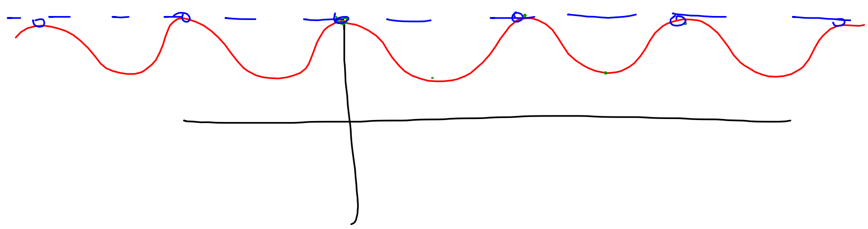
DISCRETE FUNCTIONS $\longrightarrow$ DFT

$$f(x) = e^{+i\pi\left(\frac{x}{b_0}\right)^2} \longrightarrow \xi(x) = \frac{1}{2\pi}\frac{d\Phi}{dx} = \frac{x}{b_0^2}$$



SAMPLING $\longrightarrow$ INTERPOLATION (LSI)

LSV QUANTIZATION (NL, SI)

$f(x) \geq 0$

IDEAL SAMPLING $\longrightarrow$ "GRAB" AMPLITUDES AT DISCRETE LOCATIONS

$$f(x) \cdot \underbrace{\frac{1}{\Delta x}\,\mathrm{comb}\left(\frac{x}{\Delta x}\right)}_{\text{IDEAL SAMPLING}} = f_s(x;\Delta x)$$

$$= f(x) \sum_{n=-\infty}^{+\infty} \frac{1}{\Delta x}\,\delta\left(\frac{x}{\Delta x} - n\right)$$

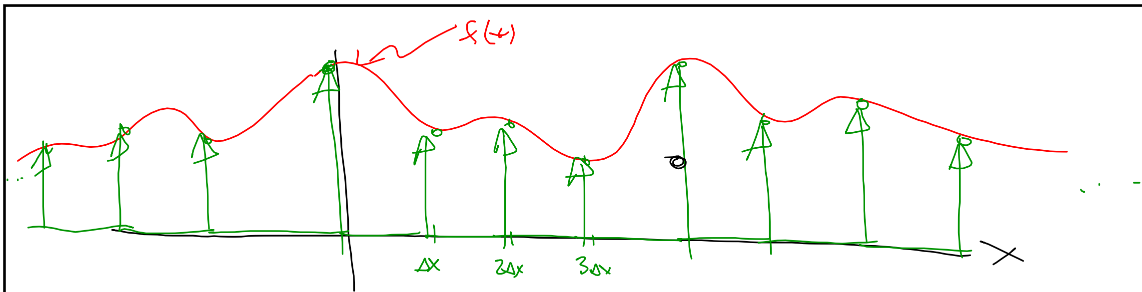
$$= \frac{1}{\Delta x} f(x) \sum_n \delta\left[\frac{1}{\Delta x}\left(x - n\cdot\Delta x\right)\right]$$

$$= \sum_n f(x)\cdot\delta(x - n\cdot\Delta x)$$

$$= \sum_n \underbrace{f(n\cdot\Delta x)}\,\delta(x - n\cdot\Delta x)$$

$f(n\cdot\Delta x) \longrightarrow$ "AMPLITUDES" OF SAMPLES

"AREAS" OF $\delta(x - n\cdot\Delta x)$ FOR EACH n



$$\underbrace{\left(f(x)\cdot\frac{1}{\Delta x}\mathrm{comb}\left(\frac{x}{\Delta x}\right)\right)}_{f_s(x;\Delta x)}\ \mathrm{RECT}\left(\frac{x+\epsilon}{N\cdot\Delta x}\right)\longrightarrow \text{"}N\text{" SAMPLES OF } f(x)$$

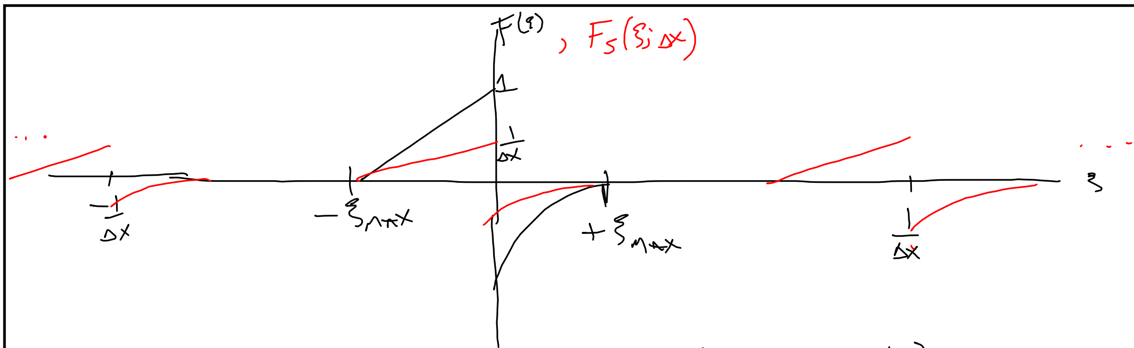
$$f_s[x;\Delta x] = f[x] \cdot \frac{1}{\Delta x}\,\mathrm{COMB}\left[\frac{x}{\Delta x}\right]$$

$$F_s[\xi;\Delta x] = F[\xi] * \mathrm{COMB}[\Delta x \cdot \xi]$$
$$= F[\xi] * \sum_{k=-\infty}^{+\infty} \delta[\Delta x \cdot \xi - k]$$
$$= F[\xi] * \sum_{k} \delta\left[\Delta x\left(\xi - \frac{k}{\Delta x}\right)\right]$$
$$= \frac{1}{\Delta x}\sum_{k} F[\xi] * \delta\left(\xi - \frac{k}{\Delta x}\right)$$

USE $\left(f[x] * \delta(x-x_0) = f(x-x_0)\right)$

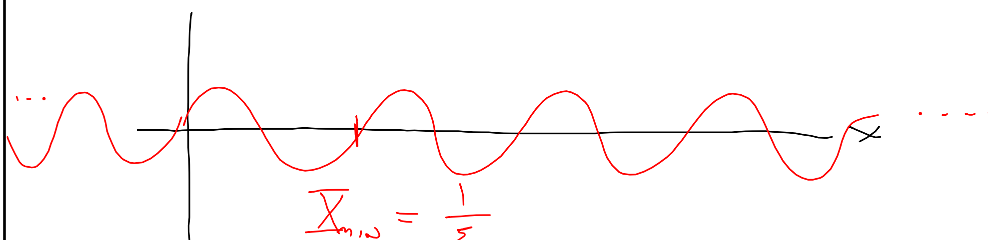
$$F_s[\xi;\Delta x] = \frac{1}{\Delta x}\sum_{k} F\left[\xi - \frac{k}{\Delta x}\right]$$

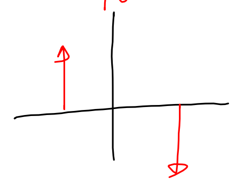
REPLICAS ~~OF~~ OF $F(\xi)$ CENTERED AT MULTIPLES OF $\frac{1}{\Delta x}$



$$F_S[\xi;\Delta x] = \frac{1}{\Delta x}\sum_k F\left(\xi - \frac{k}{\Delta x}\right)$$

AS DRAWN, $F(\xi)$ IS COMPACT





$$X_{min} = \frac{1}{\xi_{MAX}}$$

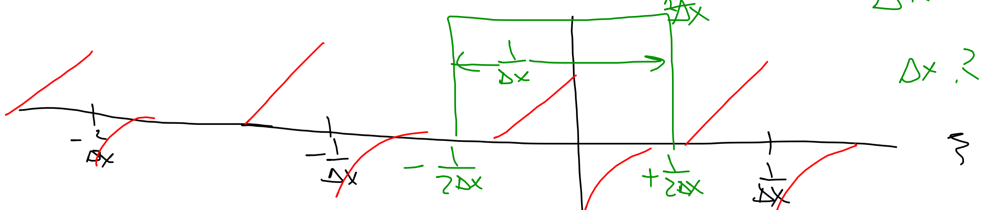
COMPACT SPECTRUM ⇒ INFINITE-SUPPORT $f(x)$

$GAUS(x) \longleftrightarrow GAUS(\xi)$

$$F_s(\xi;\Delta x) = \frac{1}{\Delta x}\sum_k F\left(\xi - \frac{k}{\Delta x}\right)$$

— $\Delta x \,\mathrm{RECT}\left(\frac{\xi}{1/\Delta x}\right)$

$\Delta x \,\mathrm{RECT}(\Delta x \cdot \xi)$

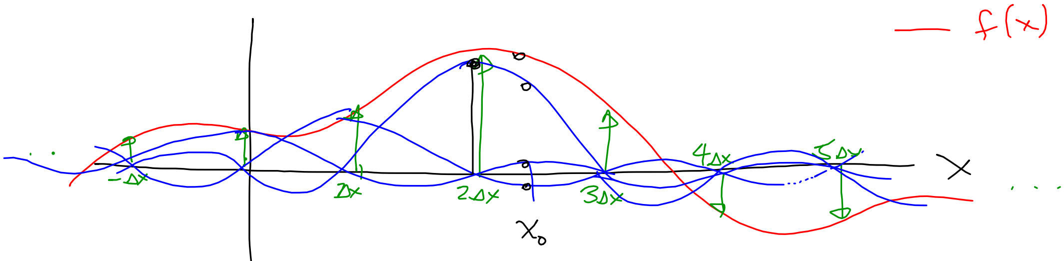


$$F_s(\xi;\Delta x) \quad \Delta x\,\mathrm{RECT}(\Delta x \cdot \xi)$$

$$\left(F(\xi) * \mathrm{COMB}(\Delta x \cdot \xi)\right) \Delta x\,\mathrm{RECT}(\Delta x \cdot \xi) = F(\xi)$$

$$\underbrace{\left(f(x) \cdot \frac{1}{\Delta x}\mathrm{COMB}\left(\frac{x}{\Delta x}\right)\right)}_{f_s(x;\Delta x)} * \mathrm{SINC}\left(\frac{x}{\Delta x}\right) = f(x)$$

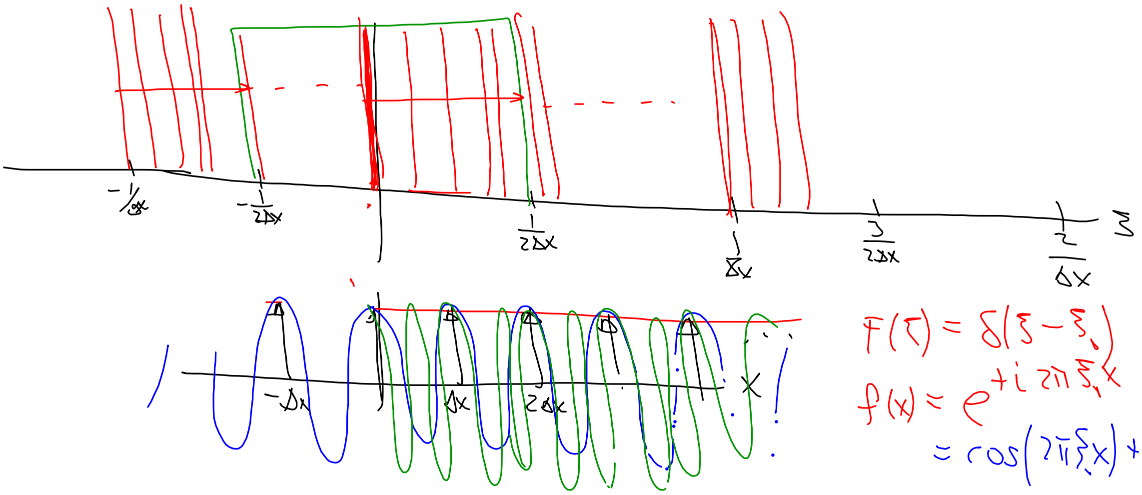
$$\underbrace{* \;\mathrm{SINC}\left(\frac{x}{\Delta x}\right)}_{\text{IDEAL INTERPOLATOR}} = f(x)$$



" SPLINES "

$$\hat{f}(x) = \left(f(x) \cdot \frac{1}{\Delta x} \mathrm{COMB}\left(\frac{x}{\Delta x}\right) \right) * \mathrm{SINC}\left(\frac{x}{\Delta x}\right)$$

ALIASING $\Rightarrow$ $F(\xi)$ IS NOT "SUFFICIENTLY" COMPACT



$F(\xi) = \delta(\xi - \xi_0)$

$f(x) = e^{+i 2\pi \xi_0 x}$

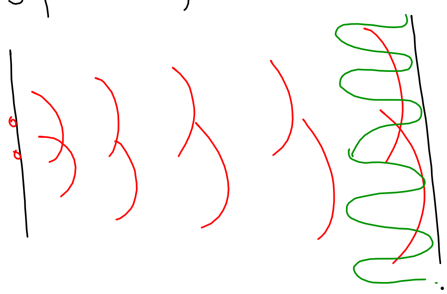
$= \cos(2\pi \xi_0 x) + i \sin(2\pi \xi_0 x)$

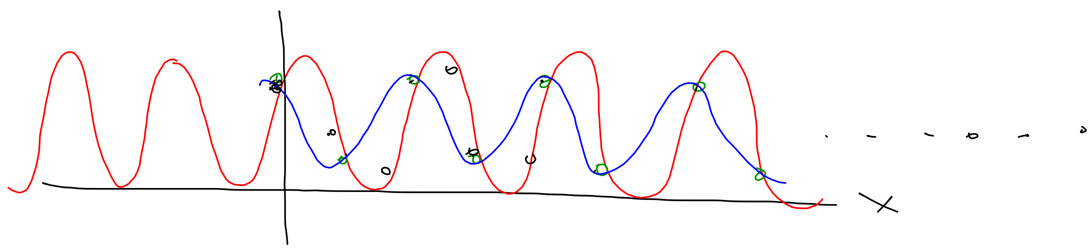
PREFILTER

$f(x)$

"INTERFERENCE"

$$\cos(2\pi \xi_0 x) \cdot \cos(2\pi \xi_1 x) = \frac{1}{2}\cos\big(2\pi \overset{\text{SUM}}{(\xi_0 + \xi_1)}\, x\big) + \frac{1}{2}\cos\big(2\pi \underset{\text{DIFF}}{(\xi_0 - \xi_1)}\, x\big)$$





$$\xi_{MAX} < \frac{1}{2\Delta x}$$

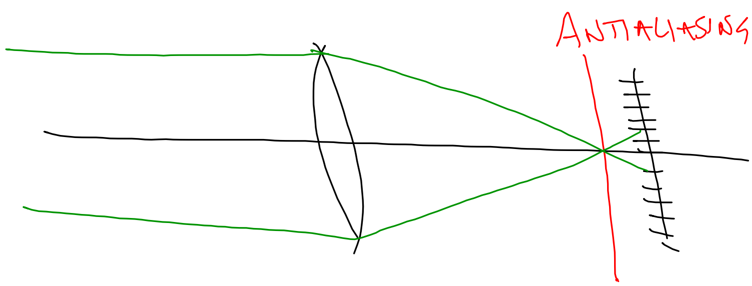
$$\Delta x < \frac{1}{2\xi_{MAX}}$$

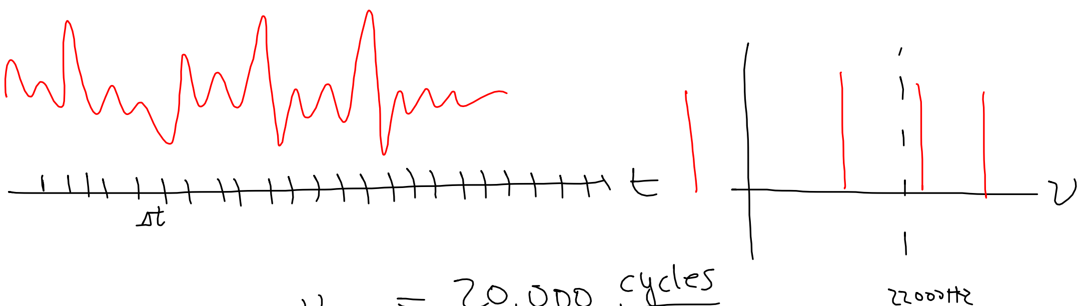
$f(x) \longrightarrow$ REMOVE FREQS THAT WOULD BE ALIASED

IDEAL LPF ;; $H(\xi) = \mathrm{RECT}\left(\frac{\xi}{1/\Delta x}\right)$

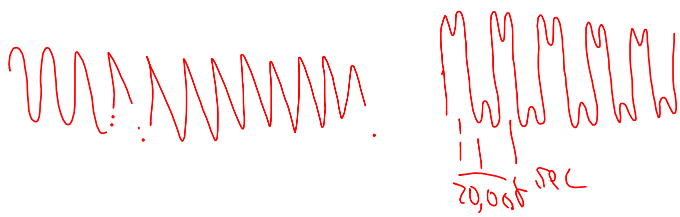
$$\left(\underbrace{\left(f(x) \ast \frac{1}{\Delta x}\,\mathrm{SINC}\left(\frac{x}{\Delta x}\right)\right) \cdot \frac{1}{\Delta x}\,\mathrm{COMB}\left(\frac{x}{\Delta x}\right)}_{f_s(x;\Delta x)} \right) \ast \mathrm{SINC}\left(\frac{x}{\Delta x}\right)$$

$$= f(x) \ast \frac{1}{\Delta x}\,\mathrm{SINC}\left(\frac{x}{\Delta x}\right)$$





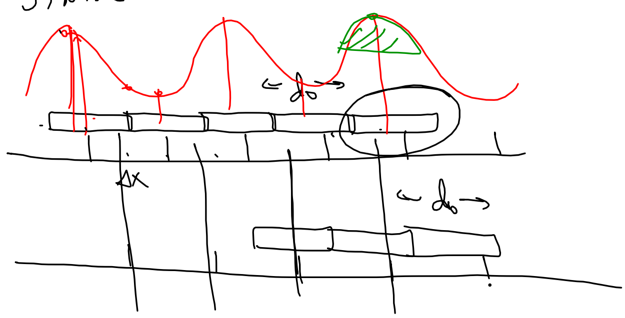
$$\nu_{max} = 20{,}000 \frac{\text{cycles}}{\text{second}}$$



Realistic Sampling

Sensor $\rightarrow h(x) = \frac{1}{d_0} \mathrm{Rect}\left(\frac{x}{d_0}\right)$

Sinar 54P



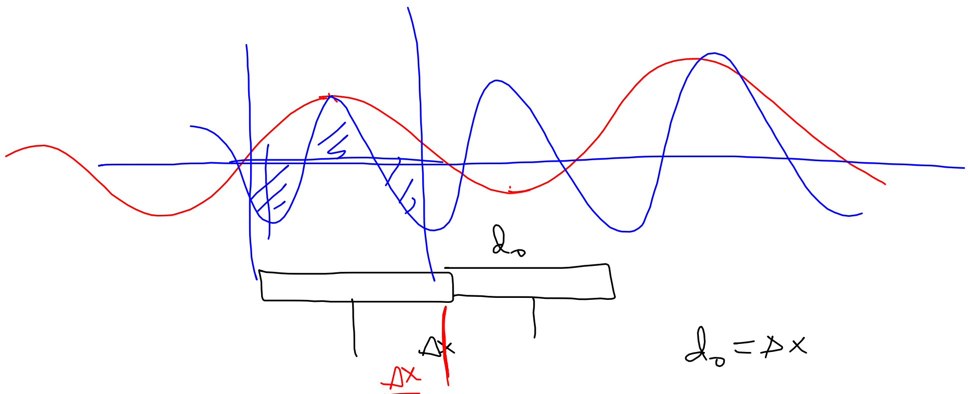
$\Delta x \longrightarrow \frac{\Delta x}{2}$

$\xi_{MAX} = \frac{1}{2\Delta x} \rightarrow \frac{1}{2 \cdot \frac{\Delta x}{2}} = \frac{1}{\Delta x}$

$d_0 \leq \Delta x$

$f(x) \longrightarrow$ AVERAGE OVER d_0

SAMPLE AT ΔX



$d_0 = \Delta x$

$d_0 \leq \Delta x$

$$\left(f(x) * \frac{1}{d_0}\,\mathrm{RECT}\left(\frac{x}{d_0}\right)\right) \cdot \frac{1}{\Delta x}\,\mathrm{COMB}\left(\frac{x}{\Delta x}\right) = f_s(x;\Delta x)$$

$$* \,\mathrm{SINC}\left(\frac{x}{\Delta x}\right)$$

$$* \frac{1}{\Delta x}\,\mathrm{SINC}\left(\frac{x}{\Delta x}\right) \; ?$$

$$\left(F(\xi) \cdot \mathrm{SINC}\left(\frac{\xi}{1/d_0}\right)\right) * \underbrace{\mathrm{COMB}\left(\frac{\xi}{1/\Delta x}\right)}_{\text{PERIODIC}}$$

MTF