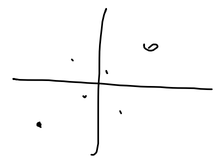
even: $f[-x,-y] = f[x,y]$

ODD: $f[-x,-y] = -f[x,y]$



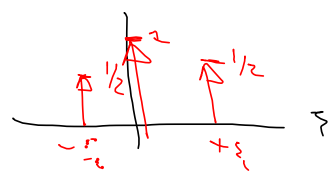
NONLINEAR OPERATIONS

$$f(x) \longrightarrow F(\xi)$$

$$\mathcal{N}\{f(x)\} = g(x) \longrightarrow G(\xi)$$ WIDER SUPPORT THAN $F(\xi)$

$$f(x) = \frac{1}{2}\left(1 + \cos(2\pi\xi_0 x)\right)$$

$$F(\xi) = \frac{1}{2}\left(\delta(\xi) + \frac{1}{2}\delta(\xi+\xi_0) + \frac{1}{2}\delta(\xi-\xi_0)\right)$$

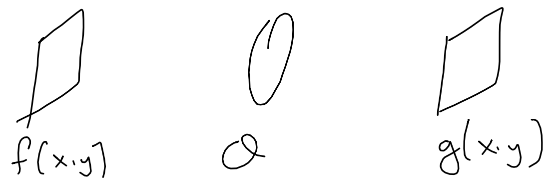


$$g(x) = \left(f(x)\right)^{\frac{11}{10}}$$

$$\left(f(x)\right)^{0.9}$$

2-D FUNCTIONS

$f(x,y)$



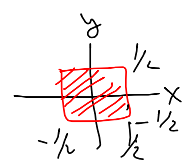
(1) CARTESIAN ~~SEPARABLE~~ SEPARABLE $\boxed{f(x,y) = f_1(x)\, f_2(y)}$

(2) POLAR SEPARABLE (CIRCULAR) $f(x,y) = f_R\left(\sqrt{x^2+y^2}\right) 1(\theta)$

(1) $F(\xi,\eta) = F_1(\xi) \cdot F_2(\eta)$

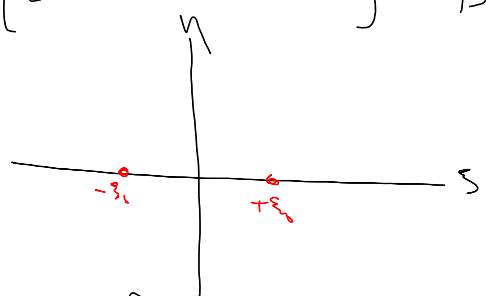
$\mathrm{RECT}(x)\,\mathrm{RECT}(y) \equiv \mathrm{RECT}(x,y)$

$\mathrm{SINC}(\xi)\,\mathrm{SINC}(\eta) = \mathrm{SINC}(\xi,\eta)$

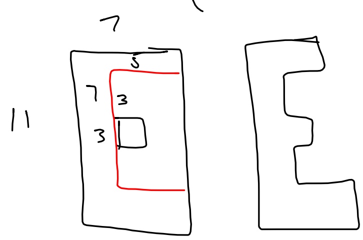


$$f(x,y) = \cos(2\pi\xi_0 x)\, 1(y) \longrightarrow \cos(2\pi\xi_0 x)$$

$$F(\xi,\eta) = \left[\tfrac{1}{2}\delta(\xi+\xi_0) + \tfrac{1}{2}\delta(\xi-\xi_0)\right]\delta(\eta)$$

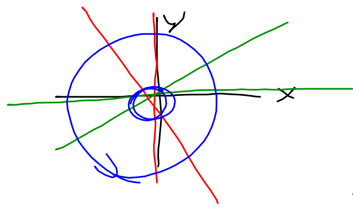


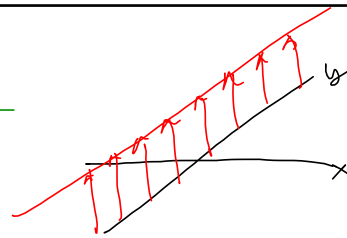
$$\cos(2\pi\xi_0 x)\,\delta(y) \longrightarrow \left[\tfrac{1}{2}\delta(\xi+\xi_0) + \tfrac{1}{2}\delta(\xi-\xi_0)\right] 1(\eta)$$



$$\mathrm{Rect}\left(\frac{x}{7},\frac{y}{11}\right) + \mathrm{Rect}\left(\frac{x}{3},\frac{y}{3}\right) - \mathrm{Rect}\left(\frac{x-1}{5},\frac{y}{7}\right)$$

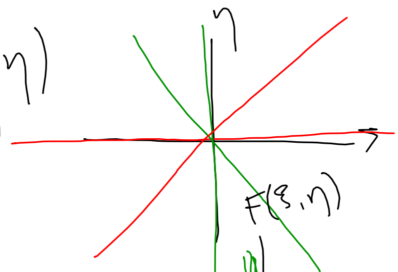
$\delta(x)\,1(y)$

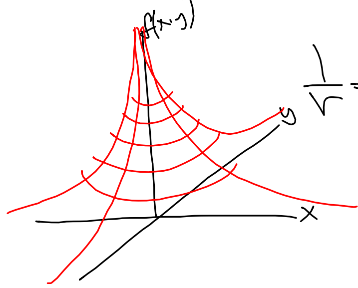




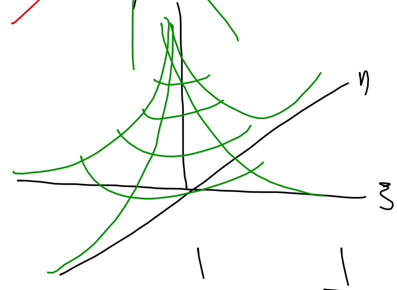
$\delta(x)\,1(y) \longrightarrow 1(\xi)\,\delta(\eta)$

$1(x)\,\delta(y) \longrightarrow \delta(\xi)\,1(\eta)$

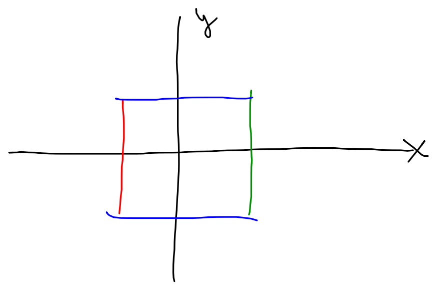




$\frac{1}{r} = \frac{1}{\sqrt{x^2+y^2}}$



$\frac{1}{\sqrt{\xi^2+\eta^2}} = \frac{1}{\rho}$



$$\delta\left(x+\tfrac{1}{2}\right)\mathrm{Rect}(y) + \delta\left(x-\tfrac{1}{2}\right)\mathrm{Rect}(y) + \mathrm{Rect}(x)\left[\delta\left(y+\tfrac{1}{2}\right)+\delta\left(y-\tfrac{1}{2}\right)\right]$$

$$\longrightarrow 2\cos\left(2\pi\xi\cdot\tfrac{1}{2}\right)\mathrm{sinc}(\eta) + \mathrm{sinc}(\xi)\cdot 2\cos\left(2\pi\eta\cdot\tfrac{1}{2}\right)$$

$$e^{-\pi x^2} e^{-\pi y^2} = e^{-\pi(x^2+y^2)} = e^{-\pi r^2}$$

$$\downarrow$$

$$e^{-\pi \xi^2} e^{-\pi \eta^2} = e^{-\pi(\xi^2+\eta^2)} = e^{-\pi \rho^2}$$

$$e^{-\pi\left(\frac{x}{d_0}\right)^2} \; e^{-\pi\left(\frac{y}{b_0}\right)^2}$$

$$e^{\pm i\pi\left(\frac{x}{b_0}\right)^2} \; e^{\pm i\pi\left(\frac{y}{d_0}\right)^2} \; ; \; b_0 = d_0 = 1 \Rightarrow e^{\pm i\pi r^2} \longrightarrow e^{\pm i\frac{\pi}{2}} e^{\mp i\pi\rho^2}$$

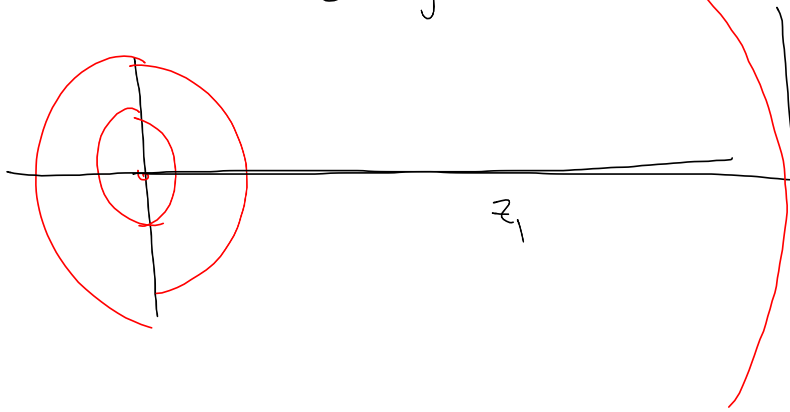
$$\pm i \; e^{\mp i\pi\rho^2}$$

$$|b_0| e^{\pm i\frac{\pi}{4}} e^{\mp i\pi(b_0\xi)^2} \; |d_0| e^{\pm i\frac{\pi}{4}} e^{\mp i\pi(d_0\eta)^2}$$

$$\pm i e^{\mp i\pi\rho^2} = \pm i\left(\cos \pi\rho^2 \mp i \sin \pi\rho^2\right)$$

$$= \pm i \cos \pi\rho^2 + \sin \pi\rho^2$$

$$= \sin \pi\rho^2 \pm i \cos \pi\rho^2$$



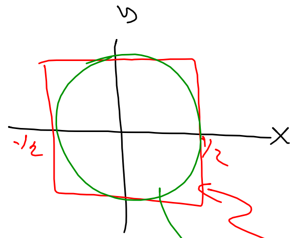
$$h(x,y) = h(r) \propto e^{+i\pi\left(\frac{r}{d_0}\right)^2}$$

$$d_0 = \sqrt{\lambda_0 z_1}$$

$$H(\xi,\eta) = H(\rho) \propto e^{-i\pi(d_0\rho)^2}$$

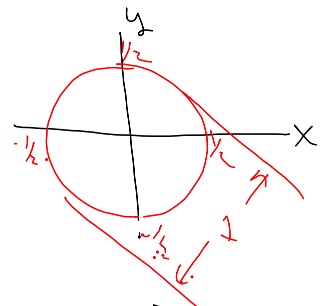
POLAR SEPARABLE → CIRCULARLY SYMMETRIC

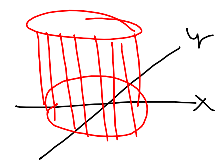
$$f(x,y) = f_r\left[\sqrt{x^2+y^2}\right] 1(\theta) = f_r(r)$$



$$\mathrm{CYL}(r) \equiv \begin{cases} 1 & r < 1/2 \\ 1/2 & r = 1/2 \\ 0 & r > 1/2 \end{cases}$$

$$\delta\left(r - \tfrac{1}{2}\right) 1(\theta)$$





$$\text{VOLUME} \quad \iint_{-\infty}^{+\infty} \delta\left(\sqrt{x^2+y^2} - \tfrac{1}{2}\right) dx\,dy$$

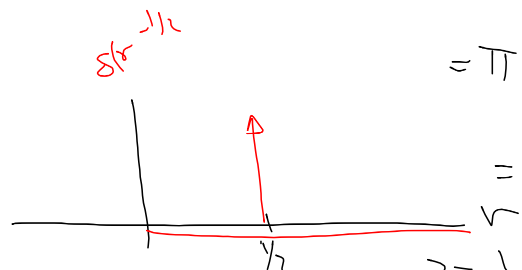
$$= \int_{r=0}^{\infty}\int_{\theta=-\pi}^{+\pi} \delta\left(r - \tfrac{1}{2}\right) r\,dr\,d\theta$$

$$= \int_{r=0}^{\infty} r\,\delta(r - 1/2) \underbrace{\left(\int_{-\pi}^{+\pi} d\theta\right)}_{2\pi} dr = 2\pi \int_0^{\infty} r\,\delta(r - 1/2)\,dr$$

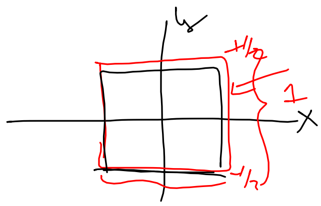
$$= 2\pi \int_0^{\infty} \tfrac{1}{2}\,\delta(r - 1/2)\,dr$$

$$= \pi \underbrace{\int_0^{\infty} \delta(r - 1/2)\,dr}_{1}$$

$$= \pi$$



$$\text{CIRCUMFERENCE} = 2\pi r = 2\pi \cdot \tfrac{1}{2} = \pi$$



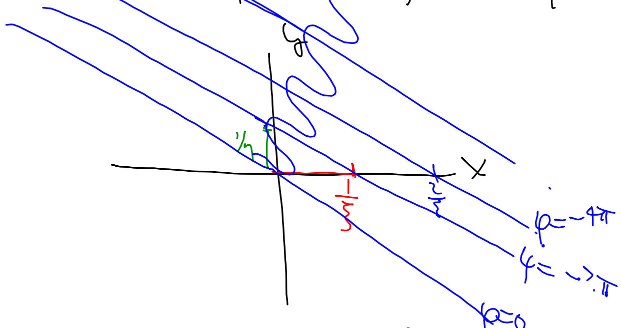
"CIRCUMFERENCE" = 4

$$\iint_{-\infty}^{+\infty} \mathrm{RECT}(x)\,\delta\left(y+\tfrac{1}{2}\right)\,dx\,dy = \underbrace{\int_{-\infty}^{+\infty} \mathrm{RECT}(x)\,dx}\ \underbrace{\int_{-\infty}^{+\infty} \delta(y+1/2)\,dy}_{1}$$

VOLUME = 4 = "CIRCUMFERENCE" · 1

$$\iint_{-\infty}^{+\infty} f(x,y)\, \underbrace{e^{-i2\pi(\xi x+\eta y)}} \, dx\, dy = \iint f_r(r)\, 1(\theta)\, e^{-i2\pi(\xi x+\eta y)}\, dx\, dy$$

$$\cos\left[2\pi(\xi x+\eta y)\right] - i\sin\left[2\pi(\xi x+\eta y)\right]$$



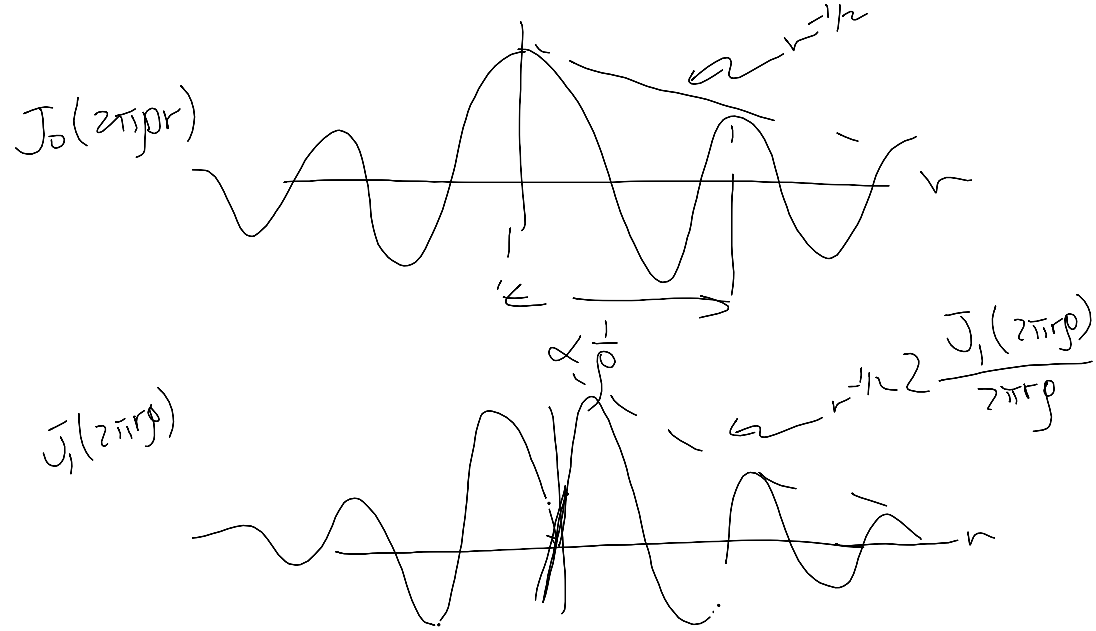
$$\xi x+\eta y = \begin{bmatrix} x \\ y \end{bmatrix} \circ \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

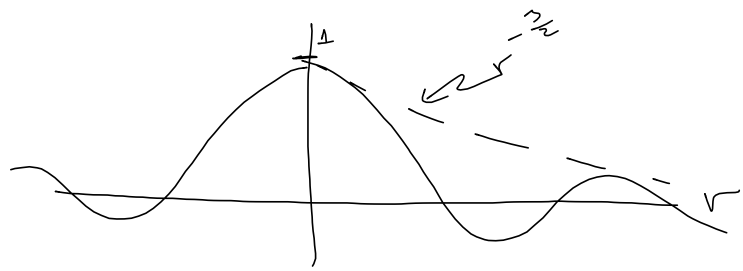
$$\underset{\sim}{r} \circ \underset{\sim}{\rho} = |\underset{\sim}{r}|\,|\underset{\sim}{\rho}| \cos\theta = r\rho\cos\psi$$

$$e^{-i2\pi(\xi x+\eta y)} = e^{-i2\pi r\rho\cos\theta} \qquad dr \cdot r\, d\theta$$

$$\mathcal{F}_2\{f_r(r)1(\theta)\} = \int_{r=0}^{\infty}\int_{\theta=-\pi}^{+\pi} f_r(r)\, 1(\theta)\, e^{-i2\pi r\rho\cos\theta}\, r\, dr\, d\theta$$

$$\int_0^{\infty} r\, f(r) \underbrace{\left(\int_{-\pi}^{+\pi} e^{-i2\pi r\rho\cos\theta}\, d\theta \right)}_{2\pi J_0(2\pi r\rho)} dr$$


$J_0(2\pi\rho r)$
$\sim r^{-1/2}$
r
$\propto \frac{1}{\rho}$
$J_1(2\pi r\rho)$
$\sim r^{-1/2}$
$2\frac{J_1(2\pi r\rho)}{2\pi r\rho}$
r


1
$r^{-3/2}$
r