delta x}\,\mathrm{comb}\left(\frac{x}{\Delta x}\right)\right)$$

$$= \mathcal{F}\left\{ \sum_n f(n\cdot\Delta x)\,\delta(x-n\cdot\Delta x)\right\}$$

$$F(\xi) * \mathrm{comb}\left((\Delta x)\xi\right)$$

$$\int_{-\infty}^{+\infty}\left(\sum_n f(n\cdot\Delta x)\,\delta(x-n\cdot\Delta x)\right) e^{-i2\pi\xi x}\,dx$$

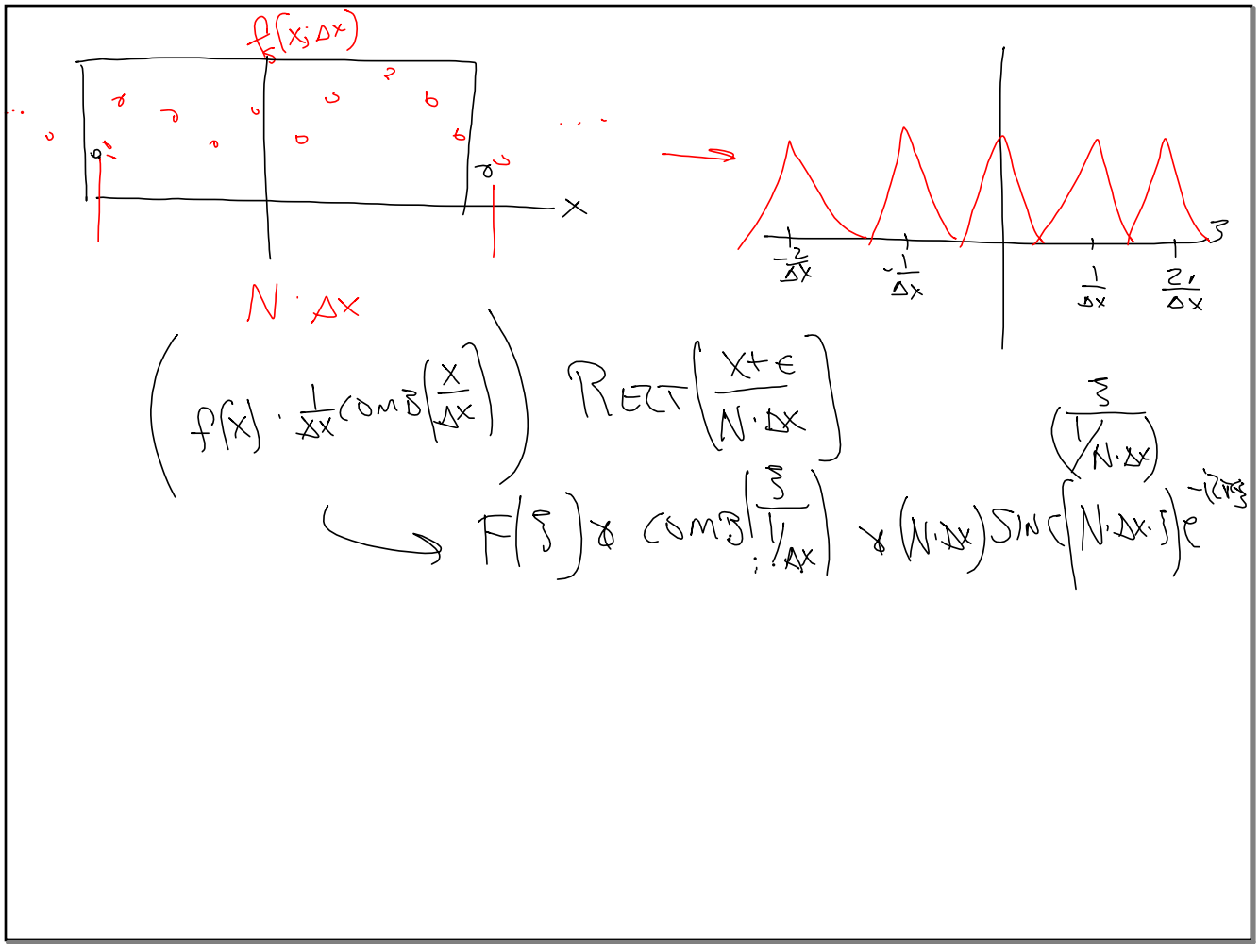
$$= \sum_n f(n\cdot\Delta x)\int_{-\infty}^{+\infty}\delta(x-n\cdot\Delta x)\,e^{-i2\pi x\xi}\,dx$$

$$\mathcal{F}\left\{f_s(x;\Delta x)\right\} = \sum_n f(n\cdot\Delta x)\,e^{-i2\pi\xi\cdot n\cdot\Delta x} = F_s(\xi;\Delta x)$$

$\xi := \frac{1}{\Delta x} \Rightarrow e^{-i2\pi n}$

CONTINUOUS VARIABLE

$-\infty < \xi < \infty$


f(x; Δx)
X
N · Δx
$\left(f(x) \cdot \frac{1}{\Delta x} \text{comb}\left(\frac{x}{\Delta x}\right)\right) \text{RECT}\left(\frac{x+\epsilon}{N \cdot \Delta x}\right)$
$F(\xi) * \text{comb}\left(\frac{\xi}{1/\Delta x}\right) * (N \cdot \Delta x)\,\text{SINC}\left(N \cdot \Delta x \cdot \xi\right) e^{-i2\pi\epsilon\xi}$